Proceedings of the 2006 IEEE International Conference on Robotics and Automation
Orlando, Florida - May 2006

Point-to-Point Planning: Methodologies for Underactuated Space Robots*

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Abstract **- In free-floating mode, space manipulator systems have their actuators turned off, and exhibit nonholonomic behavior due to angular momentum conservation. The system is underactuated and a challenging problem is to control both the location of the end effector and the attitude of the base, using manipulator actuators only. Physical limitations, imposed by system's dynamic parameters, are examined. Lower and upper bounds for base rotation, due to manipulator motions, are estimated. An analytical path planning method is briefly presented and appended by a technique which extends drastically the accessibility of final configurations and simplifies the free parameters selection. Based on this extension, a numerical approach is also derived and examples are given. The presented methodologies avoid the need for many small cyclical motions, and use smooth functions in the planning scheme, leading to smooth configuration changes in finite and prescribed time.**

Index Terms - Space free-floating robots, underactuated, nonholonomic planning

I. INTRODUCTION

Space robots will play an increasingly important role in space missions and on-orbit tasks. This is because these tasks are too risky or very costly (due to safety support systems) and many times just physically impossible to be executed by humans. Space robots consist of an on-orbit spacecraft fitted with one or more robotic manipulators. In *free-flying* mode, thruster jets can compensate for manipulator induced disturbances but their extensive use limits a system's useful life span. In many cases, as for example during capture operations, it is desired that the thrusters are turned off in order to avoid interaction with the target. In this case, the system operates in a *free–floating* mode, dynamic coupling between the manipulator and the spacecraft exists, and manipulator motions induce disturbances to the spacecraft. This mode of operation is feasible when no external forces or torques act on the system and the total system momentum is zero.

A free-floating space robot is an under-actuated system and exhibits a nonholonomic behavior due to the nonintegrability of the angular momentum, [1]. This property complicates the planning and control of such systems, which have been studied by a number of researchers. Vafa and Dubowsky have developed a technique called the Virtual Manipulator, [2]. Inspired by astronaut motions, they proposed a planning technique, which employs small cyclical manipulator joint motions to modify spacecraft attitude. Papadopoulos and Dubowsky studied the *Dynamic Singularities* of free-floating space manipulator systems, which are not found in terrestrial systems and depend on the dynamic properties of the system, [1, 3]. They also showed that any terrestrial control algorithm could be used to control end-point trajectories, despite spacecraft motions, [3]. Nakamura and Mukherjee explored Lyapunov techniques to achieve simultaneous control of spacecraft's attitude and its manipulator joints, [4]. To limit the effects of a certain null space, the authors proposed a *bidirectional* approach, in which two desired paths were planned, one starting from the initial configuration and going forward and the other starting from the final configuration and going backwards. The method is not immune to singularities and yields non-smooth trajectories with the joints coming to a stop at the switching point.

A method that allows for Cartesian motion of a manipulator's end point and avoiding dynamic singularities, has been proposed in [5]. The method involved small end-effector Cartesian cyclical motions designed to change the attitude of the spacecraft to one that was known of avoiding dynamic singularities, [5]. Caccavale and Siciliano used quaternions, to avoid representational singularities, and developed kinematic control schemes for a redundant manipulator mounted on a free-floating spacecraft, [6]. Franch et al. used flatness theory, to plan trajectories for free-floating systems, [7]. Their method requires selection of robot parameters so that the system is made controllable and linearizable by prolongations.

In this paper, two approaches, i.e. an analytical and a numerical one, are presented. The methods allow for endpoint Cartesian point-to-point control with simultaneous spacecraft attitude control, using manipulator actuators only. First, the physical limitations, imposed by system dynamic parameters, are examined. Lower and upper bounds for base rotation, due to manipulator motions, are estimated. Then, an analytical path planning method is briefly presented and appended by a technique which extends drastically the accessibility of final configurations and simplifies the estimation of free parameters, as mentioned in [11]. The key idea for this technique is the use of high order polynomials for the joint angles, as arguments in cosine functions. Based on this idea, a

* Support by the EPAN Cooperation Program 4.3.6.1.b (Greece-USA 035) of the Hellenic General Secretariat for Research and Technology and the NTUA Senate Committee of Basic Research Programme "Protagoras", R.C. No. 10, is acknowledged.

numerical approach is derived and examples are given. The presented methodologies avoid the many small cyclical motions required in the past, and use smooth functions such as polynomials in the planning scheme, leading to smooth configuration changes in finite and prescribed time.

