ON INERTIA AND STIFFNESS EFFECTS DURING IMPACT DOCKING

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ABSTRACT

Space debris removal and mitigation using space robots are complex missions, which require extensive analysis prior to launch. An important aspect during such a mission is the capturing task; any unsuccessful attempt may create more problems than solve. In this paper, the modelling of the impact docking between two multibody systems is studied. The effects of mass ratios on the resulting changes of relative velocities are analysed and discussed. An extension of the rigid body impact theory to multibody systems is developed, where the effect of system mass ratios to the change of the relative velocities is quantified, and its significance is discussed. Velocity requirements leading to a successful latching at first impact will be identified. Simulation results are presented that validate the proposed analytical approach. Future work is discussed.

1 INTRODUCTION

Space exploration and exploitation require strengthening of the human and robotic infrastructure on orbit and beyond. To this end, tasks like satellite servicing, orbital debris removal and construction of large assemblies on Earth or other planetary orbits will be of critical importance in the near future. Since On-Orbit Servicing (OOS) is expected to play a critical role in future of space programs, space agencies have already incorporated OOS activities in their roadmaps, with notable examples JAXA's ETS-VII program, NASA's Orbital Express and Robotic Refuelling Mission (RRM), as well as in a number of research activities in the Clean Space initiative and the Automation and Robotics group of ESA.

However, to achieve these goals, prior extensive analysis of any OOS mission is required. An important part of any robotic servicing mission is reaching and capturing a target (satellite or debris). Assuming a space robot already on orbit, this procedure includes the phases of far and close rendezvous, mating (docking or berthing) - which incorporates capturing of some kind - and servicing, [1]. Of those, docking to a target by a space robotic system, consisting of a satellite base and of one or more manipulators mounted on it, is an especially demanding task, due to the dynamic coupling between the base and the manipulator, [2].

Additionally, docking and capturing procedures inevitably are associated with impact forces as the chaser and the target come into contact. This task is more challenging when the robotic system and the target have comparable masses. To minimize these forces, body impulses are minimised using the Extended Inertia Tensor [3]. The concepts of virtual mass and impedance matching of systems were studied [4]. Notable works focus on the problem of taking into account the system dynamics, post impact, e.g. [5], or prior to impact, e.g. by incorporating an optimal approach method [6].

In the common case of passive docking, known as *impact docking*, impact forces are inevitable, as the chaser and the target come into contact in order to latch. Unsuccessful impacts may separate the servicer from the target, or damage critical subsystems. Thus the study of the behaviour of the participating systems under impact is vital. Therefore, two aspects need thorough examination: (a) adequate impact modelling of the procedure and (b) effects of mass and compliance parameters to the latching performance.

Various modelling approaches exist in studying impacts, [7]. However, as the computational systems in space have limited capabilities, while the impact is a fast process, simplified but relatively accurate models are necessary. Additionally, a method that could predict the performance with low computational effort prior to the contact, should lead to useful insights. In view of the above, the lumped body analysis constitutes a useful approach; however until now its use was restricted to cases in which the impacting systems can be considered as two rigid bodies, [8], [9]. Modelling multibody systems during impact is still a complicated problem, in which the existing approaches sometimes result in chaotic responses, not to mention the ambiguous problem of multiple or simultaneous impacts, e.g. [10], [11] and [12]. An approach similar to the one in this paper has been proposed in [13]; however the authors have not recognised the importance of the mass ratios of the bodies involved, especially when all systems are free-floating and not attached to a solid base.

In this work the impact docking is modelled as impact between two multibody systems. The effect of the masses during the impact is analysed to determine the post impact behaviour of the systems. A coefficient of effective masses is proposed which can help in the identification of the post impact behaviour prior to impact. A number of interesting impact cases are examined. Accordingly, the minimum impact velocity is determined in order to achieve latch at first impact; a typical mechanism for docking includes the existence of a spring-loaded latch. This could enable the design of mechanisms allowing simpler docking procedures, especially during autonomous OOS. Simulations are presented. The integration of theory with future work is discussed.

2 MODELLING BODIES UNDER IMPACT

Although the impact docking has mainly to do with systems where the probe-drogue mechanisms are not connected to appendages (e.g. ATV docking on ISS), in this more general case, it is assumed that they can be attached to manipulators. By reference to **Figure 1a**, suppose that the probe and drogue are both connected to a manipulator, and each manipulator to a free-floating base. This can be simplified if examined as a 1D case, see **Figure 1b**.

More specifically, the Chaser is a two-body system, where mass, m_1, represents the Chaser body and mass m_2, its manipulator with the probe. These are connected via a lumped parameter system, (a spring and a damper), modelling the internal compliance of the system; for example this is the case when the manipulator is controlled by an impedance controller. Similarly for the Target, a system of two masses (m_3 and m_4) connected by lumped parameters is employed. Specifically for the 1D case, the latching mechanism is regarded to be a spring-latch system, which is normal to the motion of the bodies under impact. This method of modelling is similar to known approaches such as those in [1] and [12].

