A PATH PLANNING METHOD FOR UNDERACTUATED SPACE ROBOTS

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ABSTRACT

In free-floating mode, space manipulator systems have their thrusters off, and exhibit nonholonomic behavior due to angular momentum conservation. The system is underactuated and a challenging problem is to control both the end effector location and the base attitude, using manipulator actuators only. Here, a path planning methodology satisfying this double goal is presented. The key idea is to use *high order polynomials* as *arguments* in cosines, specifying a desired path directly in joint-space, drastically extending configuration accessibility. The initial problem is converted to one of satisfying the motion integrals by optimization techniques. This approach always leads to a path, provided that the desired change in configuration lies between physically permissible limits. The method is presented for 3D systems. The resulting paths are smooth, avoid the need for small cyclical motions, and lead to smooth changes in finite and prescribed time.

1. INTRODUCTION

Space robots on orbit, will play an increasingly important role in space missions and on-orbit tasks, since these tasks are too risky or very costly (due to safety support systems) or just physically impossible to be executed by humans. Space robots consist of an on-orbit spacecraft fitted with one or more robotic manipulators. In free-flying mode, see Fig. 1, thrusters can compensate for manipulator induced disturbances but their extensive use limits a system's useful life span. In many cases, as for example during capture operations, it is desired that the thrusters are turned off to avoid interaction with the target. Then, the system operates in a free–floating mode, dynamic coupling between the manipulator and the spacecraft exists, and manipulator motions induce disturbances to the spacecraft. This mode is feasible when no external forces or torques act on the system and the total system momentum is zero. In this mode, the space robot is an under-actuated system exhibiting nonholonomic behavior due to the nonintegrability of the angular momentum [1]. This property complicates the planning and control of such systems, which have been studied by a number of researchers. Vafa and Dubowsky have developed a technique called the Virtual Manipulator, [2]. Inspired by astronaut motions, they proposed a planning technique, which employs small cyclical manipulator joint motions to modify spacecraft attitude. Papadopoulos and Dubowsky studied the *Dynamic Singularities* of free-floating space manipulator systems, which are not found in terrestrial systems and depend on the dynamic properties of the system, [1, 3]. They also showed that any terrestrial control algorithm could be used to control end-point trajectories, despite spacecraft motions, [3]. Nakamura and Mukherjee explored Lyapunov techniques to achieve simultaneous control of spacecraft's attitude and its manipulator joints, [4]. The method is not immune to singularities and yields non-smooth joint trajectories.

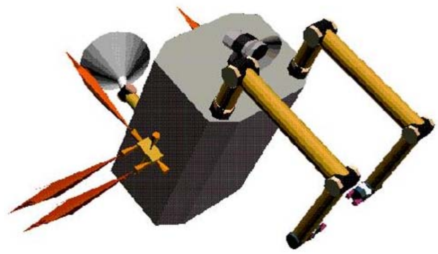

Figure 1. A Free-Flying Space Manipulator System.

A method employing small end-effector Cartesian cyclical motions to yield end point Cartesian motion, base attitude control, and avoidance of dynamic singularities, has been proposed, [5]. Quaternions were used to avoid representational singularities, and develop kinematic control schemes for a redundant manipulator mounted on a free-floating spacecraft, [6]. Franch et al. used flatness theory, to plan trajectories for free-floating systems, [7]. The method requires parameter selection so that the system is made controllable and linearizable by prolongations. Yoshida et al. tried zero reaction maneuvers on the Japanese experimental space robot ETS-VII, and presented flight data, [8]. Lampariello, et al. presented a motion planning method for free-flying space robots, which gives optimal solutions for spacecraft actuation and movement duration, and avoids unnecessary spacecraft actuation, [9]. However, the final spacecraft attitude is unknown beforehand, and is obtained only after the optimal solution is implemented.

An analytical approach to the path-planning problem was developed by the authors, [10]. The method allows for endpoint Cartesian point-to-point control with simultaneous control of the spacecraft's attitude, using manipulator actuators only, and was based on a transformation of the angular momentum to a space where it could be satisfied trivially. Further work showed that final configuration accessibility improved drastically, when *high order polynomials* for the joint angles were used as arguments in cosine functions. The planning problem was reduced to solving a non-linear equation, representing the integral of motion. Based on this idea, a numerical approach was derived, using high order polynomials, to specify the desired path directly in joint-space, [11]. In addition, lower and upper bounds for base rotation, due to manipulator motions, were analytically estimated for planar systems.