II. Physical Constraints of Motion

Free-floating space manipulator systems consist of a spacecraft (base) and a manipulator mounted on it, as shown in Fig. 1. In this mode, the system consumes electric energy which is available and renewable through solar panels and not hydrazine which is scarce.

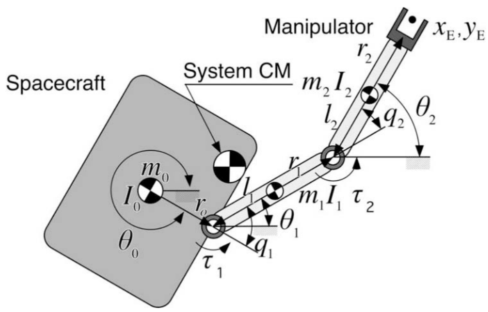

Fig. 1. A Free-Floating Space Manipulator System.

The manipulator has revolute joints and an open chain kinematic configuration, so that, in a system with N-degree-of-freedom (dof) manipulator, there are $N+6$ dof (under-actuated system). Since no external forces act on the system, and the initial momentum is zero, the system Center of Mass (CM) remains fixed in space, and the coordinates origin, O, can be chosen to be the system's CM. The equations of motion, have the form, [3],

$$\mathbf{H}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{C}(\mathbf{q},\dot{\mathbf{q}}) = \boldsymbol{\tau} \tag{1}$$

where $\mathbf{H}(\mathbf{q})$ is a positive definite symmetric matrix, called the reduced system inertia matrix, and $\mathbf{C}(\mathbf{q},\dot{\mathbf{q}})$ contains nonlinear velocity terms. The $N \times 1$ column vectors $\mathbf{q},\dot{\mathbf{q}}$ and $\boldsymbol{\tau}$ represent manipulator joint angles, velocities, and torques. The base attitude is computed using the conservation of angular momentum, [1],

$${}^{0}\boldsymbol{\omega}_{0} = -{}^{0}\mathbf{D}^{-1}\,{}^{0}\mathbf{D}_{\mathbf{q}}\dot{\mathbf{q}} \tag{2}$$

where ${}^{0}\boldsymbol{\omega}_{0}$ is the base angular velocity in the spacecraft 0^{th} frame, and ${}^{0}\mathbf{D}$, ${}^{0}\mathbf{D}_{\mathbf{q}}$ are inertia-type matrices.

For simplicity, we focus on a free-floating robotic system consisting of a two-dof manipulator mounted on a three-dof spacecraft. The spacecraft is constrained to move in the plane perpendicular to the axis of the manipulator rotation. For the planar free-floating space manipulator, the conservation of angular momentum equation can be written as,

$$D_0(\mathbf{q})\dot{\theta}_0 + D_1(\mathbf{q})\dot{\theta}_1 + D_2(\mathbf{q})\dot{\theta}_2 = 0 \tag{3}$$

where $\theta_0, \theta_1, \theta_2$ are spacecraft attitude and manipulator absolute joint angles. The D_0, D_1 and D_2 are functions of system inertial parameters and of the manipulator joint angles q_1 and q_2, see Fig. 1. The angular momentum, given by (3), cannot be integrated to analytically yield the spacecraft orientation θ_0 as a function of the system's configuration. This nonintegrability property introduces nonholonomic characteristics to free-floating systems, and results from the dynamic structure of the system; it is not due to kinematics, as is the case of nonholonomic constraints in wheeled systems.

If the joint angle trajectories are known as a function of time, then (3) can be integrated numerically to yield the trajectory for the base orientation. In this case though, a particular path is taken. Integration $(\theta_0^{in}, \theta_1^{in}, \theta_2^{in})$ along a different one will lead to a different final configuration.