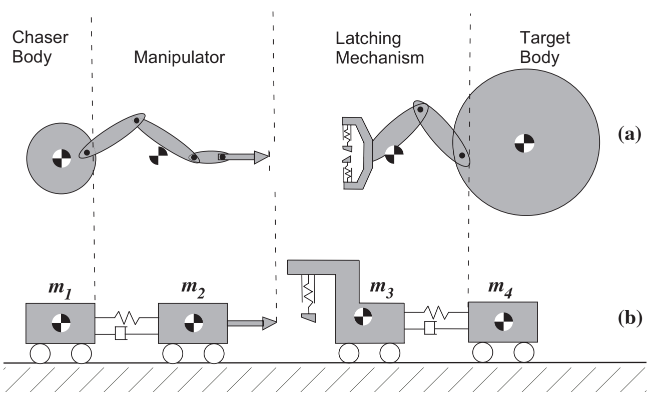

Figure 1. Model rationale of impact docking between multibody systems.

3 EFFECT OF MASSES DURING IMPACT DOCKING

3.1 ASSUMPTIONS

In our analysis, the following assumptions apply, see also [14]: (a) Impacts are between rigid bodies. The contact area remains small in comparison to other dimensions. Thus the compliance of the contact area can be represented by using lumped elements of springs and dampers, (b) Impact forces are very high and for short duration, therefore the impulse of forces like gravity is negligible, (c) During an impact, it is assumed that there is no considerable change in the system configuration. This applies also in zero-g even if there is no fixed base, because each joint appears as fixed in a position during impact (*"quasi-fixed")*, (d) Usually the probes and latching mechanisms are made from metallic materials. As such it can be considered that the impact stiffness is at least one order of magnitude stiffer than the lumped parameters of the Chaser and the Target. (e) The elastic wave speed of the impact for aluminium or steel is more than $5000\, m/s$ meaning that for an impact duration of 5 ms or more, the impact wave will travel more than 25m. Thus for two multibody systems which come into contact, this is more than enough to assume that the entire systems are affected by the impact simultaneously.

3.2 RIGID MULTIBODY IMPACT

The common multibody impact models use techniques, which are by design computationally cumbersome. Even though novel algorithms and current increased computational power can help, the computation of the impact behaviour of a n-body system takes time and is avoided for predicting impact behaviour in space systems. This is true especially in the case in which two multibody systems come into contact. For this reason a different approach is proposed which makes use of the rigid body theory approach, termed Rigid Multibody Impact (RMI). A difference from other multibody impact approaches is that in this one, the bodies are considered both as entire systems (Chaser and Target) and as multibody systems (two masses for Chaser, two masses for Target) simultaneously, see **Figure 2**.

During an impact, the masses m_2 and m_3 come into contact, thus the impact characteristics are inevitably connected with these two bodies. However, at the same time the impact occurs between the total masses of the two multibody systems, m_c and m_t which include the masses under impact m_2 and m_3. In other words during impact there is an interaction which exchanges energy between both the masses under impact as well as the total masses, [15]. The challenge is how to develop a fast procedure, with the help of which one can find the behaviour of the systems after impact, without large computational requirements.

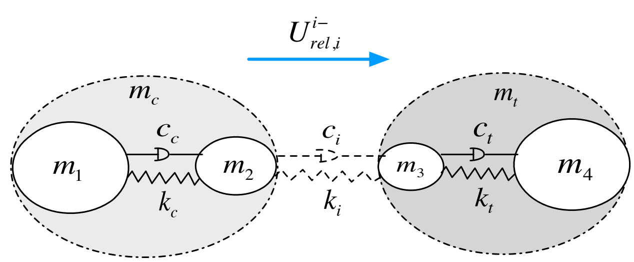

Figure 2. Concept for Multibody Contact Model.

With the help of **Figure 2**, four different effective masses are defined. More specifically, the effective mass of the total Chaser and Target systems (*total system effective mass*) is:

$$m_{i,ef} = m_c \cdot m_t / (m_c + m_t) \tag{1}$$

the effective mass of the bodies under impact (the masses that come first into contact) is

$$\mu_{i,ef} = m_2 \cdot m_3 / (m_2 + m_3) \tag{2}$$

and the effective masses of each of the Chaser and Target are

$$\mu_c = m_1 \cdot m_2 / (m_1 + m_2) \tag{3}$$

$$\mu_t = m_3 \cdot m_4 / (m_3 + m_4) \tag{4}$$

By considering an impact as a half-period oscillation, (i.e. [1], [4]) the impulse P_{imp}^i during the impact instant "i" is given by,

$$P_{imp}^i = \left(1+e^*\right) \cdot U_{rel,i}^{i-} \cdot \mu_{i,ef} \tag{5}$$

where $U_{rel,i}^{i-}$ is the relative velocity of the bodies under impact prior to impact "i", and e^* is the coefficient of restitution – any damping characteristic and loss of energy during impact is connected with the latter. Note that the signs used in superscripts have the following meaning: "-" represents a value just prior to impact and "+" represents a value just after the impact. Additionally, the same impulse P_{imp}^i is developed between m_2 and m_3, and between m_c and m_t; this is due to the fact that this impulse represents the energy exchange which occurs between the two masses under impact, which also are parts of their corresponding systems (Chaser or Target).

