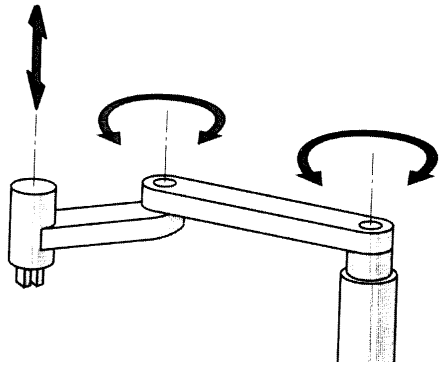
Assignment #2

Problem #1

The following manipulator is of SCARA type. This type of manipulator is used in assembly operations, and especially in the production of electronic boards, where chips are "surface mounted".



For this manipulator, sketch the endpoint reachable and dexterous workspace.

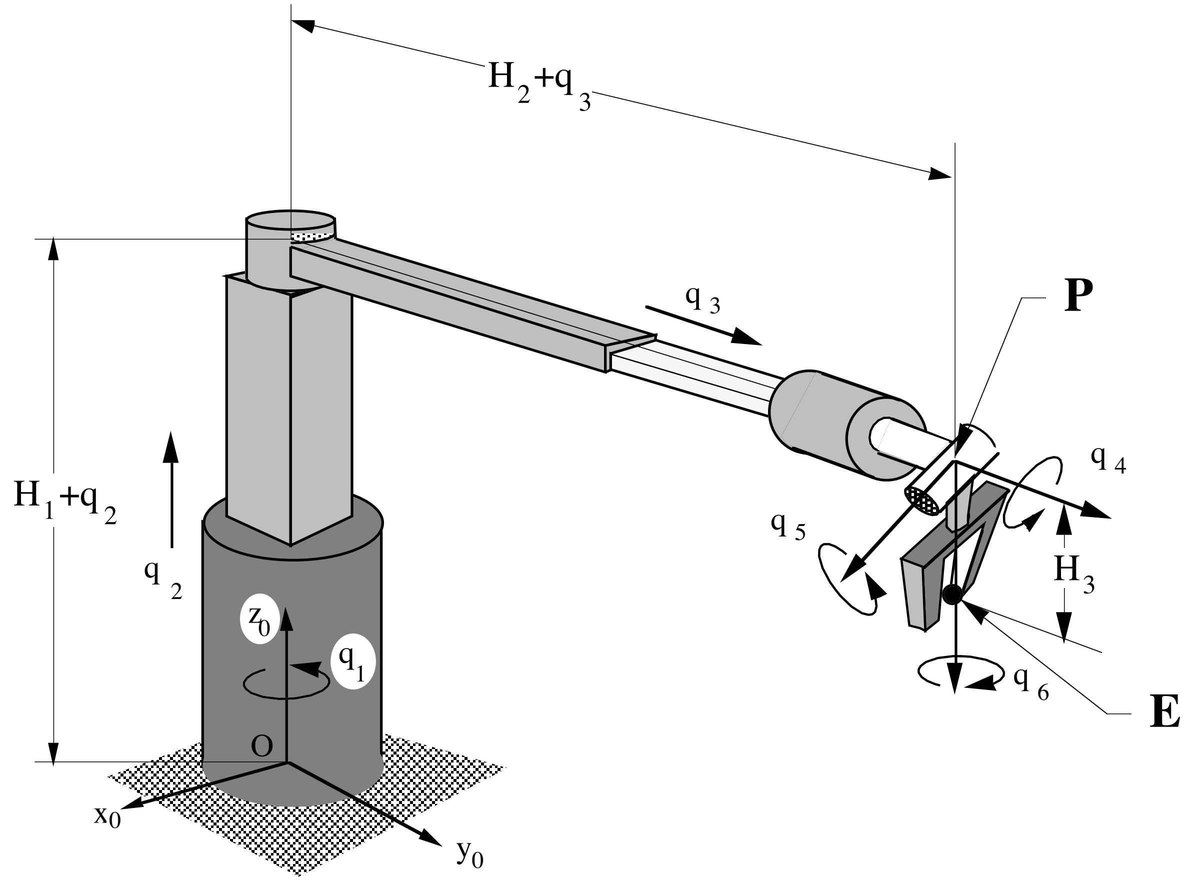
Problem #2

Using the obtained rotation matrix, find the following and explain clearly their physical meaning

i. the 3-2-1 (ZYX) Euler angles
ii. the 3-2-3 (ZYZ) Euler angles
iii. the equivalent axis-angle pair
iv. the Euler parameters

Problem #3

Consider the cylindrical manipulator with a spherical wrist, shown below. The depicted directions for the joint variables **q** are all positive. The end-effector is shown at $q_4 = 90°$, $q_5 = -90°$, $q_6 = 0°$.