The main problem we address here is to find a path which connects a given initial configuration to a final one $(\theta_0^{fin}, \theta_1^{fin}, \theta_2^{fin})$, by actuating the two manipulator joints only. The endpoint location x_E and y_E is given by, [1],

$$\begin{aligned} x_E &= a\cos\theta_0 + b\cos\theta_1 + c\cos\theta_2 \\ y_E &= a\sin\theta_0 + b\sin\theta_1 + c\sin\theta_2 \end{aligned} \tag{4}$$

where, a, b, c, are constant terms, functions of the mass properties of the system, [1]. It is expected that for a given change in manipulator joints, only a limited change in base attitude would occur, due to dynamic system's properties. In other words not all configurations are reachable from an initial one. For example, it is not rational to expect large base rotations with small manipulator displacements. Similarly, it may not be possible to move to any Cartesian point and have the final spacecraft attitude unchanged.

Using (3), the scleronomic constraint can be written in the Pfaffian form, as

$$D_0 d\theta_0 + D_1 d\theta_1 + D_2 d\theta_2 = 0 \tag{5}$$

$$\underbrace{(D_0 + D_1 + D_2)}_{D} d\theta_0 + (D_1 + D_2)dq_1 + D_2\, dq_2 = 0 \tag{6}$$

Next, we specify lower and upper bounds for base rotation $\Delta\theta_0$, caused by $(\Delta q_1, \Delta q_2)$. Using (6), we can write

$$d\theta_0 = -\frac{(D_1 + D_2)}{D}dq_1 - \frac{D_2}{D}dq_2 \triangleq g_1(\mathbf{q})dq_1 + g_2(\mathbf{q})dq_2 \tag{7}$$

where $g_1(\mathbf{q}), g_2(\mathbf{q})$ are defined by (7) and (A.1). Integrating (7) we have

$$\Delta\theta_0 = \int g_1(\mathbf{q})dq_1 + \int g_2(\mathbf{q})dq_2 \triangleq \Delta\theta_{01} + \Delta\theta_{02} \tag{8}$$

where $\Delta\theta_{01}, \Delta\theta_{02}$, represent the contribution of Δq_1 and Δq_2 in $\Delta\theta_0$ respectively. Functions $g_1(\mathbf{q}), g_2(\mathbf{q})$ are bounded, so

$$g_{1,\min} \le g_1(\mathbf{q}) \le g_{1,\max} \tag{9}$$

$$g_{2,\min} \le g_2(\mathbf{q}) \le g_{2,\max} \tag{10}$$

where the bounds in (9) and (10) are given by

$$\begin{aligned} g_{1,\min} &= \min_{\mathbf{q}} g_1(\mathbf{q}), \quad g_{1,\max} = \max_{\mathbf{q}} g_1(\mathbf{q}) \\ g_{2,\min} &= \min_{\mathbf{q}} g_2(\mathbf{q}), \quad g_{2,\max} = \max_{\mathbf{q}} g_2(\mathbf{q}) \end{aligned} \tag{11}$$

Resulting bounds for $\Delta\theta_{01}$ are estimated, by (8) and (9), as

$$\begin{cases} g_{1,\min}\Delta q_1 \le \Delta\theta_{01} \le g_{1,\max}\Delta q_1, & (\Delta q_1 > 0) \\ g_{1,\max}\Delta q_1 \le \Delta\theta_{01} \le g_{1,\min}\Delta q_1, & (\Delta q_1 < 0) \end{cases} \tag{12}$$

while bounds for $\Delta\theta_{02}$ are estimated, using (8) and (10), as

$$\begin{cases} g_{2,\min}\Delta q_2 \le \Delta\theta_{02} \le g_{2,\max}\Delta q_2, & (\Delta q_2 > 0) \\ g_{2,\max}\Delta q_2 \le \Delta\theta_{02} \le g_{2,\min}\Delta q_2, & (\Delta q_2 < 0) \end{cases} \tag{13}$$

Finally, bounds for $\Delta\theta_0$ are estimated by adding the corresponding terms of (12) and (13):

$$\Delta\theta_{0,\min} < \Delta\theta_0 < \Delta\theta_{0,\max} \tag{14}$$

We note here that absolute maximum base rotation is achieved, as expected, when the arm is fully extended $(q_2 = 0^o)$, while absolute minimum base rotation occurs when the arm is retracted $(q_2 = 180^o)$. The bounds presented in this section are valid in the sense that there is no path connecting a given initial with a desired (final) configuration, leading to a $\Delta\theta_0$ out of the limits given above.