Let us now define the relative velocity between the systems $U_{rel,s}^{i\pm}$ before or after impact "i" (according to the sign) as,

$$U_{rel,s}^{i\pm} = V_c^{i\pm} - V_t^{i\pm} \tag{6}$$

where $V_j^{i\pm}$, $j = c,t$ is the absolute velocity of the Chaser (c) or Target (t) before or after impact instant "i" with respect to the same inertia coordinate system. The following relationships apply, [8],

$$P_{imp}^i = m_c \cdot \left(V_c^{i-} - V_c^{i+}\right) \tag{7}$$

$$P_{imp}^i = m_t \cdot \left(V_t^{i-} - V_t^{i+}\right) \tag{8}$$

Therefore the relative velocity of Chaser and Target CoMs after the impact is

$$\begin{aligned} U_{rel,s}^{i+} &= V_c^{i+} - V_t^{i+} = \left(V_c^{i-} - \frac{P_{imp}^i}{m_c}\right) - \left(V_t^{i-} - \frac{P_{imp}^i}{m_t}\right) = \\ &= \left(V_c^{i-} - V_t^{i-}\right) - P_{imp}^i \cdot \left(\frac{1}{m_c} + \frac{1}{m_t}\right) \Rightarrow U_{rel,s}^{i+} = U_{rel,s}^{i-} - \frac{P_{imp}^i}{m_{i,ef}} \end{aligned} \tag{9}$$

Using Eq. (5), we obtain the following expression,

$$U_{rel,s}^{i+} = U_{rel,s}^{i-} - \frac{P_{imp}^i}{m_{i,ef}} = U_{rel,s}^{i-} - \frac{\left(1+e^*\right) \cdot U_{rel,i}^{i-} \cdot \mu_{i,ef}}{m_{i,ef}} \tag{10}$$

It is important to distinguish the difference between the two relative velocities $U_{rel,s}^{i-}$ and $U_{rel,i}^{i-}$: The first refers to the relative velocity of the total masses m_c and m_t, while the second refers to the relative velocity of the bodies under impact, namely of m_2 and m_3. Generally, these two relative velocities are not equal. For example in the case we examine, if m_2 and/or m_3 are oscillating with respect to their body coordinate system these differ. In order for

$$U_{rel,s}^{i-} = U_{rel,i}^{i-} \tag{11}$$

to apply, there must be no internal relative motion between the bodies of Chaser and between the bodies of Target. This means that Chaser masses have the same velocity (and therefore the same velocity with their system CoM); also that Target masses have the same velocity (and therefore the same velocity with their system CoM). Thus the internal springs of Chaser and Target are at their free lengths. This case is usually reasonable prior to first impact.

Generally the relative impact velocity $U_{rel,i}^{i-}$ can be expressed as

$$U_{rel,i}^{i-} = U_{rel,s}^{i-} + \delta U_{rel}^{i-} \tag{12}$$

where δU_{rel}^{i-} is the relative difference of velocities between the impact bodies (m_2 and m_3) due to their motion within their systems (i.e. oscillations), when the relative velocity of the systems has been subtracted. Using Eqs. (10) and (12), one can find

$$U_{rel,s}^{i+} = \left(1 - \frac{\left(1+e^*\right) \cdot \mu_{i,ef}}{m_{i,ef}}\right) U_{rel,s}^{i-} - \frac{\left(1+e^*\right) \cdot \mu_{i,ef}}{m_{i,ef}} \cdot \delta U_{rel}^{i-} \tag{13}$$

Applying the notation e_I for the ratio of effective masses between bodies under impact and total system

$$e_I = \mu_{i,ef} / m_{i,ef} \tag{14}$$

and combining with the coefficient of restitution in Eq. (13)

$$e_I^* = \left(1+e^*\right) \cdot e_I \tag{15}$$

one can write Eq. (13) as

$$U_{rel,s}^{i+} = \left(1 - e_I^*\right) U_{rel,s}^{i-} - e_I^* \cdot \delta U_{rel}^{i-} \tag{16}$$