(a) Carefully assign link frames to the manipulator.

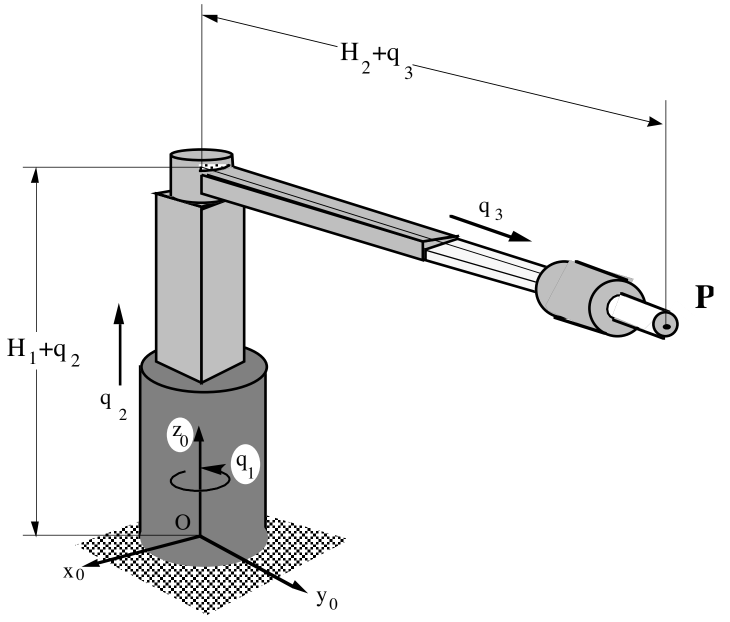
(b) Determine the manipulator’s Denavit-Hartenberg parameters. Identify the joint variables. How many Degrees-of-Freedom (DOF) does this manipulator have?

(c) Write the (intermediate) link 4×4 homogeneous transformation matrices.

(d) Write an expression for the homogeneous transform ${}^{o}\mathbf{T}_{E}$ which describes the position and orientation of the end-effector, E, whose origin in the last link frame is at $\left[0,0,H_3\right]^T$, and its axes are parallel to the those of the last frame.

Problem #4

In the previous problem, you derived the forward kinematics for the cylindrical manipulator with a spherical wrist. Here, we will solve the inverse kinematics problem. Note that the two prismatic joints can displace the links only in the positive direction shown below.



(a) First solve for the inverse kinematics of the positioning part of the manipulator, i.e. the manipulator without its spherical wrist. Assume that the target position for point P is at $[x, y, z]^T$, and then find the joint variables that correspond to it.
(b) Does this problem have always solution? Does it have multiple solutions?
(c) Sketch the reachable workspace.
(d) Now add the spherical wrist, and do the complete inverse kinematics. Assume that the desired end-effector frame is given by

$$
{}^{0}\mathbf{T}_{\mathbf{g}} = \begin{bmatrix} r_{11} & r_{12} & r_{13} & b_x \\ r_{21} & r_{22} & r_{23} & b_y \\ r_{31} & r_{32} & r_{33} & b_z \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

Does this inverse problem have always a solution? If not, why? Hint: Use Pieper's method.

Problem #5

For the RPPRRR manipulator discussed in problem 4, find the Jacobian ${}^{o}\mathbf{J}_{v}$ which relates $\dot{\mathbf{q}} = d/dt[q_1, q_2, q_3, q_4, q_5, q_6]^T$ to ${}^{o}V_E = [{}^{o}\mathbf{v}_E, {}^{o}\mathbf{w}_E]^T$. Use the general method discussed in class.