III. Nonholonomic Path Planning

A. An Analytical Approach

A derived analytical approach to the planning problem, is based on a transformation of the constraint (5), to one containing two differentials. This is in principle possible and requires appropriate functions u, v, w, of $\theta_0, \theta_1, \theta_2$, for which (5) can be transformed to [8-10],

$$du + v dw = 0 \tag{15}$$

Then, we choose functions $w = w(t)$, $u = u(w)$ and setting

$$v(w) = -u'(w) = -(du/dw) \tag{16}$$

equation (15) is satisfied identically. The method was applied in [11] to a free-floating space robot, with the manipulator mounted on the spacecraft's CM $(r_0 = 0)$. This case leads to a more compact transformation and eliminates Dynamic Singularities from the workspace, [1, 3]. The coefficients of the nonholonomic constraint (5) and the transformation used are given in Appendix A (for $r_0 = 0$) and Appendix B, respectively. Initially, a fifth order polynomial in t was used for $w = w(t) = \theta_1(t)$, with boundary conditions $(\theta_1^{in}, \theta_1^{fin})$ and zero initial and final velocities and accelerations. The choice for $u = u(w)$ was a fourth order polynomial of w, which could satisfy four constraints $(u, v)^{in,fin}$, representing initial and final system configurations, and leaving a free parameter to satisfy additional inequality constraints, implied by the transformation used. The inverse transform, is defined iff

$$\left|h(v)\right| < 1 \tag{17}$$

where $h(v) = (v - \alpha_1)/(2\alpha_3) = \cos(q_2)$, see (B.2). The problem was reduced in determining the free parameter, i.e. shaping the path in (u, v, w) space, so that (17) is satisfied for all $w \in [w_{in}, w_{fin}]$, for details see [11].

Selection of (free) parameters which satisfy (17), is a task which requires the satisfaction of at least four inequalities of two variables. In order to automate and simplify the estimation of free parameters, and to extend the accessibility of final configurations, the method is improved here using a new definition of v, as follows.

Pivot function $w = w(t)$ is specified as already mentioned and varies monotonically in $w \in [w_{in}, w_{fin}]$. We assume that q_2 is given by a kth order polynomial of w, i.e.

$$q_2 = p_k(w, \mathbf{b}) = b_k w^k + \cdots + b_1 w + b_0 \tag{18}$$

where $\mathbf{b} = [b_k \ \cdots \ b_0]^T$ the $(k+1)$ coefficients to be specified. Using (18) and (B.1), $v(w)$ can be written as

$$v(w) = \alpha_1 + 2\alpha_3\cos(q_2) = \alpha_1 + 2\alpha_3\cos(p_k(w, \mathbf{b})) \tag{19}$$

Since the path in (u, v, w) space should satisfy (16), we have

$$u(w) = u_{in} - \int_{w_{in}}^{w} v(w)dw \tag{20}$$

The problem reduces to determining the unknown vector $\mathbf{b}$, so that (18) satisfies three equality constraints, i.e. two resulting from the requirement to satisfy the initial and final values of q_2, and one from the final value of (20):

$$I_1(\mathbf{b}) \triangleq \int_{w_{in}}^{w_{fin}} v(w, \mathbf{b})dw = u_{in} - u_{fin} \triangleq u_{des} \tag{21}$$

To satisfy the three necessary constraints, it should be $k \ge 2$. Using (19) for (v_{in}, v_{fin}), two of the $(k+1)$ parameters to be determined are expressed as functions of the remaining $n_f = (k-1)$ free parameters. In the general case, where $k > 2$, vector $\mathbf{b}$ (containing free parameters) can be specified using optimization techniques, with the desired equality constraint (21), as shown

$$\left\|\mathbf{b}\right\| \to \min : I_1(\mathbf{b}) = \int_{w_{in}}^{w_{fin}} v(w, \mathbf{b})dw = u_{des} \tag{22}$$

For the special case where $k=2$, there is only one free parameter which can be calculated solving (21) analytically or numerically (e.g. using Newton Raphson method). If no solution exists for $k=2$, one can gradually increase k and try to specify $\mathbf{b}$, using (22), as previously mentioned.

