Control Systems Laboratory
Department of Mechanical Engineering
National Technical University of Athens

On Attitude Dynamics and Control of Legged Robots Using Tail-Like Systems

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Overview

- Motivation
- Tails in legged animals and robots
- Modeling and dynamics
- Angular momentum conservation
- Holonomy and Integrality
- Body attitude control with tails and reaction wheels
- Simulation experiments varying key design parameters
- Motor selection
- Conclusions
- Future work

Motivation

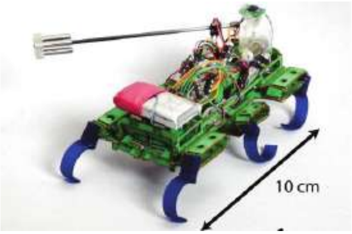
- Legged robots are highly underactuated hybrid systems.
- Their gaits include stance and aerial phases.
- Difficult tasks require precise control of the body attitude.
- Leg motions cause changes in system angular momentum.
- Control the body attitude by counteracting these changes.
- Dedicated appendages with large inertia can be used.
- We focus on attitude dynamics and control in aerial phases.

Learning from Biology

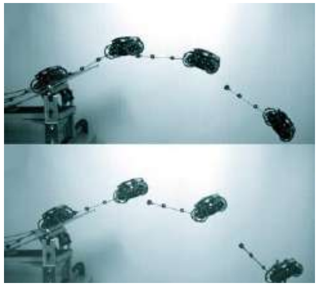
- Many legged animals have long tails which aid in balance and maneuverability at high speeds.
- Tail motion is effective for adjustments to unexpected perturbations when the legs are otherwise occupied.

Legged Robots with Tails

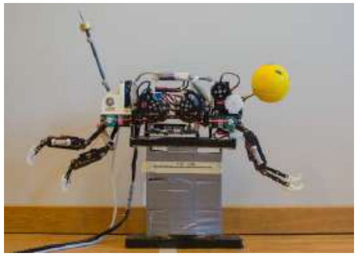
- A minority of the existing legged robots include appendages for angular momentum management, such as tails, or reaction wheels.



TAYLRoACH, Univ. of California, Berkeley



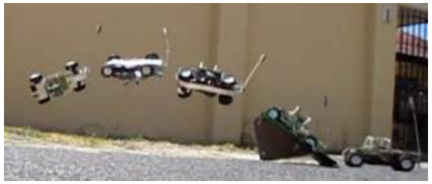
Tailbot, University of California, Berkeley



Cheetah-Cub EPFL

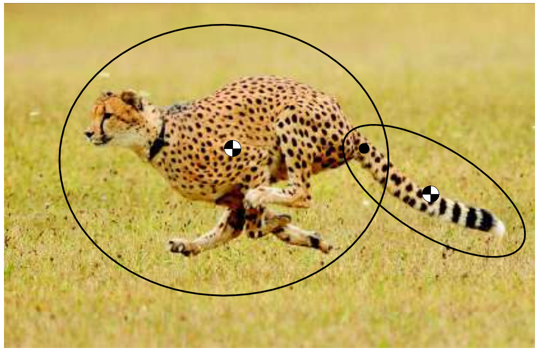
Penn Jerboa, Upenn

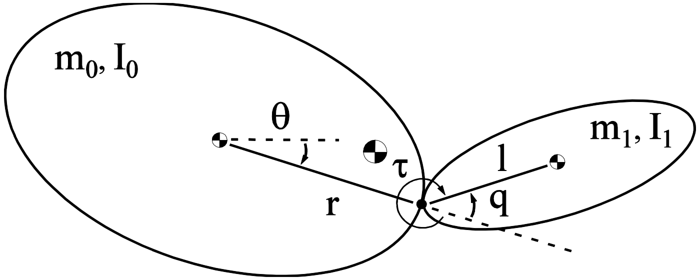
XRL, Upenn



FlipBot, University of Cape Town

Dynamic model in Aerial Phase





- We use the simplest dynamic model possible.
- Equations of Motion (reduced to the system CoM frame).
- System *effective mass*: $\mu = (m_1 m_2)/(m_1 + m_2)$