In this paper, a novel path planning methodology is presented, which solves the above-mentioned problem. The method is presented for a 3D system with a N degree-of-freedom (dof) manipulator. The dynamics of a free-floating space manipulator system is given, and the path-planning problem is formulated as an optimization one. The applicability of the method is illustrated by examples, both planar and spatial.

2. SYSTEM DYNAMICS

Free-flying space manipulator systems consist of a spacecraft (base) and one or more manipulators mounted on it, see Fig. 1. In free-floating mode, no external forces and torques act on the system, and the spacecraft translates and rotates in response to manipulator motions. It is assumed that manipulators have revolute joints in an open chain kinematic configuration, so that, in a system with a N dof manipulator, there are $(N+6)$ dof in total. The system is underactuated, and controlling both the end-effector position and the attitude of the base, using manipulator actuators only, is a non-trivial task, which can be achieved by exploiting the nonholonomic nature of the system, [2,4]. Since no external forces act on the system, and the initial momentum is zero, the system Center of Mass (CM) remains fixed in space, and the inertial origin, O, can be chosen to be the system's CM. This removes three of the six underactuated dof of the system, and is a result of the integrability of the translational equations of motion written for the system CM. The equations of motion have the form, [3],

$$\mathbf{H}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{C}(\mathbf{q},\dot{\mathbf{q}}) = \boldsymbol{\tau} \tag{1}$$

where $\mathbf{H}(\mathbf{q})$ is a positive definite symmetric matrix, called the reduced system inertia matrix, and $\mathbf{C}(\mathbf{q},\dot{\mathbf{q}})$ contains nonlinear velocity terms. The $(N\times 1)$ column vectors $\mathbf{q},\dot{\mathbf{q}}$ and $\boldsymbol{\tau}$ represent manipulator joint angles, velocities, and torques. The base attitude is computed using the conservation of angular momentum,

$${}^{0}\boldsymbol{\omega}_0 = \underbrace{-{}^{0}\mathbf{D}^{-1}(\mathbf{q})\,{}^{0}\mathbf{D}_{\mathbf{q}}(\mathbf{q})}_{\mathbf{F}_1(\mathbf{q})}\,\dot{\mathbf{q}} \tag{2}$$

where ${}^{0}\boldsymbol{\omega}_0$ is the base angular velocity in the spacecraft 0[th] frame, and ${}^{0}\mathbf{D}(3\times 3)$, ${}^{0}\mathbf{D}_{\mathbf{q}}(3\times N)$ are inertia-type matrices, [1].

Eq. (1) describes the reduced system dynamics in joint space N (dof). However, integration of these equations does not yield the attitude of the spacecraft, as these equations are independent of the spacecraft attitude. Therefore, to compute the resulting spacecraft attitude, one needs to append to (1) the angular momentum equation, given by (2). If we use ZYX Euler angles (Yaw–Pitch–Roll), to represent the attitude of the base $\psi_0 = [\theta_1 \quad \theta_2 \quad \theta_2]^T$, then the base angular velocity can be expressed as a function of the Euler rates, [12,13],

$$\begin{aligned} {}^{0}\boldsymbol{\omega}_0 &= \mathbf{E}(\psi_0)\dot{\psi}_0 = \begin{bmatrix} -s_2 & 0 & 1 \\ c_2 s_3 & c_3 & 0 \\ c_2 c_3 & -s_3 & 0 \end{bmatrix} \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{\theta}_3 \end{bmatrix} \Leftrightarrow \\ \dot{\psi}_0 &= \mathbf{E}^{-1}(\psi_0)\,{}^{0}\boldsymbol{\omega}_0 = \frac{1}{c_2}\begin{bmatrix} 0 & s_3 & c_3 \\ 0 & c_2 c_3 & -c_2 s_3 \\ c_2 & s_2 s_3 & s_2 c_3 \end{bmatrix} {}^{0}\boldsymbol{\omega}_0 \end{aligned} \tag{3}$$