Assuming no oscillation prior to first impact, Eq. (11) applies, therefore

$$\delta U_{rel}^{i-} = 0 \tag{17}$$

and dropping "i"" for clarity in the rest of this paper, Eq. (16) is simplified to

$$U_{rel,s}^{+} = \left(1 - e_I^*\right) U_{rel,s}^{-} \tag{18}$$

If Eq. (14) is analysed one can find

$$\begin{aligned} e_I &= \frac{\mu_{i,ef}}{m_{i,ef}} = \frac{m_2 \cdot m_3 / (m_2 + m_3)}{m_c \cdot m_t / (m_c + m_t)} \Rightarrow \\ &\Rightarrow 0 \le e_I = \frac{A}{A+B} \le 1 \end{aligned} \tag{19}$$

where $m_1, m_2, m_3, m_4 \ge 0$ and

$$\begin{aligned} A &= m_1 \cdot m_2 \cdot m_3 + m_2^2 \cdot m_3 + m_2 \cdot m_3^2 + m_2 \cdot m_3 \cdot m_4 \\ B &= m_1 \cdot m_3^2 + m_1 \cdot m_2 \cdot m_4 + m_1 \cdot m_3 \cdot m_4 + m_2^2 \cdot m_4 \end{aligned} \tag{20}$$

Thus

$$0 \le \mu_{i,ef} \le m_{i,ef} \tag{21}$$

The coefficient e_I plays a significant role in order to determine whether the Chaser will continue, stop or change its direction of motion after the impact as a system. This cannot be found using the simple rigid body theory, because it examines only the bodies under impact (in this case m_2 and m_3) without considering the mass ratio between the individual masses of the two multibody systems under impact (thus all the masses under consideration, m_1, m_2, m_3 and m_4).

To examine the significance of the coefficient, let a perfectly elastic impact occurs $\left(e^* = 1\right)$ and use it in Eqs. (15) and (18),

$$U^+_{rel,s} = \left(1 - e^*_I\right) U^-_{rel,s} = \left(1 - 2 \cdot e_I\right) U^-_{rel,s} \tag{22}$$

The following alternative cases can be identified:

i) $e_I = 0 \Rightarrow U^+_{rel,s} = U^-_{rel,s}$: No impact occurs.

ii) $e_I = 1 \Rightarrow \mu_{i,ef} = m_{i,ef} \Rightarrow U^+_{rel,s} = -U^-_{rel,s}$: Resembles an impact between two rigid bodies. Therefore, the well-known theoretical case is obtained, while the rest of equations are simplified as the relative velocity between the two systems is equal to the relative velocity of two simple rigid bodies.

iii) $e_I = \dfrac{1}{2} \Rightarrow U^+_{rel,s} = 0$: The two multibody systems move with the same velocity. This situation is further examined later.

iv) $0 < e_I < \dfrac{1}{2} \Rightarrow U^+_{rel,s} \cdot U^-_{rel,s} > 0$: The two systems will move in the same direction after impact. Practically the Chaser will continue its direction of motion, and the Target will move towards the same direction. This is the favourable situation during docking.

v) $\dfrac{1}{2} < e_I < 1 \Rightarrow U^+_{rel,s} \cdot U^-_{rel,s} < 0$: The two systems will move in a different direction. Practically the Chaser will change its direction of motion, and the Target will move towards the initial direction of the Chaser. This would prevent docking.

The previous results show that the behaviour during impact depends on the ratio of the masses, and not on the masses per se. This is important both for the design of an approach strategy on orbit, but also for the design of the controller to be used.

3.3 More on the Coefficient of Effective Masses

A number of interesting cases for the Coefficient of Effective Masses are examined next. For this reason the ratios between the masses are defined as

$$\lambda_i = m_2 / m_3 , \quad \lambda_c = m_1 / m_2 , \quad \lambda_t = m_4 / m_3 \tag{23}$$

3.3.1 Equal Mass Ratios of Chaser and Target

If one assumes that the ratios, -not the absolute masses-, of the Chaser and the Target are equal that is

$$\lambda_c = \lambda_t = \lambda \tag{24}$$

so that Eq. (19) becomes,

$$e_I = 1 / \left(\lambda + 1\right) \tag{25}$$

then, using Eq. (15) and (18) one can find

$$U^+_{rel,s} = \left[\left(\lambda - e^*\right) / \left(\lambda + 1\right)\right] \cdot U^-_{rel,s} \tag{26}$$

The ratio λ can be positive only; therefore the numerator of Eq. (26) can be positive (and the systems will continue to move in the same direction because $U^+_{rel,s} \cdot U^-_{rel,s} > 0$) if and only if the ratio of the masses of the bodies is larger than the coefficient of restitution. Note however that $0 \le e^* \le 1$, therefore if $\lambda > 1$, then this situation is trivial. In other words Eq. (26) must be examined especially when the Target has larger m_2 than m_1. Finally one can easily see that as the ratio λ increases, that is $m_1 \gg m_2$, the coefficient e_I tends to zero, therefore the Chaser keeps its direction after impact and the relative velocity of the systems is decreased partly.

3.3.2 Impact of three masses

If the Chaser or the Target must be modelled with a single mass, then either $m_1 = 0$ or $m_4 = 0$, thus $\lambda_c = 0$ or $\lambda_t = 0$ correspondingly.