$$(I_0 + I_1 + \mu(l^2 + r^2 + 2rl\cos q))\ddot{\theta} + (I_1 + \mu(l^2 + rl\cos q))\ddot{q} - \mu rl\sin q\dot{q}^2 - 2\mu rl\sin q\dot{q}\dot{\theta} = 0$$

$$(I_1 + \mu l^2 + \mu rl\cos q)\ddot{\theta} + (I_1 + \mu l^2)\ddot{q} + \mu rl\sin q\dot{\theta}^2 = \tau$$

Angular Momentum (1)

- The body angle ϑ does not appear in the Lagrangian L.
- It is a *cyclic* or *ignorable* coordinate.
- The EoM can be written without ϑ.
- The tail angle q is a *palpable* or *positional* coordinate for the general case.
- The *generalized momentum* associated to ϑ is conserved.
- The conservation of angular momentum equation about the system CoM (h_0 is the initial angular momentum) is:

$\partial L / \partial$ [redacted]

Angular Momentum (2)

- An *integral of motion* exists.
- One of the EoM can be replaced with the first order conservation equation, giving a set of a second-order and a first-order differential equations.
- Can we integrate the conservation equation once more?
- This would yield an analytical expression between the body angle ϑ and the hinge angle q.

Conservation Equation: A Dynamic Constraint

- A constraint is *dynamic* if it is a consequence of the EoM, and not externally imposed.
- Angular momentum conservation is a dynamic constraint.
- A dynamic constraint can be:
  - integrable/ *holonomic* (geometric constraint), or
  - nonintegrable/ *nonholonomic* (constraint on velocities).
- The two-body system is often incorrectly considered nonholonomic for every case.
- We show that the system holonomy depends on the geometry and the initial angular momentum.

Integrability of the Constraint

- First, we write the conservation of angular momentum as an *acatastatic Pfaffian constraint:*

$$(I_0+I_1+\mu(l^2+r^2+2rl\,\mathrm{cq}))d\theta+(I_1+\mu(l^2+rl\,\mathrm{cq}))dq=h_0dt \quad \text{or} \quad P(q)d\theta+Q(q)dq+Rdt=0$$

- The necessary and sufficient condition for integrability is:

$$I=P(\frac{\partial Q}{\partial t}-\frac{\partial R}{\partial q})+Q(\frac{\partial R}{\partial \theta}-\frac{\partial P}{\partial t})+R(\frac{\partial P}{\partial q}-\frac{\partial Q}{\partial \theta})=0 \quad \text{or} \quad h_0 rl\sin q=0$$

- Nonintegrable only when $r,\, l\neq 0$ and $h_0\neq 0$ at the same time, or:

Integrability of the Constraint

- Nonintegrable only when a tail is hinged at distance *r* from the body CoM and init. ang. momentum is nonzero.

- In other words:
  - For zero initial angular momentum,
    - Always integrable, independent of the hinge position.

  - For nonzero initial angular momentum,
    - Integrable only if the tail is hinged at the body CoM or if the appendage is a reaction wheel.

Integrable Cases

- Zero initial ang. Momentum (same for tails and wheels):

$$\theta = \theta_0 - \frac{1}{2}(q - q_0) - \frac{A}{C}\tan^{-1}\left(\frac{B}{C}\tan\frac{q}{2}\right) + \frac{A}{C}\tan^{-1}\left(\frac{B}{C}\tan\frac{q_0}{2}\right)$$

$$A = I_1 + \mu l^2 - I_0 - \mu r^2, \; B = I_0 + I_1 + \mu(l - r)^2$$
$$C = \sqrt{(I_0 + I_1 + \mu l^2 + \mu r^2)^2 - (2\mu r l)^2}$$

- Nonzero initial angular momentum:
  - Tail hinged at the body CoM:

