

Large Force-task Planning for Mobile and Redundant Robots

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ABSTRACT

This paper analyzes the application of large force/torques by robotic systems with limited force/torque actuators. It is shown that such system may be able to apply a force/torque in some configurations only; therefore its useful force workspace is limited. To improve the force capabilities of a system, base mobility and/or redundancy can be employed. A planning algorithm is proposed which results in proper base positioning relative to a large-force quasi-static task. To plan redundant manipulator postures during large force-tasks, a new method based on a min-max optimization scheme is developed. Unlike norm-based methods, this method guarantees that no actuator capabilities are exceeded, and that the force/torque of the most loaded joint is minimized. Examples that demonstrate the validity and usefulness of the proposed methods are included.

I. INTRODUCTION

Animals and humans can develop and apply large forces, compared to their muscle force capabilities. During a task requiring application of large forces, a body position is assumed first, by moving to some location with respect to a task, and then some arm posture is assumed. The body position, or arm posture may change during the execution of the task, and both mobility and redundancy are utilized efficiently. On the other hand, robotic manipulators exhibit limited force/torque capabilities, even in static or quasi-static tasks. This issue becomes very important in mobile applications of robotic systems, where typically development of large forces is expected. In these applications, the position of a mobile system's base can be relocated with respect to a task, providing system redundancy. In cases where repetitious tasks are being planned, the robot can be positioned initially such that its posture is optimal for the given force-task. In space, highly redundant systems are being built to operate in a gravity-less environment. Efficient application of forces becomes very important because actuators are typically small due to lack of static loads, and to weight restrictions.

Actuator limitations have been considered in studies of time optimal motion planning, and in resolving manipulator redundancy in motion control [1,2]. The force distribution problem in multi-limb systems has been studied using linear programming techniques, and energy and load balancing performance indexes [3]. The necessary and sufficient conditions for applying a force to the environment were presented in [4]. Posture control in motion or force-tasks has been considered using a task compatibility measure [5]. The direction and magnitude of maximum force/torque that can be applied at some given end-effector location has been analyzed [6]. A configuration-space-based Force Workspace approach, originally proposed in [7], was used to plan motions of multi-limb systems without violating actuation and joint limits, or frictional constraints [7]. Redundancy resolution criteria were introduced based on desired motion or force-task requirements [8].

In this paper, we study the application of large force/torques by robotic systems with limited force/torque actuators. It is shown that the useful force workspace of such systems is limited since it may be able to apply a desired force in some configurations only. To improve a system's force capabilities, base mobility and/or redundancy are employed. A planning method is described that allows *object or base positioning* such that desired large forces can be applied without actuator saturation. For force-tasks that require application of forces *along a given path*, it is shown that there exists a workspace, called here the *task workspace*, with the property that if the end-effector's initial position is in it, the force-task at hand can be executed. To plan force-tasks for *redundant* manipulators, a new method based on a min-max optimization approach is employed, which guarantees that all joint torques will be within their bounds. This new method is compared to a typical norm-based method, and is found superior to it since the latter may yield planned postures at which individual actuators saturate. Finally, illustrative examples are presented.

II. FORCE WORKSPACE

In this paper we focus on tasks which require the application of large forces or torques on the environment. For example, such tasks include holding or lifting large payloads, pushing heavy containers in warehouse operations, or removing of an Orbital Removable Unit (ORU) during some contingency operation in space. In such cases, inertial forces are relatively small or non-existent, and therefore such tasks can be considered as quasi-static or even static. Then, the equivalent torques/forces $\boldsymbol{\tau}$ required to apply a force/torque $\mathbf{F} = [\mathbf{f}^T, \mathbf{n}^T]^T$, where \mathbf{f} and \mathbf{n} are the force and moment applied by the end-effector to the environment, are given by

$$\boldsymbol{\tau} = \mathbf{J}(\mathbf{q})^T \mathbf{F} \quad (1)$$

where $\mathbf{J}(\mathbf{q})$ is the Jacobian of the manipulator, and \mathbf{q} its configuration. We assume here that the manipulator structure is such that the force \mathbf{F} can be applied in any desired direction. Since actuators are not ideal sources of force or torque, the forces/torques $\boldsymbol{\tau}$ are subject to constraints such as torque-speed characteristics. In static or quasi-static cases, these result in maximum actuator force/torque limits given by

$$|\boldsymbol{\tau}| \leq \boldsymbol{\tau}_{\max} \quad (2)$$

For quasi-static tasks, dynamic terms can be neglected. However, gravity terms affect maximum available actuator force/torques, so $\boldsymbol{\tau}_{\max}$ is a function of \mathbf{q} . As the magnitude of the applied force \mathbf{F} is increased, it is expected that one or more actuators will saturate, i.e. they will not be able to provide the force/torque required by the equivalent torques given by Eq. (1). If the magnitude of the force is further increased, then either the manipulator will have to change posture, have its base move to another position, or fail in its task, sometimes with disastrous consequences.

