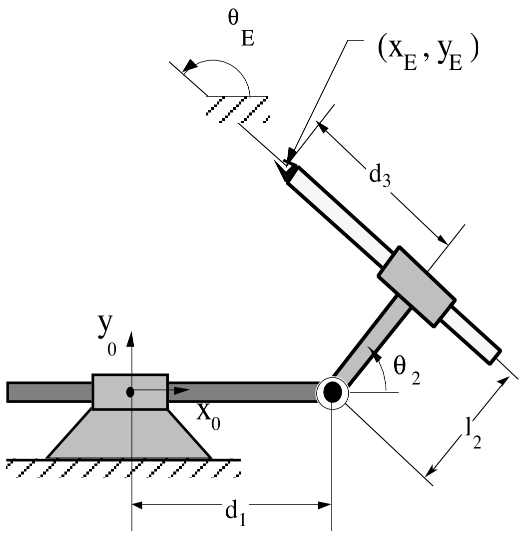
Assignment #3

Problem #1

For the 3 DOF **PRP** manipulator shown below, it is known that $0 \le d_1 \le 1$m, and $0 \le d_3 \le 1$m. The length of the second link, l_2 = 1m. There are no limits for θ_2. For this manipulator



(a) Sketch the reachable workspace. Does this manipulator have a dexterous workspace? Explain.

(b) Solve the forward kinematics problem, i.e. derive expressions for x_E, y_E, θ_E, as functions of the joint variables. Use your favorite approach.

(c) Solve the inverse kinematics problem, i.e. given x_E, y_E, θ_E, find the corresponding joint variables. Be careful to state the conditions under which solution(s) exist.

(d) Find the Jacobian that maps the joint rates $[\dot{d}_1, \dot{\theta}_2, \dot{d}_3]^T$ to end-effector velocities $[\dot{x}_E, \dot{y}_E, \dot{\theta}_E]^T$ by *direct differentiation* of the kinematic equations found in part (b).

(e) Verify the result of your part (d) by solving for the Jacobian using the *general method* discussed in class.

(f) Now that you know your Jacobian is right, familiarize yourself with the *velocity propagation* method and apply it here to again derive the Jacobian.

(g) What is the contribution of joint 2 to the linear velocity of the end-effector?

(h) Does this manipulator have singularities? If yes, what is their physical significance?

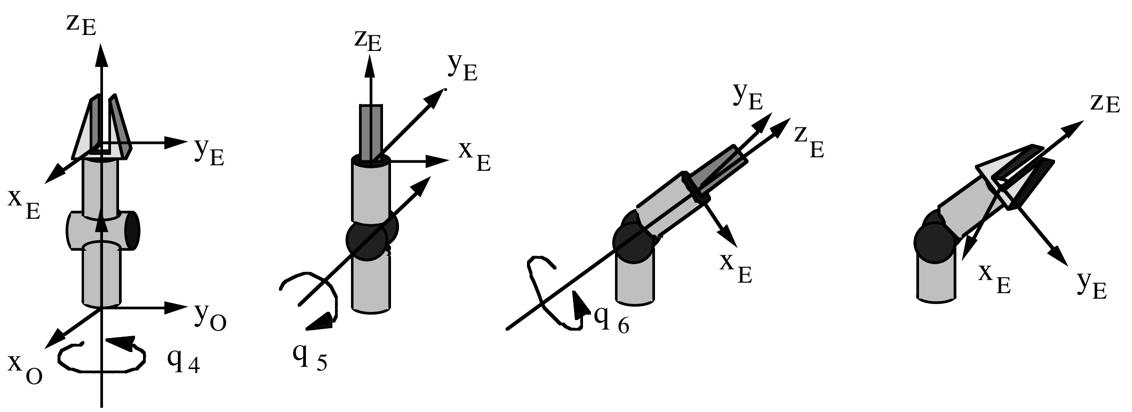
Problem #2

We analyze the differential kinematics of a very common robotic mechanism, the 3R wrist, shown below. The end-effector frame (x_E, y_E, z_E) is rigidly attached to the end-effector. A fixed reference frame (x_o, y_o, z_o) is attached to the base of the wrist as shown.

