Assignment #1

Problem #1

(a) How many degrees of freedom must a manipulator have to be able to arbitrarily position and orient its end-effector in Cartesian space? How many degrees-of freedom (d.o.f.s) are required if the end-effector is something like a welding torch and we don't care about its roll orientation?

(b) Give three physical examples each of a line vector (i.e. a position vector) and a free vector.

(c) How many reference frames are required to specify the orientation of an arbitrary rigid body with respect to another arbitrary rigid body?

(d) What is the physical meaning of the columns of a rotation matrix ${}^{A}\mathbf{R}_{B}$? Similarly, what is the physical meaning of the rows of ${}^{A}\mathbf{R}_{B}$?

(e) Show by induction that the product of any number of rotation matrices is itself a rotation matrix.

Problem #2

(a) A frame $(Oxyz)_B$, which is initially coincident with a frame $(Oxyz)_A$, is rotated about x_B by θ degrees then about y_B by ϕ degrees. Give the rotation matrix which will change the description of vectors from ${}^{B}\mathbf{P}$ to ${}^{A}\mathbf{P}$.

(b) A frame $(Oxyz)_B$, which is initially coincident with a frame $(Oxyz)_A$, is rotated about x_B by θ degrees then about the original y_A axis by ϕ degrees. Give the rotation matrix which will change the description of vectors from ${}^{B}\mathbf{P}$ to ${}^{A}\mathbf{P}$.

(c) In part (a) if frame $(Oxyz)_B$ is first translated by $^A\mathbf{P}$, then rotated as specified, and then again translated by $^A\mathbf{P}$, what is the homogeneous transform which maps vectors expressed in $(Oxyz)_B$ to vectors expressed in $(Oxyz)_A$?

(d) In part (a) if frame $(Oxyz)_B$ is rotated as specified, and then translated by $^B\mathbf{P}$, what is the homogeneous transform which maps vectors expressed in $(Oxyz)_B$ to vectors expressed in $(Oxyz)_A$?

Problem #3

(a) Show that the determinant of a rotation matrix is +1. (Hint: Note that $\hat{\mathbf{x}}_1 \times \hat{\mathbf{y}}_1 = \hat{\mathbf{z}}_1$)

(b) If $\mathbf{R}$ is a rotation matrix, show that +1 is an eigenvalue of $\mathbf{R}$. Let $\hat{\mathbf{k}}$ be a unit eigenvector corresponding to the eigenvalue +1. Give a physical interpretation of $\hat{\mathbf{k}}$. What are the other two eigenvalues of $\mathbf{R}$?

(c) Write expressions for the rotation matrices which correspond to the three principal rotations. Find the rotation matrix $^0\mathbf{R}_3$ which is obtained from frame 0 after a ZYX Euler sequence of rotations (a series of consecutive principal rotations, the first around z_0, the second around y_1, and the third around x_2, see Craig, pg. 48). What is the physical meaning of the column vectors of the resulting rotation matrix $^0\mathbf{R}_3$? Be precise.

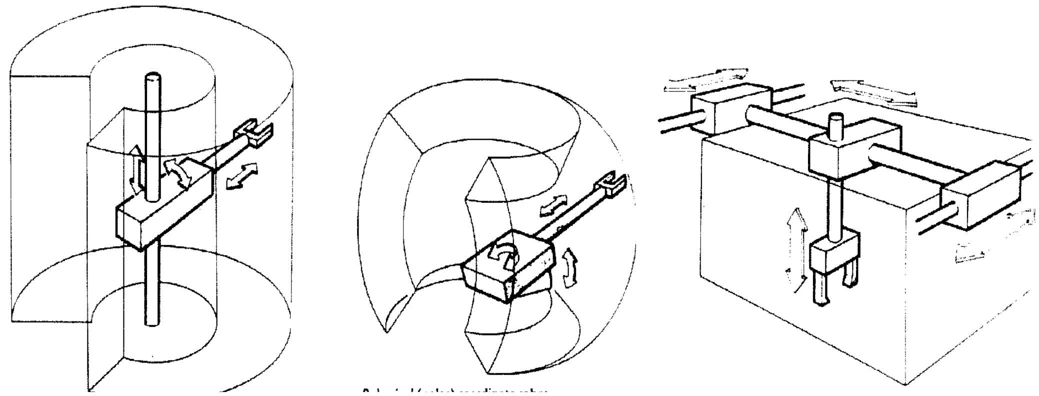
(d) Translations *can* be represented by vectors: Show that translations are commutative, i.e. that the position of a point which underwent two consecutive translations does not depend on the order of the two translations. Finite angle rotations *cannot* be represented by vectors. Prove this statement using an example. Then show that in contrast to finite rotations, infinitesimal rotations *can* be represented by a vector $\delta\boldsymbol{\Theta}$. Use the same example as before. (Hint: See Craig's 2.21, 2.22)