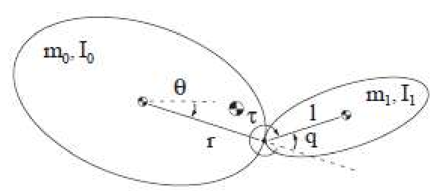
$$(I_0 + I_1 + \mu l^2)(\theta - \theta_0) + (I_1 + \mu l^2)(q - q_0) = h_0(t - t_0)$$

  - Reaction Wheel at distance *r* from the body CoM:

$$(I_0 + I_1 + \mu r^2)(\theta - \theta_0) + I_1(q - q_0) = h_0(t - t_0)$$

EoM for Different Geometries

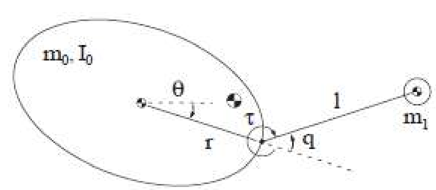
When both coordinates are ignorable the inertia matrix becomes independent of the shape variable q, the constraint is integrable, the EoM can be written decoupled, and analytical solutions can be derived.



General case: 2-body free floating system
θ: ignorable, q: palpable

$$(I_0+\mu r^2+I_1+\mu l^2+2\mu rl\cos q)\ddot{\theta}+(I_1+\mu l^2+\mu rl\cos q)\ddot{q} -\mu rl\sin q(\dot{q}^2+2\dot{q}\dot{\theta})=0$$

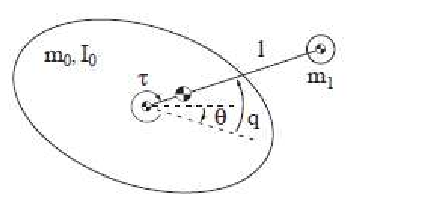
$$(I_1+\mu l^2+\mu rl\cos q)\ddot{\theta}+(I_1+\mu l^2)\ddot{q}+\mu rl\sin q\dot{\theta}^2=\tau$$



Tail hinged at distance r from body CoM (I_1=0)
θ: ignorable, q: palpable

$$(I_0+\mu r^2+\mu l^2+2\mu rl\cos q)\ddot{\theta}+(\mu l^2+\mu rl\cos q)\ddot{q} -\mu rl\sin q(\dot{q}^2+2\dot{q}\dot{\theta})=0$$

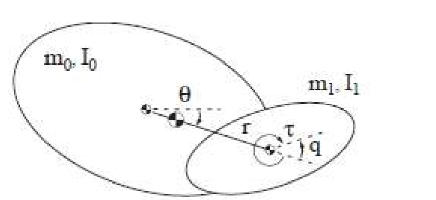
$$(\mu l^2+\mu rl\cos q)\ddot{\theta}+\mu l^2\ddot{q}+\mu rl\sin q\dot{\theta}^2=\tau$$



Tail hinged at body CoM (I_1=0, r=0)
θ: ignorable, q: ignorable

$$I_0\ddot{\theta}=-\tau$$

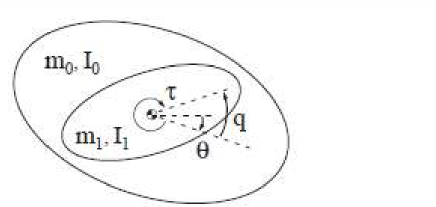
$$\frac{I_0\mu l^2}{I_0+\mu l^2}\ddot{q}=\tau$$



Reaction wheel hinged at distance r from body CoM (l=0)
θ: ignorable, q: ignorable

$$(I_0+\mu r^2)\ddot{\theta}=-\tau$$

$$\frac{I_1(I_0+\mu r^2)}{I_0+I_1+\mu r^2}\ddot{q}=\tau$$



Reaction wheel hinged at body CoM - "Elroy's Beanie", [12], (l=r=0)
θ: ignorable, q: ignorable

$$I_0\ddot{\theta}=-\tau$$

$$\frac{I_1I_0}{I_0+I_1}\ddot{q}=\tau$$

Design Principles – Tail Mass Selection

General Principles

- Tail mass selection determines the maximum body maneuver $\Delta\vartheta$ that can be performed for zero initial ang. momentum.
- The torque provided by the tail determines the maneuver duration – the higher the torque, the faster the maneuver.