In (3), $\mathbf{E}^{-1}$ is non-singular except at discrete configurations, i.e. for pitch-angle, around y-axis: $\theta_2 = \pm\pi/2 \Leftrightarrow c_2 = 0$. If such a configuration occurs, this can be dealt with the use of an alternative set of Euler angles. Alternatively, Euler parameters can be used to describe the base attitude. However, the analysis that follows is independent of the method chosen to represent the base attitude. Using (2) and (3), the conservation of angular momentum is rewritten as

$$\dot{\psi}_0 = \mathbf{E}^{-1}(\psi_0)\mathbf{F}_1(\mathbf{q})\dot{\mathbf{q}} \tag{4}$$

where $\mathbf{F}_1(\mathbf{q})$ is a function of the configuration defined in (2). It is well known that (4) cannot be integrated to analytically yield the spacecraft orientation ψ_0 as a function of the system's configuration, [1, 5]. However, if the joint angle trajectories are known as a function of time, then (4) can be integrated numerically to yield the trajectory for the base orientation. This nonintegrability property introduces nonholonomic characteristics to free-floating systems, and results from the dynamic structure of the system.

3. NONHOLONOMIC PATH PLANNING

The main problem we address here is to find a path, which connects a given (initial) configuration $(\psi_0^{in},\mathbf{q}^{in})$ to a desired (final) one $(\psi_0^{fin},\mathbf{q}^{fin})$, by actuating manipulator joints only. The problem must be solved in finite time and with simple motions, i.e. a large number of small cyclical motions is undesirable. It is well known that this problem is not trivial, since one must satisfy (4) which introduces nonholonomic behavior, and at the same time, achieve a change in a $(N+3)$ dimensional configuration space with only N controls (underactuated system). Next, a planning methodology is described that allows for a systematic approach in the planning of systems subject to nonholonomic constraints of the form of (4). We use high order polynomials for $\mathbf{q}(t)$, in order to specify a trajectory directly in joint-space. Let

$$q_i = q_i(t, \mathbf{b}_i), \quad \mathbf{b}_i \ ((k_i+1)\times 1), \quad i=1,\cdots,N \quad (5)$$

be polynomials of time t, of order k_i, and $\mathbf{b}_i$ the corresponding coefficients to be specified for joint-i, i.e. there are $n_f = (k_1 + k_2 + \cdots + k_N + N)$ free parameters. The minimum number of constraints to be satisfied include 6 boundary conditions per joint, i.e. the desired initial and final positions, and zero initial and final velocities and accelerations, plus at least three for the integrals of motion, representing the angular momentum conservation equation in three axes, i.e. $n_c \geq 6N+3$. Since it should be $n_f \geq n_c$, we have

$$\begin{aligned}(k_1 + k_2 + \cdots + k_N) &\geq 5N+3 \\ k_i &\geq 5, (i=1,\ldots,N)\end{aligned} \quad (6)$$

For each joint, the 6 boundary conditions mentioned above are imposed. If $k_i = 5$, then the corresponding trajectory of joint-i is determined. In a different case, where additional freedom is introduced to joint-i, i.e. $k_i \geq 6$, $\mathbf{b}_i$ contains $(k_i - 5)$ additional free parameters to be determined, which contribute to the satisfaction of the integrals of motion. All the other parameters in $\mathbf{b}_i$, are expressed as linear functions of these $(k_i - 5)$ free parameters. Let $\mathbf{b} \in \mathbb{R}^k$, the vector containing the remaining free parameters of all $\mathbf{b}_i \ (i=1,\cdots,N)$, after boundary conditions for all joints are satisfied. Then (5), can be written in vector form as

$$\mathbf{q} = \mathbf{q}(t, \mathbf{b}) \quad (7)$$

Using (4) and (7), we can write

$$\dot{\boldsymbol{\psi}}_0 = \mathbf{F}(\boldsymbol{\psi}_0, \mathbf{b}, t) \quad (8)$$

These free parameters $\mathbf{b} \in \mathbb{R}^k$, as said earlier, should be at least three $(k \geq 3)$ and satisfy the integrals of motion

$$\begin{aligned}&\boldsymbol{\psi}_0^{fin}(\mathbf{b}) = \boldsymbol{\psi}_0^{fin}(des), \ or \\ &\mathbf{h}(\mathbf{b}) \triangleq \boldsymbol{\psi}_0^{fin}(\mathbf{b}) - \boldsymbol{\psi}_0^{fin}(des) = \mathbf{0}\end{aligned} \quad (9)$$