(b) Assuming first that Euler angle rates $\dot{\mathbf{q}} = d/dt[q_4, q_5, q_6]^T$ are used to describe the rotational velocity of the end-effector, what is the Jacobian ${}^{o}J_r$? You should be able to identify it by inspection.

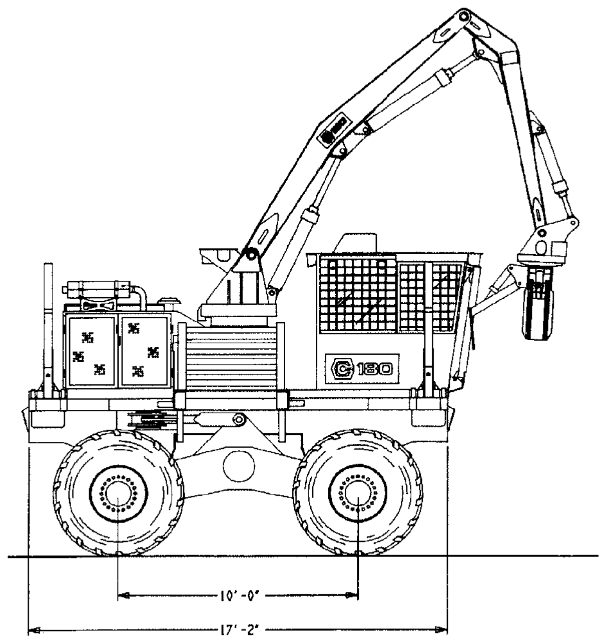
(c) Next, we assume that the rotational velocity of the end-effector is described by the angular velocities ${}^{o}\mathbf{w}_{E.}$ Find an expression relating the angular velocity of the end-effector, ${}^{o}\mathbf{w}_E$, to the ZYZ Euler angle rates $\dot{\mathbf{q}} = d/dt[q_4, q_5, q_6]^T$. Identify the Jacobian J_A. What is the physical meaning of its columns?

(d) Assume that at some configuration (q_4, q_5, q_6) the end-effector must have some specified angular velocity ${}^{o}\mathbf{w}_E = \mathbf{w}$. Find the required joint rates $\dot{\mathbf{q}}$. Can you always find these? Explain.

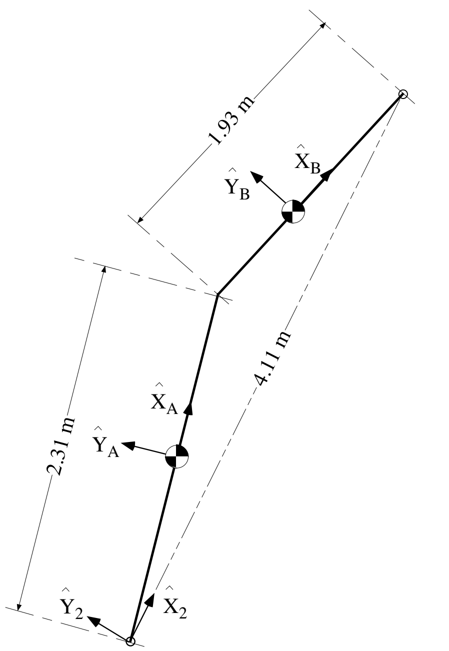


Problem #3

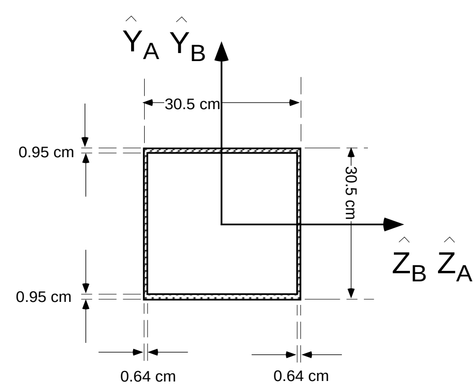
A drawing of a state-of-the-art vehicle manipulator is shown below. We examine here the computation of the link inertias for the first link called the boom (the second link is called the stick). The system equations of motion have been derived in general terms and it is now desired to substitute numerical values for the boom inertia tensor.