It was found that this approach leads always to a path, provided that the desired change in configuration lies between physically permissible limits, as specified in Section II. The main advantages of this modification follows: (a) Using cosine of polynomials, in the new definition of $v(w)$, leads to a wider range of shapes for $v(w)$ which in turn improves the accessibility of the final configurations, (b) it is possible to obtain paths leading to almost maximum permissible $\left|\Delta\theta_0\right|$, in *one* point-to-point motion, (c) determination of $\mathbf{b}$ is automated, (d) the inverse transform is defined for all $v(w)$, since (17) is always true, (e) the order k can be increased, to meet additional requirements, such as joint limits or obstacles, and finally (f) the presented improvement is still valid, if we increase the order of $w(t)$ and have the additional free parameters specified by a similar manner. In this way $w(t)=\theta_1(t)$ should not be necessarily monotonic in $[w_{in}, w_{fin}]$. With these in mind, a derived numerical approach is developed next.

B. A Numerical Approach

Here, we use high order polynomials for $\mathbf{q}(t)$, in order to specify a trajectory directly in joint-space. Let

$$\begin{aligned} q_1 &= q_1(t,\mathbf{b}_1), \quad \mathbf{b}_1\ ((k_1+1)\times 1) \\ q_2 &= q_2(t,\mathbf{b}_2), \quad \mathbf{b}_2\ ((k_2+1)\times 1) \end{aligned} \tag{23}$$

be polynomials of t, of order k_1, k_2 respectively, and $\mathbf{b}_1, \mathbf{b}_2$ the corresponding coefficients to be specified, i.e. the free parameters $n_f=(k_1+k_2+2)$. The minimum number of constraints (n_c) to be satisfied are: 6-per-joint (by desired initial and final positions, and zero initial and final velocities and accelerations), plus at least one for the integral of motion, i.e. $n_c \geq 13$. Since it should be $n_f \geq n_c$, we have

$$(k_1+k_2+2)\geq 13 \Leftrightarrow (k_1+k_2)\geq 11 \tag{24}$$

Using (7), we obtain the following equation

$$\Delta\theta_0^{des} = \int_{q_1^{in}}^{q_1^{fin}} g_1 dq_1 + \int_{q_2^{in}}^{q_2^{fin}} g_2 dq_2 = \int_{t_{in}}^{t_{fin}} \underbrace{(g_1\dot{q}_1 + g_2\dot{q}_2)}_{g(t,\mathbf{b}_1,\mathbf{b}_2)}\, dt \triangleq I_2(\mathbf{b}) \tag{25}$$

which represents the integral constraint to be satisfied. Here $\mathbf{b}=[\mathbf{b}_1^T\mathbf{b}_2^T]^T$ and can be specified numerically, either by solving the nonlinear equation (25), if there is only one free parameter to be estimated, or by optimization techniques, as shown in (26). One could introduce additional freedom, increasing the order of the polynomial for the first joint, the second joint or both, to meet additional requirements, if necessary.

$$\left\|\mathbf{b}\right\| \rightarrow \min : I_2(\mathbf{b}) = \int_{t_{in}}^{t_{fin}} (g_1\dot{q}_1 + g_2\dot{q}_2)\, dt = \Delta\theta_0^{des} \tag{26}$$

After joint-space trajectories are specified, $\theta_0(t)$ is given by

$$\theta_0(t) = \theta_0^{in} + \int_{t_{in}}^{t} (g_1\dot{q}_1 + g_2\dot{q}_2)\, dt \tag{27}$$

All the other configuration variables can be calculated using (23) and (27). We should note here that the presented approach can be easily extended to the 3-D case, with a 6-dof manipulator. Once $\mathbf{b}$ is estimated, initial and final values for base attitude (θ_0) are satisfied. Also initial and final velocities and accelerations of θ_0 are zero, since the same constraints are imposed to joint variables $\mathbf{q}(t)$ and the fact that $\theta_0(t)$ satisfies (7), where $D(\mathbf{q})\neq 0$ for all $\mathbf{q}$. Finally, Dynamic Singularities in the workspace are avoided since the path is defined directly in the joint space.