Equation (8) represents a system of very complex, highly non-linear and dynamically coupled differential equations, which, as said earlier, cannot be integrated analytically. However, these equations can also be seen as a dynamic system with a constant input $\mathbf{b}$, state vector $\boldsymbol{\psi}_0$, and t as a parameter. Then, for given joint trajectories, i.e. given $\mathbf{b}$, and initial base attitude $\boldsymbol{\psi}_0^{in}$, (8) can be numerically integrated, on $[t_{in}, t_{fin}]$, yielding the final base attitude $\boldsymbol{\psi}_0(\mathbf{b}, t_{fin}) \triangleq \boldsymbol{\psi}_0^{fin}(\mathbf{b})$. In this way the problem reduces to determining the unknown vector $\mathbf{b}$, numerically, so that (9) is satisfied.

We should note here the existence of a physical limitation, which is that for a given change in manipulator joints, only a limited change in base attitude is expected, due to dynamic system's properties. In other words, not all configurations are reachable from an initial one, in prescribed finite time and with simple motions (without the use of many small cyclical motions), [11]. The path planning method, presented here, is expected to give at least one solution, provided that the desired change in base attitude, due to a given change in manipulator's configuration, is feasible. If the number of the free parameters to be determined is three, i.e. if $\mathbf{b} \in \mathbb{R}^3$, (9) can be solved numerically. If the desired change in base attitude is feasible, but (9) does not yield to a solution for $k=3$, additional freedom to one or more joints can be introduced. Then, $\mathbf{b} \in \mathbb{R}^k$, with $k \geq 3$, is determined using optimization techniques, as follows, [14]:

$$\|\mathbf{b}\| \rightarrow \min : \mathbf{h}(\mathbf{b}) = \mathbf{0} \quad (10)$$

We should clarify here, that although it is known that an optimization problem may have multiple local minima, all these minima still satisfy (9), i.e. they still solve the problem of finding a parameter vector $\mathbf{b}$ that will lead to the desired system configuration. In other words, we are mostly interested in finding solution(s) satisfying the equality constraints (9), which represent the integrals of motion, rather than finding the global minimum for $\mathbf{b}$.

Some observations of the method outlined above are presented here: First, additional requirements such as attitude change maximization, and joint limits or obstacle avoidance can be achieved by adding more freedom in the end-point path via the use of higher order polynomials for one or more joints. Also, even if there are only three parameters to be determined, (9) may result in multiple solutions, due to the periodicity of the trigonometric functions involved, elements of $\mathbf{F}_1(\mathbf{q})$, see (2) and (4). This physically means that we can have alternative or multiple rotations for some joints, leading to the same final configuration. These solutions result in end-effector paths of different length. In other words, the method does not exclude multiple rotations as solutions. More generally, joint paths have additional freedom, leading even to multiple extremes for one or more $q_i(t)$. Joint trajectories are not necessarily monotonic, between initial – final values, and it is possible to obtain more complex paths, or even closed loops (in joint-space). However, long paths are in general undesired, because for given motion duration, they result in high joint velocities. Determination of $\mathbf{b}$ according to (10) is expected to yield shorter paths.

Once joint-space trajectories are specified, the base attitude ψ_0 is calculated by integrating (8). Following the estimation of $\mathbf{b}$, initial and final values for the base attitude ψ_0 are satisfied. Also, the initial and final velocities and accelerations of ψ_0 are necessarily zero because the joint variables have zero initial and final velocities and accelerations, and in addition, $\dot{\psi}_0$ satisfies (4). Finally, since the path is defined directly in the joint space, it is always feasible and will never be subject to Dynamic Singularities problems in the system's workspace, [1].

4. PLANAR IMPLEMENTATION

In this section, we implement the methodology outlined earlier on a free-floating robotic system consisting of a two-dof manipulator mounted on an arbitrary point of a three-dof spacecraft, which is constrained to move in the plane perpendicular to the axis of manipulator joint rotations, see Fig. 2.