It is interesting to examine what are the possible configurations \mathbf{q} (postures) at which the end-effector can apply a force with given magnitude and direction. These configurations map to a set of Cartesian locations at which the end-effector can apply a given force \mathbf{F} . The union of all such locations gives rise to the *force workspace*. As the kinematic workspace is reduced by the existence of joint limits, the force workspace is affected by actuator limitations. If a desired force \mathbf{F} is relatively small, then none of the joint actuators will have any problem generating the necessary force/torque. In such case, the force workspace is identical to the kinematic workspace. However, as \mathbf{F} is increased, this workspace will reduce gradually and may even vanish, i.e. there may be no points in the kinematic workspace for which the end effector of the manipulator is able to apply \mathbf{F} . To demonstrate this notion, a simple two-link manipulator is studied. The link lengths are $l_1 = 1.4m$, $l_2 = 1.0m$, the torque limits $\tau_{1,\max} = 10Nm$, $\tau_{2,\max} = 6Nm$, the link masses are $m_1 = 0.30 kg$, and

$m_2 = 0.25 kg$, and the vector of gravity is $g = [0 \ -9.81]^T m/s^2$. As depicted in Fig. 1a, when the force \mathbf{F} is very small, (here 1 N), the end-effector can apply it along the x axis at any point in its reachable workspace. However, when the force magnitude increases to 8 N, the available force workspace is reduced significantly. As shown in Fig. 1a, the force workspace is delimited by two straight lines parallel to the force, at a distance $|\tau_{1,\max}/F|$ from the base of the manipulator. These boundaries are due to the saturation of the first joint. For simplicity, the second joint is restricted to positive angles. If the actuators must support the weight of the manipulator links, then the force workspace is reduced further as shown in Fig. 1b.

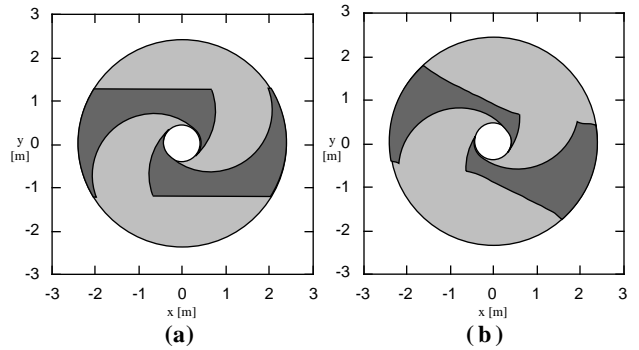


Fig. 1. Workspace regions where force \mathbf{F} can be applied. Small \mathbf{F} (light gray), Large \mathbf{F} (dark gray). (a) Without gravity. (b) With gravity.

Finding this workspace for a system as a function of the end-effector force allows planning of force-tasks. For example, a large force can be applied to an object if this is located in the corresponding force workspace. If it is not, then this workspace suggests locations at which the object must be moved prior of execution of the force-task.

The force workspace for a given system and some \mathbf{F} can be obtained analytically in simple cases, but in general requires sophisticated search techniques. The searching can be minimized by noting that at the boundaries of the force workspace region, one or more actuators will saturate, i.e. $|\tau_i| = \tau_{i,\max}$. Another way of reducing the amount of calculations required is proposed in [7]. However, the force workspace in [7] is defined as a subset of the multi-dimensional configuration space. In contrast to this, the force workspace defined in this work is a subset of the Cartesian space. Since the mapping from configuration space to Cartesian space is not always one-to-one, each Cartesian location must be validated by examining all possible configurations that correspond to a given end-effector position, to find at least one valid configuration; i.e. one at which none of the actuators saturate. The search for valid configurations can be accelerated using optimization techniques based on simulated annealing [9].