Tail Mass Selection

- Full rotation about the hinge is forbidden → q bounded.
- Which tail mass permits a maneuver $\Delta\vartheta$ when zero initial angular momentum is considered?

$$(I_0 + \mu l^2)\Delta\theta + \mu l^2 \Delta q = 0 \;\Rightarrow\; m_1 = \frac{-\Delta\theta I_0 m_0}{(\Delta\theta + \Delta q) l^2 m_0 + \Delta\theta I_0}$$

Control of the Unactuated Body Angle ϑ

- Eliminating $\ddot{q}$ from the 2nd EoM, yields a single equation:

$$D^*(q)\ddot{\theta}+C^*(q,\dot{q},\dot{\theta})=\tau$$

- A model-based controller is used for ϑ, achieving $\ddot{\theta}=\ddot{\theta}_{des}$

$$\tau=D^*(q)(\ddot{\theta}_d+k_v\dot{e}_\theta+k_p e_\theta)+C^*(q,\dot{q},\dot{\theta})$$

- A quintic polynomial is used for trajectory planning:

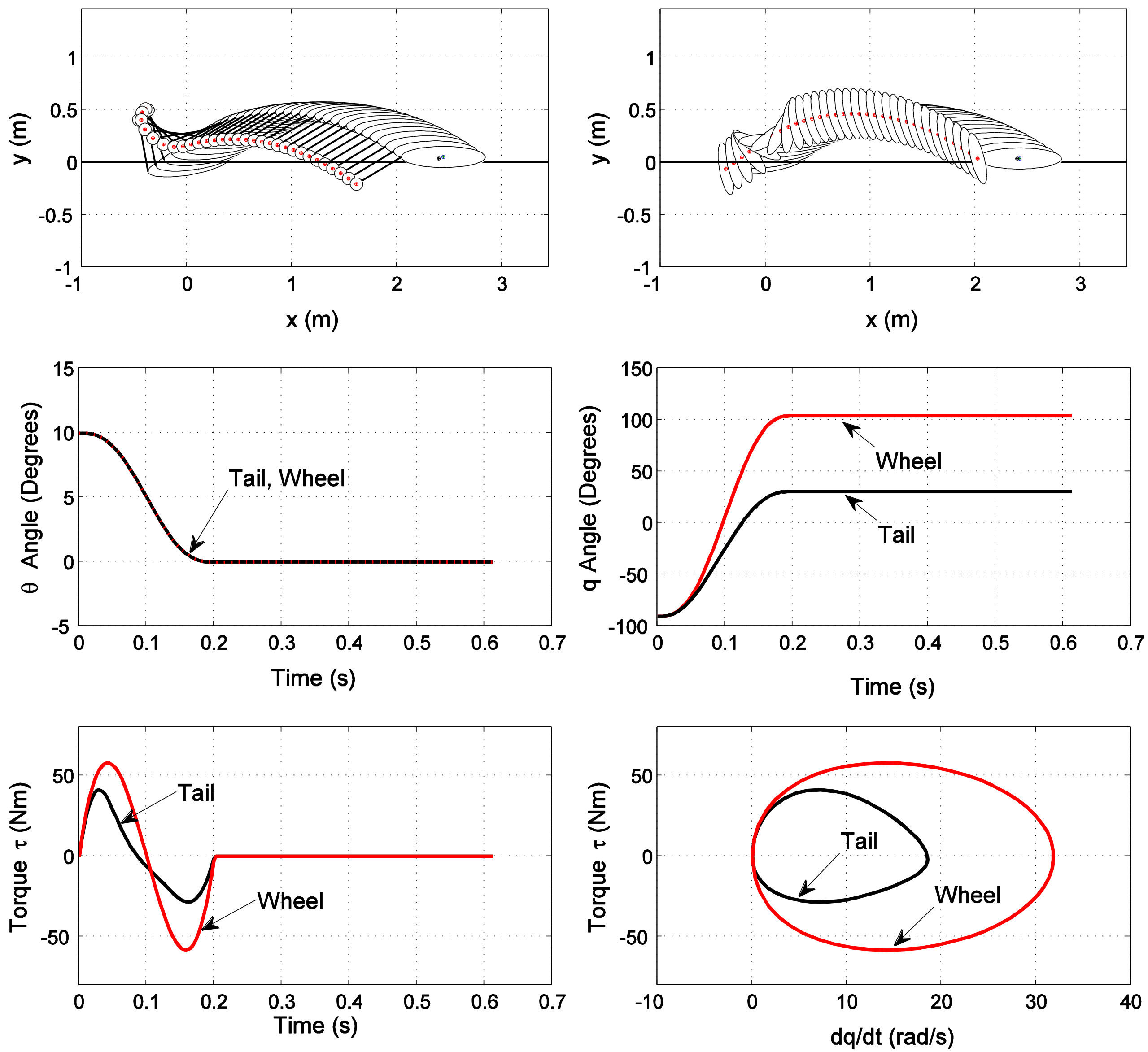
$$\theta_{des}(t)=a_0+a_1t+a_2t^2+a_3t^3+a_4t^4+a_5t^5$$