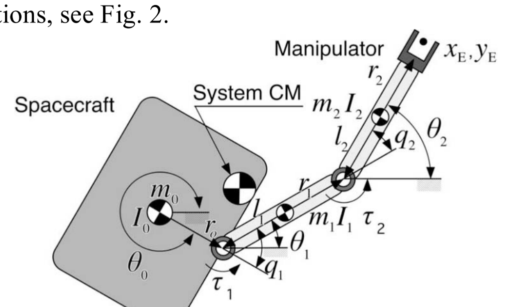

Figure 2. A Planar Free-Floating Space Manipulator System.

For this system, (2), can be written as,

$$D_0(\mathbf{q})\dot{\theta}_0 + D_1(\mathbf{q})\dot{\theta}_1 + D_2(\mathbf{q})\dot{\theta}_2 = 0 \qquad (11)$$

where $(\theta_0, \theta_1, \theta_2)$ are spacecraft attitude and manipulator absolute joint angles. The $D_0, D_1,$ and D_2 are functions of system inertial parameters and of the manipulator joint angles q_1 and q_2. The (x_E, y_E) is given by [1],

$$\begin{aligned} x_E &= a\cos\theta_0 + b\cos\theta_1 + c\cos\theta_2 \\ y_E &= a\sin\theta_0 + b\sin\theta_1 + c\sin\theta_2 \end{aligned} \qquad (12)$$

where, $a, b, c,$ are constant terms, functions of the mass properties of the system, given analytically by (13).

As mentioned earlier, for a given change in manipulator joints $(\Delta q_1, \Delta q_2)$, a limited change in base attitude $(\Delta\theta_0)$ is expected, depending on system dynamic properties. In other words, not all configurations are reachable from an initial one, in prescribed finite time and with simple motions (without the use of many small cyclical motions). An estimation of these bounds is given in [11]. In the general 3D case, the change in the base attitude, due to manipulator motions, is bounded too, although more difficult to be derived analytically.

We note here that maximum absolute base rotation is achieved, as expected, when the arm is fully extended $(q_2 = 0^o)$, while absolute minimum base rotation occurs when the arm is fully retracted $(q_2 = 180^o)$.

The path planning method presented in Section 3 is applied. Here (9) is reduced to a single integral of motion, so the unknown vector $\mathbf{b}$, should contain at least one free parameter, i.e. $k \geq 1$, to satisfy this unique integral of motion. After $\mathbf{b}$ is determined, by methods presented in Section 3, joint-space trajectories are known, and $\theta_0(t)$ is given by integration of (11), details in [11].

TABLE 1. Planar System Parameters

Body	$l_i\ [m]$	$r_i\ [m]$	$m_i\ [kg]$	$I_i\ [kgm^2]$
0	0.5	0.5	400.0	66.67
1	0.5	0.5	40.0	3.33
2	0.5	0.5	30.0	2.50

Parameters $a,\ b,$ and $c,$ in (12), are given by,

$$\begin{aligned} a &= r_0 m_0 (m_0 + m_1 + m_2)^{-1} = 0.43m \\ b &= (r_1(m_0 + m_1) + l_1 m_0)(m_0 + m_1 + m_2)^{-1} = 0.89m \\ c &= (l_2(m_0 + m_1))(m_0 + m_1 + m_2)^{-1} + r_2 = 0.97m \end{aligned} \qquad (13)$$

The reachable workspace is a disk with an external radius equal to $R_{\max,2} = a + b + c = 2.29m$. The outer ring of the Path Dependent Workspace (PDW), i.e. the subworkspace in which Dynamic Singularities may occur, is defined by $R_{\min,2} = b + c - a = 1.44m$ and $R_{\max,2}$, [1]. The duration of motion is chosen as 10 s. Since the constraints are scleronomic, increasing or decreasing this time has no effect on the path taken, but instead increases or decreases the torque requirements and the magnitude of velocities or accelerations.