In addition to investigating the feasibility of applying a force, or finding appropriate locations for an object on which the end-effector is to operate, the notion of the force workspace can be used also in positioning the base of a mobile system relative to an immobile task. If we let \mathbf{p}_b be the position of the base, and \mathbf{p}_E be the position of the end effector, then

$$\mathbf{p}_E = \mathbf{p}_b + \mathbf{p}_W \quad (3)$$

where \mathbf{p}_W locates points of the force workspace with respect to the manipulator's base for a given force/torque \mathbf{F} . Eq. (3) can be rewritten as

$$\mathbf{p}_b = \mathbf{p}_E - \mathbf{p}_W \quad (4)$$

Hence, knowing \mathbf{F} and the location at which it must be applied, both \mathbf{p}_E and \mathbf{p}_W are known, and therefore Eq. (4) can be used to compute feasible positions for the base of the manipulator. Note that we implicitly assumed that manipulator actuator limits are more severe than those of the base. This is typically the case in mobile robotic applications.

As an example, the manipulator introduced previously is used, and it is assumed that a force \mathbf{F} must be applied at point $(-2, 1)$. Fig. 2a shows the force workspace of this manipulator. Clearly, point $(-2, 1)$ is part of the workspace when the base is at $(0, 0)$. Now, to determine feasible manipulator base locations, Eq. (4) is applied to all the points of the force workspace shown in Fig. 2a. Fig. 2b displays the result; placing the base at any location in the shown feasible area, guarantees that the end-effector will be able to apply force \mathbf{F} at point $(-2, 1)$ without saturating its actuators. As expected, point $(0, 0)$ is part of the feasible area shown in Fig. 2b.

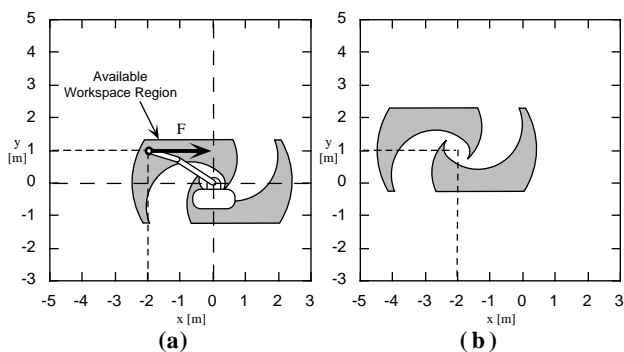


Fig. 2. (a) Workspace regions where force \mathbf{F} can be applied, (b) Feasible manipulator base positions from which \mathbf{F} can be applied at coordinates $(-2, 1)$.

Note that the force workspace changes, as the required force/torque magnitude, or direction changes. If the end effector must apply a force beyond its limits, then either the base of the manipulator must relocate, or the task must be moved with respect to the base of the manipulator. The first

case applies to mobile systems, whereas the second becomes important in repetitive operations. If any of these possibilities does not apply, the particular task becomes unfeasible.

III. TASK WORKSPACE

The above analysis focused on the application of some end-effector force/torque at some particular location. However, there are tasks for which forces/torques must be applied along some given path. In such cases, a simple force workspace is not sufficient to determine if the manipulator can perform the entire task. If the task is simple, such as applying a constant force in a constant direction, then the force workspace remains constant, and the task is feasible if the path starting at some initial location is contained within the force workspace; i.e. placing the end effector at the start location guarantees that the manipulator can accomplish the task without exceeding its actuator constraints. We call the collection of all possible initial locations from which we can perform the task of applying some force/torque along a given path the *task workspace*. According to its definition, the task workspace is a subset of the force workspace.

To illustrate this concept, the force and task workspaces are compared in Fig. 3. The light gray area in this figure corresponds to the force workspace of a two link manipulator with $l_1 = 1.0\text{ m}$, $l_2 = 0.9\text{ m}$ and $\tau_{1,\max} = 10\text{ Nm}$, $\tau_{2,\max} = 6\text{ Nm}$ applying a force of 12 N at 0° . When the force-task is to apply the same force along a straight line of 0.5 m in the negative x direction, then the force workspace is reduced to the dark gray task workspace, see Fig. 3.

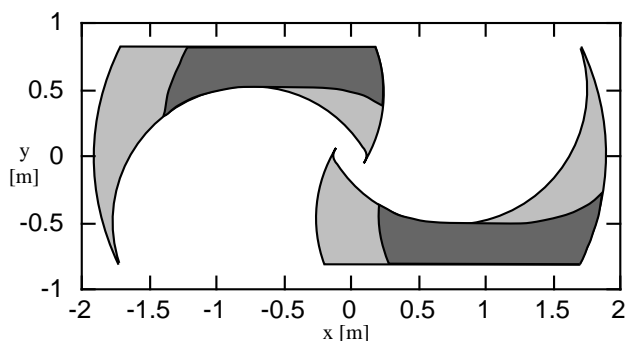


Fig. 3. Comparison of the force workspace (light gray) and the task workspace (dark gray) for a two link manipulator.