IV. Application Examples

In the following examples, the free-floating space manipulator shown in Fig. 1 is employed. The system parameters used are shown in Table 1. For this system, a, b, and c, in (4), are given by the following equations

$$\begin{aligned} a &= r_0 m_0 (m_0+m_1+m_2)^{-1} = 0.43m \\ b &= (r_1(m_0+m_1)+l_1 m_0)(m_0+m_1+m_2)^{-1} = 0.89m \\ c &= (l_2(m_0+m_1))(m_0+m_1+m_2)^{-1} + r_2 = 0.97m \end{aligned} \tag{28}$$

TABLE I. System Parameters

Body	$l_i\ [m]$	$r_i\ [m]$	$m_i\ [kg]$	$I_i\ [kgm^2]$
0	1.0	0.5	400.0	66.67
1	0.5	0.5	40.0	3.33
2	0.5	0.5	30.0	2.50

For the examples presented here, the duration of motion is chosen equal to 10 *s*. Increasing or decreasing this time has no effect on the path taken, but increases or decreases the torque requirements and the magnitude of velocities or accelerations.

A. New Configuration

We first assume that the manipulator is mounted at an arbitrary point of the base $(r_0\neq 0)$ and the reachable workspace is computed to be a hollow disk with an external radius equal to $R_{\max,2}=a+b+c=2.29m$ and an internal one $R_{\min,1}=a+b-c=0.35m$. The outer ring of the Path Dependent Workspace (PDW), i.e. the sub-workspace in which Dynamic Singularities may occur, is defined by $R_{\min,2}=b+c-a=1.44m$ and $R_{\max,2}$, [1]. The free-floater has to move its manipulator endpoint to a new location and at the same time change its spacecraft attitude to a desired one. The initial system configuration is $(\theta_0, x_E, y_E)^{in} = (-50^o, 1.53m, 0.96m)$ and the final one is $(\theta_0, x_E, y_E)^{fin} = (0^o, 1.71m, -0.29m)$. The equivalent change in q_1, q_2 is $(\Delta q_1, \Delta q_2) = (-140.0, 60.0)^o$ and bounds for base rotation, calculated using (11)-(14), are given by $\Delta\theta_0 \in (1.4, 72.2)^o$. Here, $\Delta\theta_0^{des} = 20^o$, which lies between the permissible limits, so it is expected that there is at least one path connecting the given (initial) with the desired (final) configurations. The numerical approach, presented in Section III.B., is employed here to specify the desired path. We assigned a sixth and a fifth order polynomial of t, for q_1, q_2 respectively, i.e. we assumed initially that we have $k_1 = 6$ and $k_2 = 5$, see (23). The parameters were calculated using initial and final positions of joints, with zero initial and final velocities and accelerations and solving numerically the nonlinear equation, given by (25). Fig. 2 depicts snapshots of the free-floater motion as it changes its configuration, while Fig. 3 shows system trajectories.

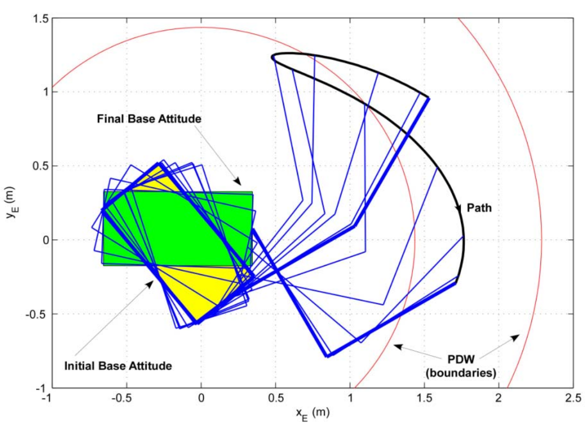

Fig. 2. Snapshots of a free-floater moving to a desired θ_0, x_E, y_E.

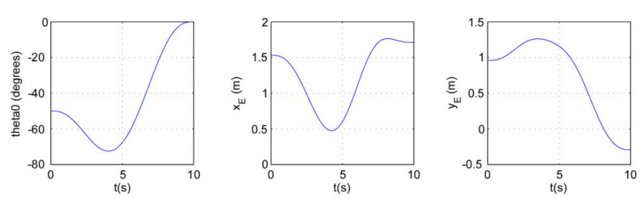

Fig. 3. Configuration variables that correspond to snapshots in Fig. 2.