Tail – Reaction Wheel Comparison

- Tail and wheel with equal MoI about the hinge.
- **Body**: m_0=40kg, I_0=2kgm^2, performing a maneuver $\Delta\vartheta$=10^o in Δt=0.2s, with appendages hinged at distance r=0.4m from the body CoM.
- **Tail**: m_1=0.5kg, l=0.5m, I_1=0.
- **Wheel**: m_1=2kg, $I_1 = m\rho^2$, ρ=0.25m.

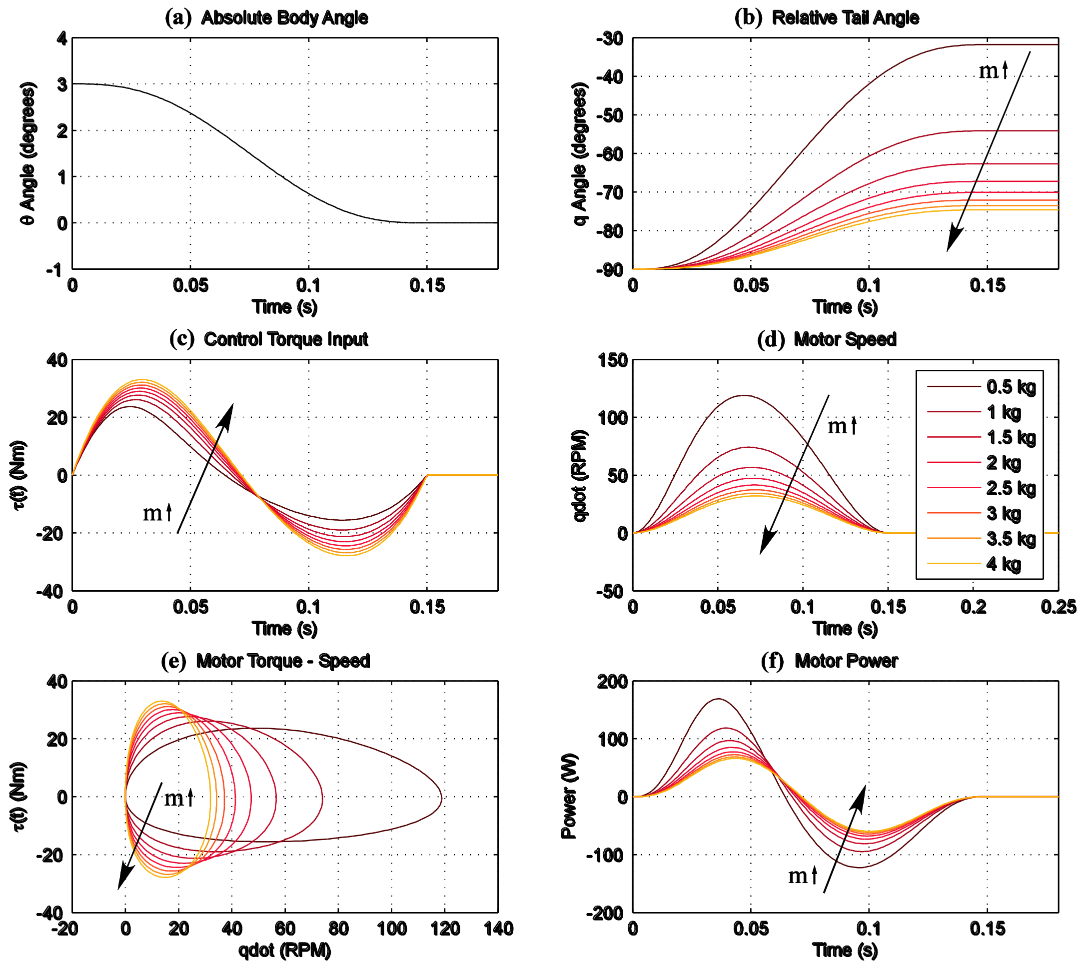
Conclusions

- Less torque required for the tail case.
- Much higher motor speed in the wheel case → for the same result, more power is requested from the motor.


y (m)
x (m)
θ Angle (Degrees)
Tail, Wheel
Time (s)
q Angle (Degrees)
Wheel
Tail
Torque τ (Nm)
dq/dt (rad/s)