In general a force-task path is arbitrary, and the direction and magnitude of \mathbf{F} changes along the path, so the task workspace is not easy to determine. To obtain an accurate task workspace, each point of the initial force workspace must be validated for the entire path. The joint torque histories from the starting point, and over the path must never exceed their respective actuator limits. Using Eq. (1), the joint torques can be found and compared to the

maximum value (Eq. (2)) as the manipulator performs the task. All the points for which none of the actuators reach their limit remain as part of the task workspace, while the rest are discarded. Again, the quadtree (for 2D) or the octree (for 3D) method can be implemented to decompose the workspace in a more efficient manner [7,10]. The test function in this case must validate each starting point over the entire task before admitting it as part of the task workspace. The area included by these starting points is the task workspace. To determine the possible base positions that will allow a manipulator to perform the task at a specific location, say (x_o, y_o) in world coordinates, Eq. (4) is used as in the case of the force workspace.

As an example, consider a two link manipulator mounted on a mobile base. The manipulator parameters are as above. The task consists of applying a constant force of 7 N directed towards the center of a quarter circle of radius 0.5 m, see Fig. 4a. Fig. 4b shows possible base positions that allow the manipulator to perform a task starting at $(x_o, y_o) = (1.0, 1.0)$. Next we consider the use of redundancy in applying large forces/torques, and we find optimal manipulator configurations that minimize actuator efforts.

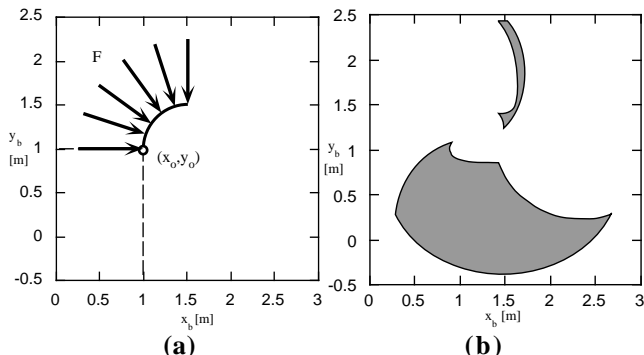


Fig. 4. Task Workspace for a two link manipulator. (a) Task. (b) Task Workspace.

IV. CONFIGURATION PLANNING FOR REDUNDANT MANIPULATORS

Optimal Configurations. A redundant manipulator has more degrees of freedom than the task requires, allowing one to choose additional conditions, such as energy minimization, or manipulability maximization. A comprehensive review of related criteria can be found in [8]. To apply these, some manipulator performance index is defined first, and then optimized to result in planning configurations and trajectories. However, these criteria affect the manipulator in an overall manner; i.e. they do not consider individual joint limitations. For example, optimizing the mechanical advantage, does not guarantee that *individual* joint actuator limits will not be exceeded. To avoid such problems, it is essential to determine first the configuration range at which

a given force can be applied, if any, and then consider criteria to be optimized within that range.

To apply this concept, assume that the actuators of a redundant manipulator are subject to limits as specified by Eq. (2). To account for differences in size and type of actuators, torque/force output is normalized with respect to the maximum available output. To avoid saturation, it is required that all actuator normalized forces/torques do not exceed 1 in absolute value

$$\left| \frac{\tau_i}{\tau_{i,\max}} \right| \leq 1 \quad (6)$$

Note that due to Eq. (1), the value of the above ratio is a function of the configuration \mathbf{q} of the manipulator. Using Eq. (6), and assuming some configuration \mathbf{q} , we can determine how much a particular joint actuator is loaded, and in addition, by computing this ratio for all joints, which actuator is loaded the most. The closer the normalized torque for some joint is to 1, the worse it is for this joint.