4.1. New System Configuration

The free-floater has to move its manipulator endpoint to a new (desired) location and at the same time change its spacecraft attitude to a new (desired) one. The initial system configuration is $(\theta_0, x_E, y_E)^{in} = (-50^o, 1.53m, 0.96m)$ and the final is $(\theta_0, x_E, y_E)^{fin} = (0^o, 1.71m, -0.29m)$, or $(\theta_0, q_1, q_2)^{in} = (-50, 80, 30)^o$ and $(\theta_0, q_1, q_2)^{fin} = (0, -60, 90)^o$. For the given change in q_1, q_2, the calculated bounds for base rotation are given by $\Delta\theta_0 \in (1.4, 72.2)^o$. Here, $\Delta\theta_0^{des} = 50^o$, (between the permissible bounds), so it is expected that at least one path exists that can connect the given (initial) to the desired (final) configurations. The path planning method presented in Section 3 is employed here to specify the desired path. A fifth and a sixth order polynomial of t, is specified for q_1, q_2 respectively, i.e. we assume initially that $k_1 = 5$ and $k_2 = 6$, see (5). Here only one free parameter exists, to satisfy the integral of motion. Solving the nonlinear equation (9) numerically, results to infinite solutions for b_{26}. Using the optimization formulation defined in (10), the minimum b_{26} is found, yielding the shortest path, as shown in Fig. 3.

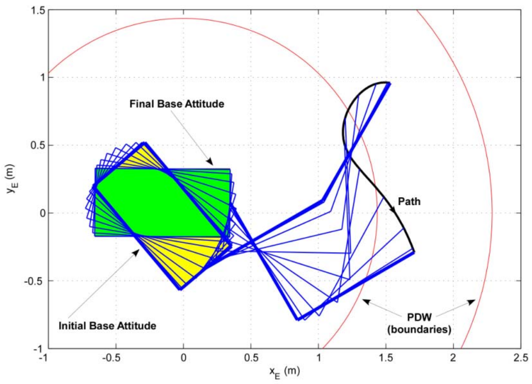

Fig. 3: Snapshots of a motion to a desired θ_0, x_E, y_E.

In Fig. 4, we can see that the desired configuration is reached in the specified time. Also, all trajectories are smooth throughout the motion, and the system starts and stops smoothly at zero velocities, as expected. The corresponding joint torques are given in Fig. 5. These torques are computed using (1) and the elements of the reduced inertia matrix, [1]. As shown in Fig. 5, the required torques are small and smooth while they can be made arbitrarily small, if the duration of the maneuver is increased. The implication of this fact is that joint motors can apply such torques with ease and therefore the resulting configuration maneuver is feasible.

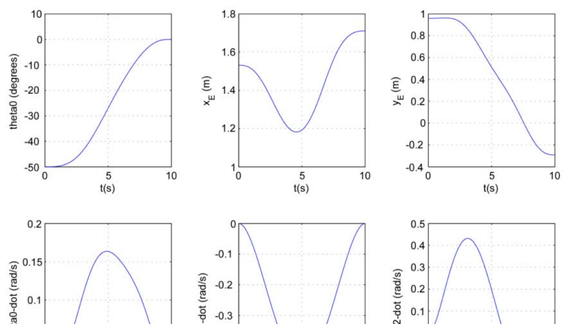

Fig. 4: End-effector coordinates, spacecraft orientation, and joint angle rate trajectories for the motion in Fig. 3.

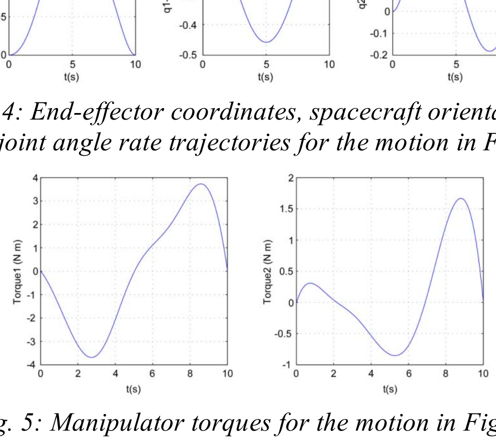

Fig. 5: Manipulator torques for the motion in Fig. 3.

5. SPATIAL IMPLEMENTATION

In this section, the problem is tackled for 3D systems. The extension from 2D to 3D is far from trivial. Here, the path planning method of Section 3, is applied to a free-floating robotic system consisting of a three-dof anthropomorphic manipulator mounted on an arbitrary point of a six-dof spacecraft (base). The system has 9–dof and only 3-actuators. The integrability of linear momentum equations eliminates three of the six underactuated dof of the system, so we have to control the remaining 6-dof, using the three available actuators. The system, for this example, is given in Fig. 6, and its parameters in Table 2. Using the system parameters, we can obtain the system inertia matrices, and finally an expression for the (3x3) matrix in this case, see (2).