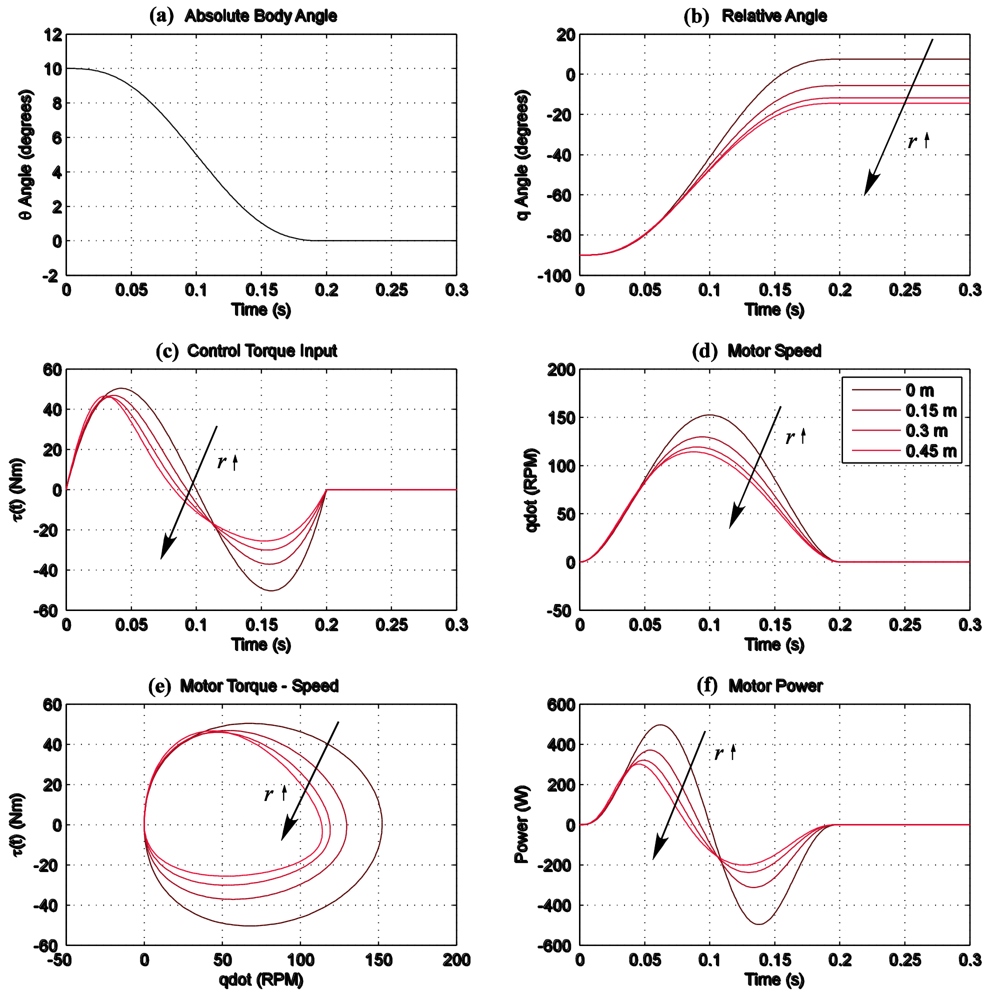
Experiments varying the tail mass

A body of m_0=30kg, I_0=2kgm^2, with a tail of length l=0.4m, MoI I_1=0, and mass m_1 varying from 0.5kg to 4kg, hinged