The next question we address is what the optimization criterion should be to guarantee that the force can be applied without violating actuator limitations. To this end, we first define the optimal configuration as the one at which all actuators ‘suffer’ the least, i.e., the normalized torques are all as away as possible from 1. Mathematically, this criterion can be set as a min-max problem defined as follows: choose the optimal configuration \mathbf{q} such that it satisfies

$$U = \min_{\mathbf{q}_r} \max_i \left| \frac{\tau_i(\mathbf{q}_r)}{\tau_{i,\max}} \right| \quad (7)$$

where \mathbf{q}_r is a set of redundant joint variables. To apply this criterion, one needs to find the maximum of the normalized torques as a function of \mathbf{q}_r and then find its minimum. This operation guarantees that all joints will have a load less or equal to U , and results in an optimum set of \mathbf{q}_r . The other joint variables are found using standard inverse kinematic equations. Usually, optimal configurations result when two actuators are equally loaded (share the load equally). This observation can speed up the optimization, since root finding algorithms like Newton’s method can be used to find the optimal \mathbf{q}_r ; this is also useful in planning manipulator postures during a task.

Force-task Planning. When a manipulator performs a task, such as pushing a heavy container, or removing an ORU, it must apply a force/torque along a path, S , parameterized by some variable s , e.g. its length. Here again, we require that the actuator limits at the joints must not be exceeded. To solve this problem, the min-max optimization is performed as a function of the path parameter s . The optimal \mathbf{q} is computed for the initial point on the path, where $s = 0$, and the procedure is repeated by increasing s , till the end of the path, where $s = 1$. It can be

shown that the resulting optimal configuration \mathbf{q} are piecewise continuous. However, there are path points at which the manipulator must change its posture while keeping its end-effector at the same location [11]. This can be explained as follows. Assume that actuator A is the most loaded one during motion along some path segment. At a switching point, actuator B becomes equally loaded to A, i.e. their normalized torques are equal. If for further motion of the end-effector along the path actuator B is the most loaded one, then Eq. (7) results in a different set of optimal configurations, and therefore in a discontinuity in \mathbf{q} . Note that during switching the end-effector may not be able to apply the force. This is analogous to trying to move large objects. Sometimes, one must stop pushing, reposition oneself, and continue pushing again.

If switching configurations is undesirable or time consuming, it can be reduced by following suboptimal configurations, i.e. ones for which the normalized torques are below 1, but not necessarily minimum. However, in some cases switching cannot be avoided. Next, the min-max optimization along a path is illustrated using an example, and compared to a norm-based method.

V. EXAMPLE

The planning method presented in Section IV is next applied to a three link manipulator, whose parameters are given in Table I. The force-task consists of applying a force of 8 N at 0° along s - a straight line connecting points A and B with coordinates $(x_A, y_A) = (0.3, -0.6)\text{m}$ and $(x_B, y_B) = (2.3, -0.6)\text{m}$, i.e. a motion parallel to the x -axis. For this system, Eq. (7) becomes

$$U = \min_{q_1} \max_{i=1,2,3} \left| \frac{\tau_i(q_1)}{\tau_{i,\max}} \right| \quad (8)$$

The angle q_1 is chosen as the redundant configuration variable. However, any other angle could have been used.

Table I. Manipulator parameters

l_1 (m)	l_2 (m)	l_3 (m)	$\tau_{1,\max}$ (Nm)	$\tau_{2,\max}$ (Nm)	$\tau_{3,\max}$ (Nm)
1.4	1.0	0.6	10.0	5.0	3.0

Fig. 5 depicts the result of the min-max optimization in the form of the normalized torque as a function of the path parameter s , $U(s)$. Fig. 6 displays the corresponding solution for q_1 . Discontinuities in this figure correspond to switchings from one configuration to another. To minimize the number of switchings, one can use a suboptimal solution as shown in Fig. 5. This suboptimal solution requires a single switching at $s = 0.7$. Fig. 7 displays the actual evolution of configurations along path S .

Next, the proposed method is compared to a frequently used method which resolves the redundancy by minimizing a sum of weighted actuator squared torques [8]

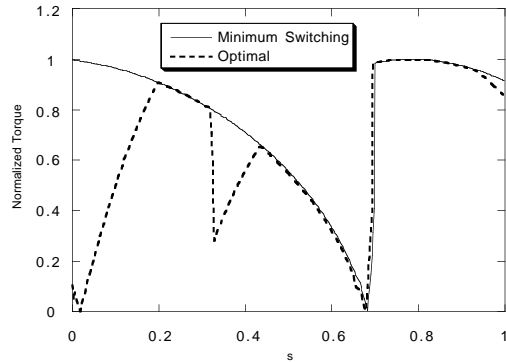


Fig. 5. Optimal normalized torque vs. path parameter s .