Let us examine the case in which the Target is modelled as a single mass. Using Eq. (19) one can find that

$$e_I = \frac{\left(\left(\lambda_c + 1\right) \cdot \lambda_i + 1\right)}{\left(\lambda_i + 1\right) \cdot \left(\lambda_c \cdot + 1\right)} \tag{27}$$

Plotting this function, **Figure 3**, it can be seen that there is tendency for the systems to change the direction of their relative velocity (as $e_I > 0.5$). This is reasonable if it is taken into account that the entire energy of the impact of the Target is received by a single mass only. Therefore the only case in which the systems retain their initial direction of relative velocity is when the Target is much larger than the mass under impact from the side of the Chaser.

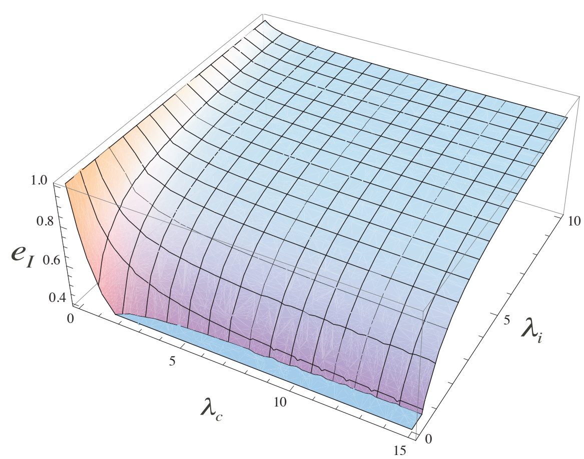

Figure 3. Impact of three masses, where the Target is only one mass.

On the other hand if the Chaser is a single mass, then Eq. (19) leads to

$$e_I = 1/(\lambda_i + 1) \quad (28)$$

Apparently Eq. (28) is similar to Eq. (25). Thus as the Chaser becomes larger, it tends to retain its initial velocity.

3.3.3 Impact with a mass connected to a fixed wall

Another extreme case is when the second mass of the Target a very large, such as if it is a fixed wall (e.g. ISS). Let $m_4 \to +\infty$. This time it is best to solve Eq. (19) again and take into account that

$$m_4 >> m_1, m_2, m_3 \quad (29)$$

resulting in,

$$e_I = \frac{\mu_{i,ef}}{m_{i,ef}} \simeq \frac{\mu_{i,ef}}{(m_1 + m_2)} = \frac{\mu_{i,ef}}{m_c} \quad (30)$$

By substituting the mass ratios it can be found that

$$e_I = 1/\left[(\lambda_i + 1)\cdot(\lambda_c + 1)\right] \quad (31)$$

In other words as the Chaser mass increases, the magnitude of the relative velocity is affected less. Plotting this function, it can be seen in **Figure 4** that after impact, the systems retain the initial direction of the relative velocity, except in cases where the Chaser has a larger mass under impact and/or the mass connected to the wall has about the same mass as the Chaser.

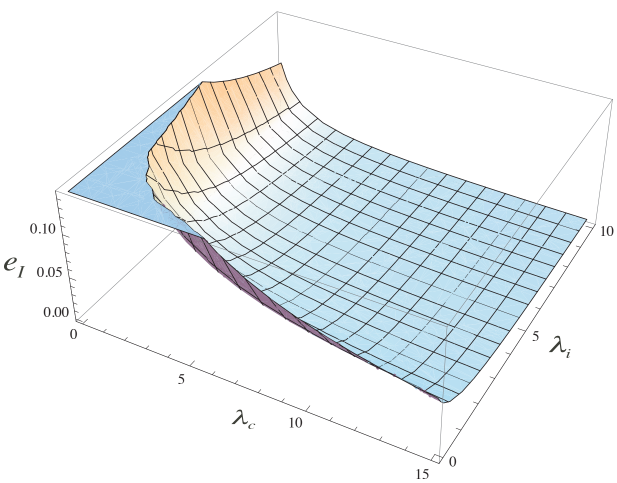

Figure 4. Target mass connected to a wall.

3.3.4 Zeroed Relative Velocity After Impact

In some cases, the relative velocity after impact can be zeroed ($e_I = 0.5$) which is a favourable situation. In fact if this can be achieved, then the Chaser and the Target will have zero relative velocity, which is ideal. Equating Eq. (19) with 0.5,

$$e_I = 0.5 \Rightarrow (\lambda_c\lambda_i + \lambda_i)(1 - \lambda_t) + \lambda_t - \lambda_c\lambda_t - \lambda_c + 1 = 0 \quad (32)$$

By substituting the ratios with a constant, the necessary equations are derived. For example, if $\lambda_c = c$ is known, then

$$\lambda_i \cdot (c+1)\cdot(1-\lambda_t) + (1-c)\cdot(\lambda_t + 1) = 0 \quad (33)$$

By plotting Eq. (33), see **Figure 5**, it can be seen that if the mass ratio of the Chaser is unity, then if the mass ratio of the target is also unity, it does not matter what is the mass ratio between the systems themselves. However in general, it is obvious that to find a relationship between the mass ratios that would zero the post impact relative velocity, the following must apply: if the mass ratio of the Chaser is larger than one, then the mass ratio of the Target should be less than one, and vice versa. Similar conclusions are derived for the cases $\lambda_i = c$ and $\lambda_t = c$.