at distance r=0.4m from the body CoM, performs a maneuver of $\Delta\vartheta$=3^o in Δt=0.15s.



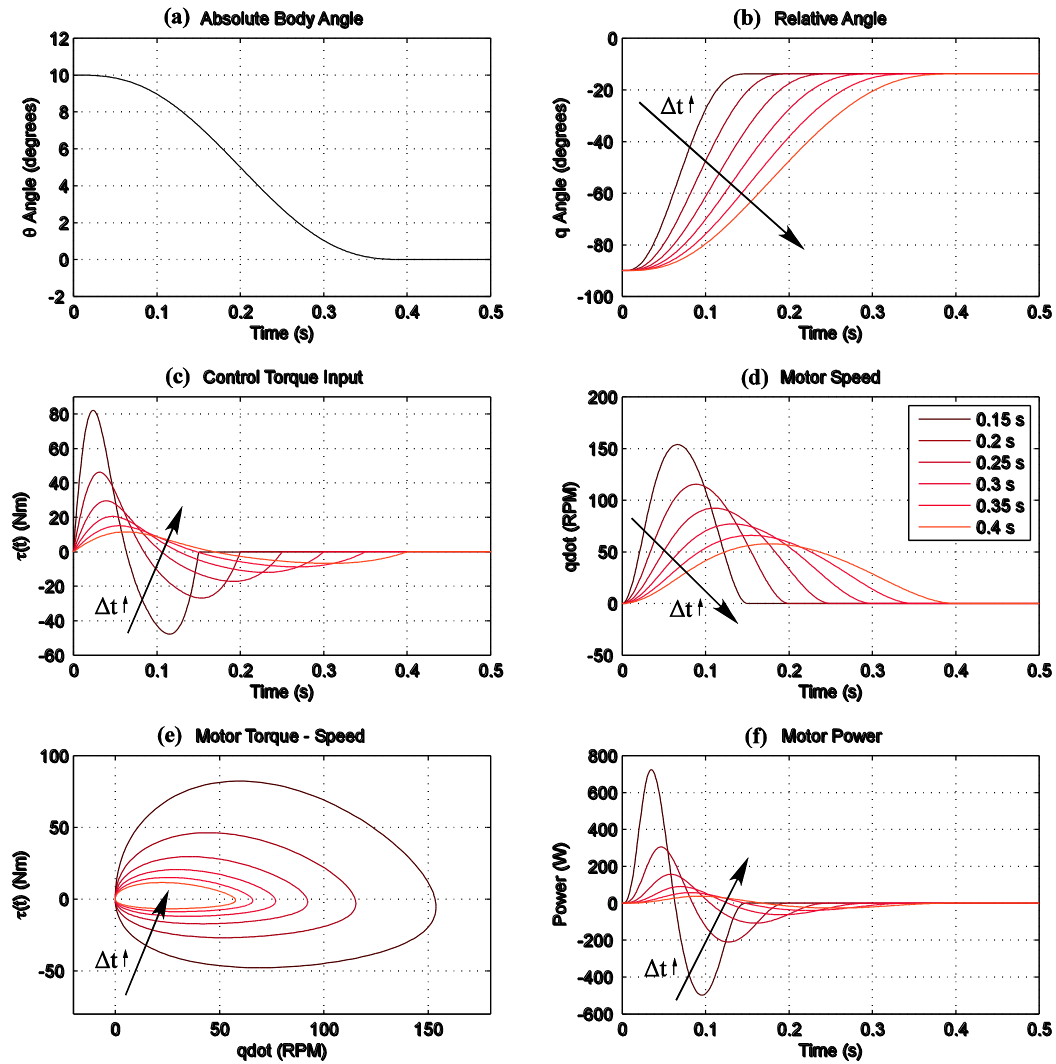
Experiments varying the Body CoM - Hinge Distance