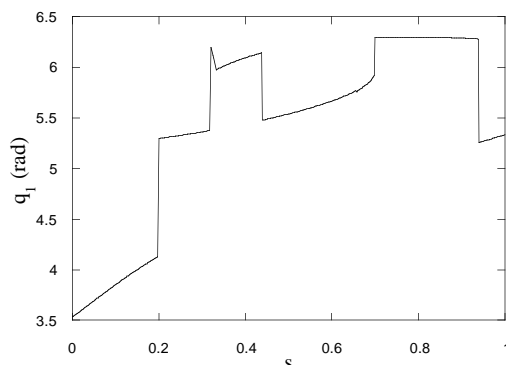


Fig. 6. Optimal q_1 -configuration vs. path parameter s .

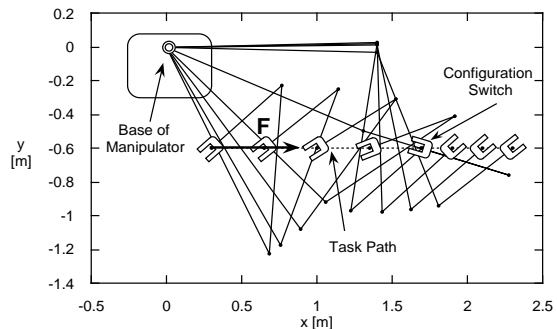


Fig. 7. Configurations that correspond to a minimum number of switchings planning.

$$q_1 \text{ satisfies } \min \{ w_1 \tau_1^2(q_1) + w_2 \tau_2^2(q_1) + w_3 \tau_3^2(q_1) \} \quad (9)$$

Dimensional weights are needed especially in the case where prismatic and rotary actuators are used, so that the criterion is uniform in terms of units. Note that the min-max method proposed here does not suffer from this limitation. Also note that the criterion in Eq. (9) does not guarantee no actuators will saturate. Instead, it guarantees that the ‘power’ required will be minimal.

To compare the solution obtained previously using the min-max criterion (Eq. (8)) to that of Eq. (9), we apply the latter using unit weights. In such case, Eq. (9) roughly minimizes total power consumption. The resulting normalized torques in this case are shown in Fig. 8, while the corresponding q_1 -configurations are shown in Fig. 9. Fig. 9 reveals that for a substantial part of the task, the required torque τ_3 exceeds the maximum available. Therefore, in the case where the desired end-effector force is large, such a method fails to yield feasible posture planning. It should be noted that changing the weights in Eq. (9) will not alleviate this problem; increasing the weight on joint 3 will result in lower torques for it, but then other joints will saturate.

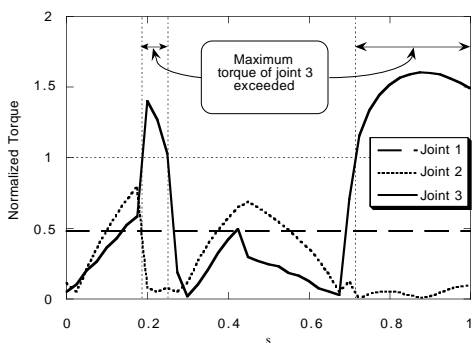


Fig. 8. Normalized torques using Eq. (9).

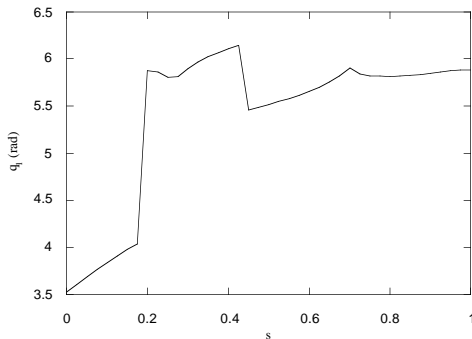


Fig. 9. Optimal angle q_1 using Eq. (9).

V. CONCLUSIONS

In this paper, the application of large force/torques by robotic systems with limited force/torque actuators was studied. It was shown that such system may be able to apply a force in some configurations only; therefore its useful force workspace is limited. For a given desired end-effector force, there exists a Cartesian workspace with the property that if the end-effector is in it, then the desired force/torque can be applied without violating actuator constraints. To improve the force capabilities of a system, base mobility and/or redundancy was employed. A planning algorithm was proposed to position the base relative to a

large-force quasi-static task. To plan redundant manipulator postures during large force-tasks, a new method based on a min-max optimization scheme was used. This method guarantees that no actuator capabilities are exceeded, and that the force/torque of the most loaded joint is minimized.

VI. ACKNOWLEDGMENTS

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