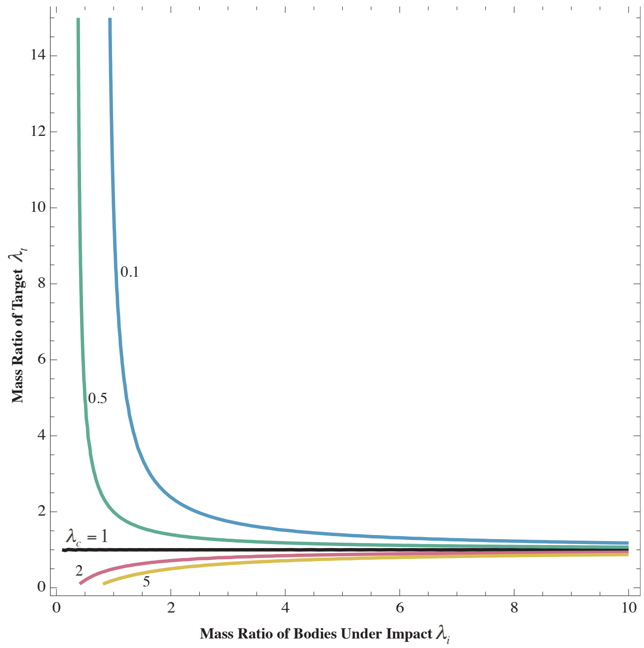

Figure 5. Mass ratio combinations which zero relative velocity after impact if the mass ratio of Chaser is known.

4 MINIMUM VELOCITY FOR IMPACT DOCKING

The objective is to examine the minimum velocity required during impact in order to perform latching during the impact docking. Although it is usual that the latching system is on the probe, due to fact that the analysis is performed between relative velocities, the results can be transferred.

In **Figure 6**, x_C is the position of the chaser, x_T the position of the Target, k is the spring constant of latching mechanism and θ is the angle of the probe according to the x-axis. It is assumed that the initial velocity of the Target at t=0 is $\dot{x}_{T,0} = 0$ and its initial position is $x_{T,0} = 0$ (without loss of generality) and the initial position of the Chaser is $x_{C,0} = -x_0$, $x_0 > 0$ where x_0 is the initial distance between the two bodies. While,

$$x_T + x_C < 0 \quad (34)$$

no impact occurs. In Eq. (34) x_T is the reference point of the Target along the x-axis, which is located at the end of the latching mechanism, and x_c is the reference point of the Chaser along the x-axis which is located at the end of the probe tip.

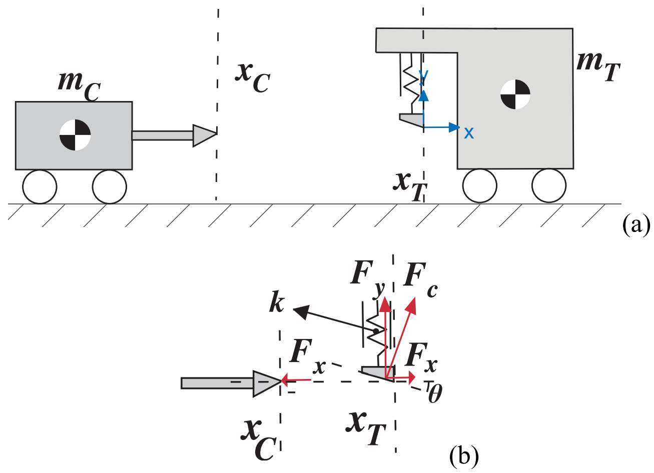

Figure 6. Simplified model for t=0.

When the two reference points are at the same position the probe starts to push the latching mechanism, and for the duration of contact the following apply: (i) the chaser begins to decelerate, (ii) the target begins to accelerate and, (iii) the spring begins to compress. Note that for this preliminary theoretical analysis the friction is disregarded.

The equations of motion for the two bodies are

$$-F_X = m_C \cdot \ddot{x}_C \tag{35}$$

$$F_X = m_T \cdot \ddot{x}_T \tag{36}$$

The compression of the spring, after the two bodies have the same position, is given by y_k

$$y_\kappa = (|x_C| - |x_T|) \cdot \tan\theta \tag{37}$$

Using Eqs. (35)-(37), and after some arithmetic manipulation, the force F_X becomes

$$F_X = k \cdot (x_C - x_T) \cdot \tan^2\theta \tag{38}$$

By subtracting Eq. (35) and (36) we can find the relative position of the two bodies.