A body of m_0=30kg, I_0=2kgm^2, with a tail of mass m_1=1.5kg, MoI I_1=0, and length l=0.4m, hinged at a distance r from body CoM, which varies from 0m to 0.45m, performs a maneuver $\Delta\vartheta$=10^o in Δt=0.2s.



Experiments varying the Time of the Maneuver

A body of m_0=30kg, I_0=2kgm^2, with a tail of mass m_1=1.5kg, MoI I_1=0, and length l=0.4m, hinged at a distance r =0.4m from the body CoM, performs a $\Delta\vartheta$=10^o maneuver in time intervals Δt varying from 0.15 to 0.4s.

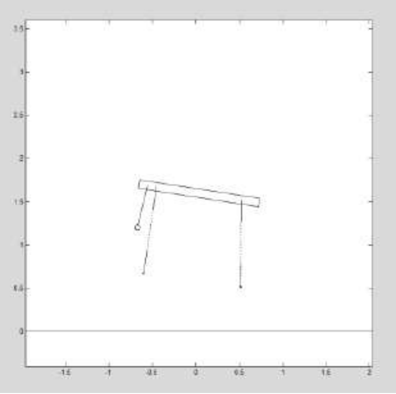


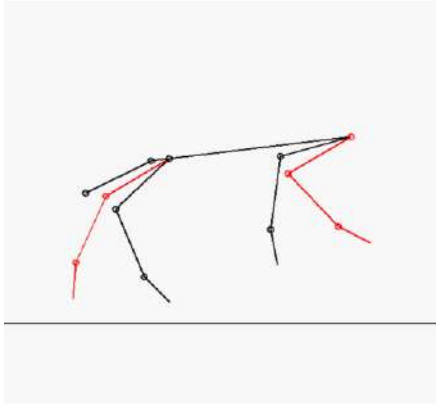
Conclusions

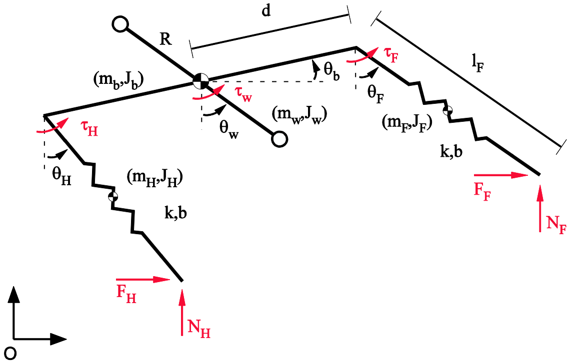
- EOM derivation for a two-body system in aerial phase.
- Analysis based on the angular momentum conservation.
- Analytical solutions for the holonomic cases.
- A tail should be preferred to a wheel – less torque and speed needed.
- Design guidelines given for a tail.
- Tail and maneuver parameters affect motor selection.
- Simulation results showed the best design choices for different maneuvers.

Future Work

- Dynamic analysis and control of more complicated models with ground contact.







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Thank you for your attention!