Forestry Vehicle



Engineering Model of Boom



Cross-section of Boom

(a) Modeling the boom as two homogeneous steel beams of uniform hollow square cross-section with dimensions as shown below, find the 3 by 3 inertia tensors of the upper and lower parts of the boom with respect to the frames located at their individual mass centers: ${}^{A}I_l^c$, ${}^{B}I_u^c$. Hint: make use of the principle of superposition and recall that for a solid rectangular parallelepiped $I_{xx} = m(a^2 + b^2)/12$, $I_{yy} = m(a^2 + L^2)/12$, and $I_{zz} = m(L^2 + b^2)/12$. Use 7.8 x10^3 kg/m^3 for the density of the steel.

(b) A reference frame $(Oxyz)_2$ has been affixed to the base of the boom using the DH convention. Find the positions of the lower and upper boom mass centers expressed in this frame, ${}^2\mathbf{P}_A$ and ${}^2\mathbf{P}_B$. Also solve for the position of the mass center of the overall boom ${}^2\mathbf{P}_C$ and affix at this point a reference frame $(Oxyz)_C$ with the same orientation as $(Oxyz)_2$.

(c) Transform the two inertia tensors ${}^A I_l^c$, ${}^B I_u^c$ so that they are expressed with respect to and in $(Oxyz)_C$. Finally, solve for the overall boom inertia tensor expressed with respect to its center of mass and in $(Oxyz)_2$.

(d) It was hoped that the cross-product inertia terms would be negligible compared to the principal mass moments of inertia to speed up and simplify calculations. Is this the case?

Problem #4

The DH parameters of a PUMA like shoulder joint (i.e. a shoulder joint with an offset) are given in the following table. Note that the base and end-effector frames do not coincide with the joint frames. Mass of links 1 and 2 are given by $m_1 = 15$ kg, and $m_2 = 30$ kg. The gravity vector is directed along the negative z_1 axis. Since a model based controller will be designed for this shoulder we need to solve for the equations of motion.

i	α_{i-1}	a_{i-1}	d_i	θ_i
1	0	0	h_1	θ_1
2	-90°	0	L_1	θ_2
E	0	L_2	0	0

(a) Sketch the shoulder joint for $\theta_1 = \theta_2 = 0$ using reference frames and rotation axes.