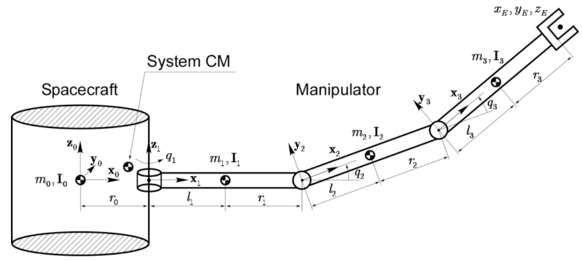

Fig. 6: A Spatial Free-Floating Space System.

TABLE 2. Spatial System Parameters

Body	l_i $[m]$	r_i $[m]$	m_i $[kg]$	I_{xx}, I_{yy}, I_{zz} $[kgm^2]$
0	0.5	0.5	400.0	66.7, 66.7, 66.7
1	0.5	0.5	30.0	0, 2.5, 2.5
2	0.5	0.5	30.0	0, 2.5, 2.5
3	0.5	0.5	20.0	0, 1.7, 1.7

5.1. New End Point Position

In this example, the manipulator end-point is desired to move to a new location, while at the end of the motion the spacecraft must be at its initial attitude, i.e. $\boldsymbol{\psi}_0^{fin} = \boldsymbol{\psi}_0^{in}$, avoiding loss of communication links. The equivalent manipulator configuration should change from $(q_1, q_2, q_3)^{in} = (20, 30, 30)^o$, to $(q_1, q_2, q_3)^{fin} = (0,0,0)^o$. We remind here that the remaining free parameters (after satisfying initial –final values for position, velocity and acceleration for all joints) should be at least three $(k \geq 3)$, in order to satisfy the three integrals of motion. We assigned polynomials to joints-1, 2, and 3, of order $k_1 = 7$, $k_2 = 6$, and $k_3 = 6$, respectively. Following the method of Section 3, we obtained the joint trajectories q_1, q_2, q_3 for such an end-effector path, along with the base attitude coordinates, $\theta_1, \theta_2, \theta_3$, using (Z-Y-X) = (Y-P-R) Euler angles. The results are depicted in Fig. 7. One can observe that while the joint angle history depicted at the top of the figure change significantly during manipulator motion, the Euler angles return to their initial values. Therefore, the spacecraft attitude remains undisturbed. In addition, since all joint accelerations are zero at the start and the end of the motion, and remain smooth throughout the motion, all manipulator joint torques are also smooth and bounded.

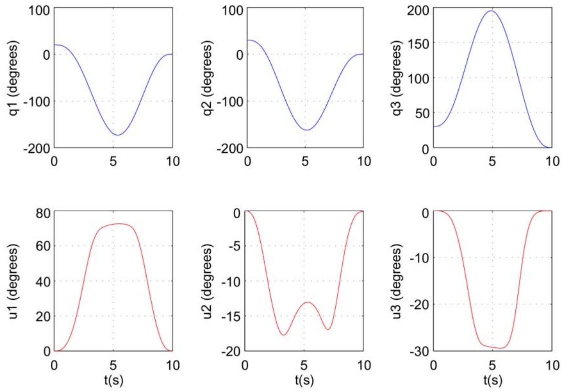

Fig. 7: Joint trajectories and resulting base attitude, for the spatial system in Fig. 6.

6. CONCLUSIONS

In this paper, a novel path planning method for underactuated free-floating space manipulator systems, exhibiting nonholonomic behavior, was developed. Using this method, one can obtain paths to control both the location of the end effector and the final attitude of the base. The key idea is the use of high order polynomials, as arguments in cosine functions. The accessibility of final configurations is extended drastically and the initial problem is transformed to one of satisfying the integrals of motion, using optimization techniques. It was found that this approach leads always to a path, provided that the desired change in configuration lies between physically permissible limits. The method is presented for general 3D systems. As shown by example, no small cyclical motions are needed, the paths are smooth, and the configuration changes occur in finite and prescribed time.

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