As shown in Fig. 3, the desired configuration is reached in the specified time. Also, all trajectories are smooth throughout the motion, and the system starts and stops smoothly at zero velocities, as expected. This is an important characteristic of the method employed and is due to the use of smooth functions, such as polynomials. The corresponding joint torques are given in Fig. 4. These torques are computed using (1) and the elements of the reduced inertia matrix, given in [1]. As shown in Fig. 4, the required torques are small and smooth while they can be made arbitrarily small, if the duration of the maneuver is increased. The implication of this fact is that joint motors can apply such torques with ease and therefore the resulting configuration maneuver is feasible.

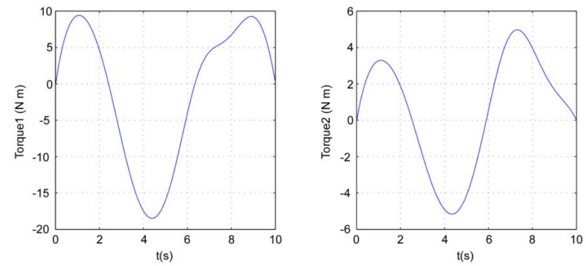

Fig. 4. Manipulator torques required for the motion shown in Fig. 2.

B. *New End-Point Position*

In this case it is assumed that the manipulator is mounted on spacecraft's CM, i.e. $r_0 = 0$. The reachable workspace is computed to be a hollow disk with an external radius equal to $R_{max} = 1.86m$ and an internal one $R_{min} = 0.08m$. The manipulator's end-point is desired to move to a new location, while at the end of the motion the base must be at its initial attitude, i.e. $\theta_0^{fin} = \theta_0^{in}$. The initial system configuration is $(\theta_0, x_E, y_E)^{in} = (0^o, 1.7m, 0.8m)$ and the final is $(\theta_0, x_E, y_E)^{fin} = (0^o, 0.6m, -0.3m)$. The equivalent change in joint angles is $(\Delta q_1, \Delta q_2) = (30.0, -150.0)^o$ and calculated bounds for base rotation (using (11)-(14)) are given by $\Delta\theta_0 \in (-22.4, 17.2)^o$. Here we have $\Delta\theta_0^{des} = 0^o$, which lies within the permissible range. The improved analytical approach, presented in Section III.A., is used to specify the desired path. Using the new definition of $v(w)$, see (19), we obtained a solution for $k = 4$, i.e. assigning a fourth order polynomial of w, to the second joint. The parameters **b**, were calculated using (22). Fig. 5 shows snapshots of the free-floater motion, and smooth trajectories are shown in Fig. 6.

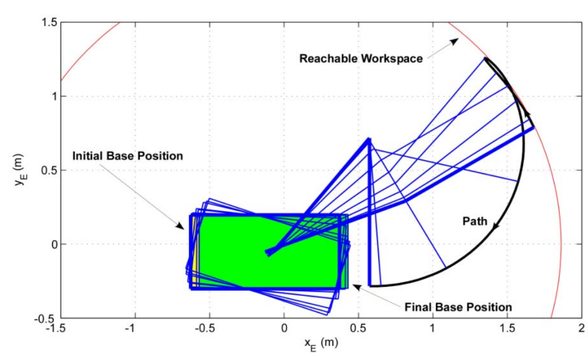

Fig. 5. Snapshots of a free-floater moving to a desired x_E, y_E.

The resulting path leads to the desired final configuration, in the specified time. The system starts and stops smoothly at zero velocities, as predicted. The required torques are also smooth and given in Fig. 7.

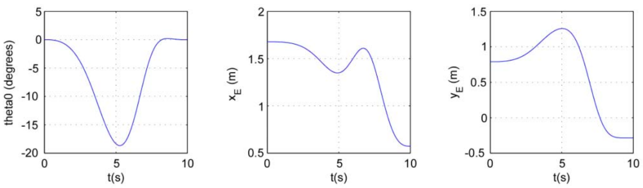

Fig. 6. Configuration variable trajectories that correspond to Fig. 5.