$$\ddot{x}_C - \ddot{x}_T = -k \cdot \tan^2\theta \cdot (x_C - x_T) \cdot (1/m_C + 1/m_T) \Rightarrow \\ \Rightarrow \ddot{x}_C - \ddot{x}_T = -K \cdot (x_C - x_T)/m_{i,ef} \tag{39}$$

where K is

$$K = k \tan^2\theta \tag{40}$$

Note that in this case

$$\omega = \sqrt{K/m_{i,ef}} \tag{41}$$

Equation (39) is a differential equation, and its solution

$$x_c - x_t = (x_c - x_t)_0 \cdot \cos(\omega \cdot t) + \\ +((\dot{x}_c - \dot{x}_t)_0/\omega) \cdot \sin(\omega \cdot t) \tag{42}$$

Accordingly the relative velocity of the bodies is,

$$\dot{x}_c - \dot{x}_t = (\dot{x}_c - \dot{x}_t)_0 \cdot \cos(\omega \cdot t) - \\ -((\dot{x}_c - \dot{x}_t)_0 \cdot (x_c - x_t)_0/\omega) \cdot \sin(\omega \cdot t) \tag{43}$$

Making the assumption that the Chaser has such initial velocity that its probe will be advanced only by its length (the probe tip) in order to latch, the time when this will be achieved depends on the compression of the latching spring, thus it must hold

$$t_{latch} = \mathrm{T}/4 = \pi/2\omega = \sqrt{\pi^2 m_{i,eff}/4K} \tag{44}$$

If the latch time is less than $T/4$ then the probe will not push the latching mechanism enough in order for the following to apply:

$$\mathrm{x}_C - \mathrm{x}_T > l_p \tag{45}$$

where l_p is the width of the probe tip, a design parameter.

By using Eq. (43) and (44), the minimum velocity in order to have latching is

$$\dot{x}_{C,0,\min} = \omega(l_p - |\mathrm{x}_0|\cos(\omega t))/\sin(\omega\,\mathrm{t}) \tag{46}$$

5 SIMULATION RESULTS

5.1 RIGID MULTIBODY IMPACT DOCKING VERIFICATION

In order to verify the proposed RMI theory, a MATLAB/ Simulink model has been created. In order to test the validity of the propositions, the model was developed with a fully analytical approach. Thus each system (Chaser and Target) has been modelled as a 2-mass spring-damper system and the contact forces between the bodies under impact were calculated using the Kelvin-Voigt model. In particular the impact was modelled by a spring-damper system which can only be compressed. As the simulation advances, Simulink calculates the velocities of the masses under impact, and calculates their interpenetration. This interpenetration is fed back to the contact model and a force is developed which tries to push away the masses under impact. Therefore prior and after the impact the simulation presents two moving 2-body systems, and during impact a 4-body system. No equation stemming from the proposed RMI was used in order to avoid bias of results. Thus the validity of the proposed theory is examined via a complete visco-elastic theoretical formulation. The user can also change the initial parameters of the bodies, however, except the initial velocity of the Chaser (m_1 and m_2 have the same velocity, therefore the internal spring and damper of the Chaser is at their free length) and its initial position, all other values have been set to zero without loss of generality.

In order to verify the theoretical calculations of post-impact relative velocity between Chaser and Target, in relation to the pre-impact corresponding velocity, various configurations were examined: a) the masses of the robotic systems of the CSL emulator, b) A situation with all masses equal and c) and d) random masses. **Table 1** presents these values and the calculations according to Eq. (18). **Figure 7** shows the relative velocities of all cases. Only the first impact (which interests) is shown for each example. It can be seen that in all cases the theoretical model finds the post-impact relative velocity with high accuracy. Note that the stiffness has been selected low in order to have more clear plots; however with higher stiffness the results are the same, and the only difference is the duration of the impact. Only the relative magnitude of the system's stiffness with respect to the contact stiffness interests according to the assumptions. The damping here was zeroed.

Table 1. Data for the first set of simulations.

Properties	A	B	C	D
m_1 (kg)	17	10	5	100
m_2 (kg)	2	10	50	20
m_3 (kg)	1.5	10	10	10
m_4 (kg)	15	10	100	200
Contact Stiffness	1000	1000	1000	1000
Chaser Stiffness	15000	15000	15000	15000
Target Stiffness	200	200	200	200
Initial Rel. Velocity	0.05	0.05	0.05	0.05
Final Rel. Velocity (Eq. (18)) (m/s)	0.0403	0	0.02728	0.0413
Final Rel. Velocity (simulation) (m/s)	0.0402	-0.000476	0.02715	0.0412
Absolute Error (m/s)	0.0001	0.000476	0.00013	0.0001

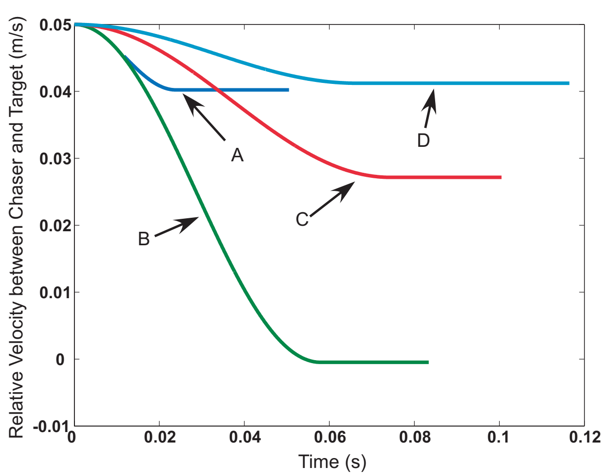

Figure 7. Relative velocities between Chaser and Target after first impact. Examples A-D.