(e) Under what condition do two rotation matrices representing finite rotations commute? Give an example. (Bonus: prove your statement.)

Problem #4

(a) For the following 3 degree of freedom manipulator configurations, identify the explicit form of the homogeneous transformations describing the position and orientation of the end-effector frame $(Oxyz)_E$ with respect to the base frame $(Oxyz)_B$. Assume that $(Oxyz)_E$ is coincident with $(Oxyz)_B$ when all joint variables are zero.

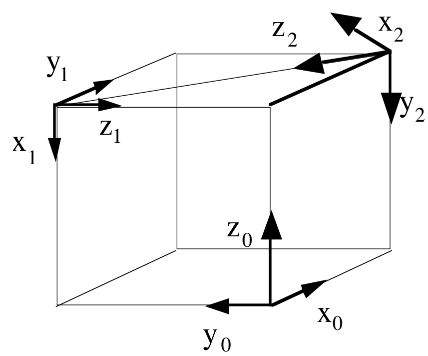
- cylindrical coordinate robot
- spherical coordinate robot
- Cartesian coordinate robot



What can you say about the space of achievable orientations for each of these manipulators?

Problem #5

(a) Given a cube of volume L^3 and a set of frames $(Oxyz)_0$, $(Oxyz)_1$, and $(Oxyz)_2$ as shown below find the homogeneous transformations ${}^0\mathbf{T}_1$, ${}^0\mathbf{T}_2$, and ${}^1\mathbf{T}_2$. Show that ${}^0\mathbf{T}_2 = {}^0\mathbf{T}_1\,{}^1\mathbf{T}_2$.



(b) Given the following transformation matrices relating frames $(Oxyz)_1$, $(Oxyz)_2$, and $(Oxyz)_3$, find ${}^2\mathbf{T}_3$

$$
{}^1\mathbf{T}_2 = \begin{bmatrix} 0.500 & 0 & -0.866 & 10 \\ 0.6124 & -0.7071 & 0.3536 & 0 \\ -0.6124 & -0.7071 & -0.3536 & 10 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad {}^1\mathbf{T}_3 = \begin{bmatrix} 0 & 0 & -1.0 & 0 \\ 0 & 1.0 & 0 & 10 \\ 1.0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

(c) Consider a frame $(Oxyz)_0$. Compute the homogeneous transformation $\mathbf{T}$ representing a translation of 3 m along the x_0-axis, followed by a rotation of $\pi/2$ about the current y_1-axis, followed by a translation of 1m along the initial z_0-axis. Sketch the final frame, $(Oxyz)_2$, and show the coordinates of its origin, O_2, with respect to frame 0.

(d) Vector $\mathbf{P} = [2.8284, 0.7071, 0.7071]^T$ is rotated resulting in vector $\mathbf{P}' = [2, 1, 2]^T$. Find $\mathbf{R}$. (Hint: make use of axis-angle representation of rotation matrix.)

Problem #6

The Canadian Space Vision System is used in an experiment to measure the position and orientation of the grapple fixture of a payload in the shuttle cargo bay. The homogeneous transform from the vision system to the payload ${}^{V}\mathbf{T}_{P}$ is therefore known. Joint encoders in the Canadarm are simultaneously used to provide the necessary data for the computation of the transform from the base of the arm to the end-effector ${}^{B}\mathbf{T}_{E}$.

It is desired to provide a display informing the astronaut who is operating the arm what the required *change* is in the end-effector position and orientation to be able to grasp the payload. This information is to be provided in two forms: expressed in the end-effector frame and expressed in the base frame. Show how this information can be computed and identify what additional homogeneous transformation information is required. Comment on how the additional homogeneous transformation information can be obtained and sketch the frames involved.