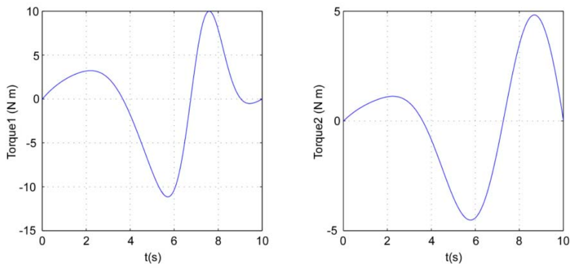

Fig. 7. Manipulator torques required for the motion shown in Fig. 5.

V. Conclusions

Point-to-point path planning methods were developed that allow endpoint Cartesian location control, and simultaneous control of the spacecraft's attitude, avoiding many small cyclical motions. Physical constraints, due to the dynamic parameters of the system, were given. The analytical method is based on a transformation of the angular momentum and is given along with an improvement, which uses high order polynomials as arguments of cosine functions and extends the accessibility of final configurations. Using optimization techniques determination of free parameters is simplified. A numerical approach is also developed, which specifies trajectories directly in joint space, and examples are given. Both methods use smooth functions such as polynomials in the planning schemes, leading to smooth configuration changes in finite and prescribed time.

Appendix A

The coefficients of (5) are given by

$$\begin{aligned} D_0(q_1,q_2) &= d_1 + d_2\cos(q_1) + d_3\cos(q_1+q_2) \\ D_1(q_1,q_2) &= d_4 + d_5\cos(q_1) + d_6\cos(q_2) \\ D_2(q_1,q_2) &= d_7 + d_8\cos(q_2) + d_9\cos(q_1+q_2) \end{aligned} \tag{A.1}$$

where $M = m_0 + m_1 + m_2$, the coefficients d_i are given by

$$\begin{aligned} d_1 &= \mathrm{I}_0 + m_0(m_1+m_2)r_0^2 / M \\ d_2 &= d_5 = m_0 r_0((m_1+m_2)l_1 + m_2 r_1) / M \\ d_3 &= d_9 = m_0 r_0 m_2 l_2 / M \\ d_4 &= \mathrm{I}_1 + (m_0 m_1 l_1^2 + m_1 m_2 r_1^2 + m_0 m_2 (l_1 + r_1)^2) / M \\ d_6 &= d_8 = m_2 l_2 (m_0(l_1+r_1) + m_1 r_1) / M \\ d_7 &= \mathrm{I}_2 + m_2(m_0+m_1) l_2^2 / M \end{aligned} \tag{A.2}$$

and all variables in (A.2) are defined in Fig. 1.

Appendix B

Assuming that the manipulator is mounted at the spacecraft CM $(r_0 = 0)$, and employing the procedure in [9], the following compact transformation is chosen among the possible ones,

$$\begin{aligned} u(\theta_0,\theta_1,\theta_2) &= \alpha_0\theta_0 + \alpha_2\theta_2 - \alpha_3\sin(\theta_1-\theta_2) \\ v(\theta_0,\theta_1,\theta_2) &= \alpha_1 + 2\alpha_3\cos(\theta_1-\theta_2) \\ w\big(\theta_0,\theta_1,\theta_2\big) &= \theta_1 \end{aligned} \tag{B.1}$$

The inverse transformation from (u, v, w) to $(\theta_0, \theta_1, \theta_2)$ is

$$\begin{aligned} \theta_1 &= w, \quad h = (v-\alpha_1)/(2\alpha_3) = \cos(q_2) \\ \theta_2 &= w + \cos^{-1}(h) \\ \theta_0 &= (u - \alpha_2 w - \alpha_2\cos^{-1}(h) - \alpha_3\sqrt{1-h^2}) / \alpha_0 \end{aligned} \tag{B.2}$$

where the coefficients α_i are expressed, using (A.2), as

$$\alpha_0 = d_1\big|_{r_0=0} = \mathrm{I}_0,\ \alpha_1 = d_4,\ \alpha_2 = d_7,\ \alpha_3 = d_6 = d_8 \tag{B.3}$$

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