5.2 Verification of Minimum Impact Velocity for Docking

In order to verify the proposed minimum velocity for impact docking, a Simulink model has been developed and various initial velocities have been tested. Here an example with the robots of the CSL Space emulator is presented, with $m_c = 15kg$, $m_t = 17kg$, $k = 100\,N/m$, $l_p = 0.02m$ and $\theta = 45^o$. By using Eq. (46), the minimum velocity for impact docking is 0.07m/s. In **Figure 8** an unsuccessful case is examined, where the initial relative velocity is 0.05m/s and this leads to the probe not to insert fully to the latching mechanism. In the contrary in **Figure 9** the initial velocity is 0.07m/s which leads to a successful docking.

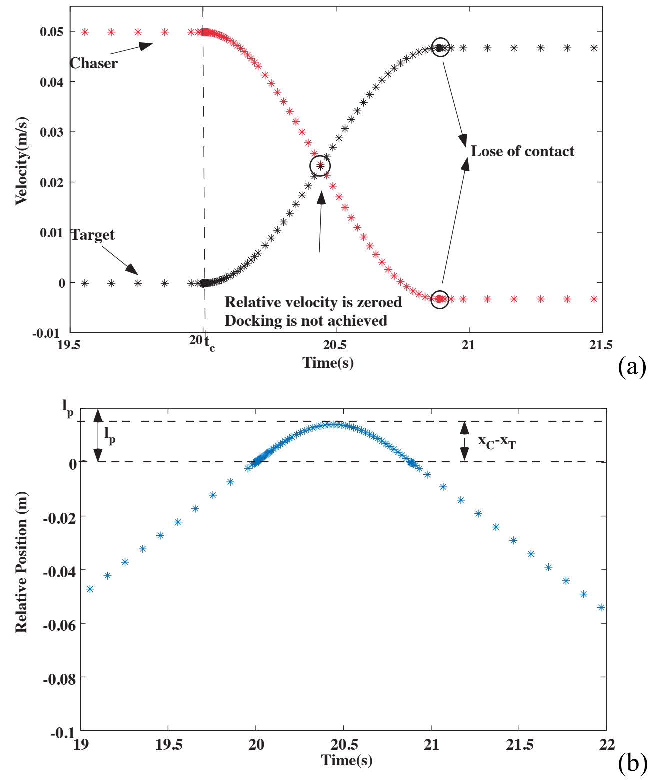

Figure 8. Docking Unsuccessful: (a) Velocities of the two bodies before and after impact and (b) Relative position of probe/latch mechanisms in contrast to the required for docking.

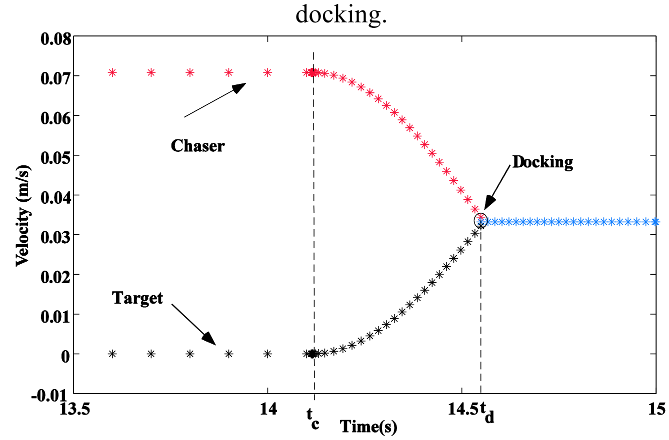

Figure 9. Docking Successful: Velocities of the two bodies before and after impact.

6 Conclusions and Future Work

In this work the impact docking between two multibody systems was examined. As the masses of the systems can be of the same level of magnitude, it was of interest to examine how this affects their behaviour during docking impacts. As it has been proven, it is the mass ratios and not their magnitude that will determine the post impact behaviour of the systems. The coefficient of effective masses was proposed which can help in the identification of the impact behaviour prior to impact. A number of interesting cases were examined. Accordingly the minimum impact velocity was determined in order to perform latch at first impact. For both situations simulations are presented which are in line with the theoretical approach. In the future the effect of friction during latching as well as the integration of the approaches will be examined. Additionally experiments on the air bearing space emulator of our laboratory will take place in order to establish the exact mechanism behind the effects of the various parameters during the impact docking.

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