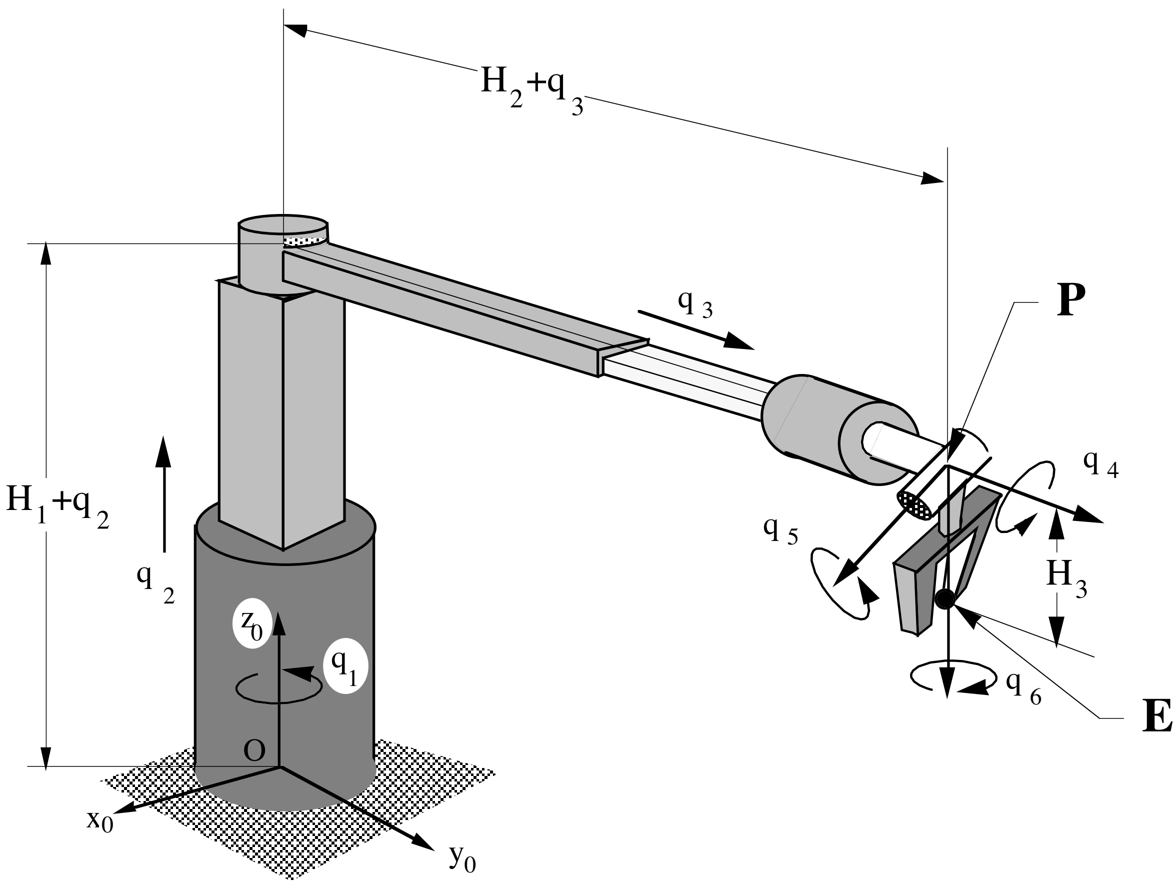
(b) For a conservative calculation we shall lump mass of link i at origin of frame i+1. Using the closed form Newton Euler method solve for the system equations of motion, i.e. systematically solve for the required position and velocity vectors in the base frame, write force and moment equations for links 1 and 2 in the base frame, and then solve for the joint torques.

(c) What is the physical meaning of the different terms in the equations of motion?

Problem #5

(a) Using the Jacobian for the RPPRRR manipulator obtained in PS 2, Problem 5, find the actuator forces and torques, $\mathbf{t} = [\tau_1, f_{2,a}, f_{3,a}, \tau_4, \tau_5, \tau_6]^T$, required to produce a force moment couple along the joint 6 axis given by ${}^6\mathbf{f}_E = [0,0,f]^T$, and ${}^6\mathbf{n}_E = [0,0,n]^T$.

(b) Assume that the actuator force and torque limits are given by $|\tau_1| \leq 25$ Nm, $|f_{2,a}| \leq 30$ N, $|f_{3,a}| \leq 15$ N, $|\tau_{4,5,6}| \leq 10$ Nm. Also assume that the revolute joints limits are all $\pm$ 180°, and that the limits on the prismatic joint ranges are given by $0 \leq q_2 \leq 1$m, and $0 \leq q_3 \leq 1$m with $H_1 = H_2 = 0.5$ m and $H_3 = 0.1$ m. At what configuration(s) in the joint space can the manipulator exert the largest force along the joint 6 axis? At what configuration(s) can the manipulator exert the largest moments about the joint 6 axis?



Problem #6

(a) Explain in your own words the iterative Newton-Euler method and describe briefly how it differs from the closed-form Newton-Euler method.

It is desired here to determine the required size of the prismatic actuator for the simple but fast pick&place RP robot shown below. The links have mass m_1 and m_2, and inertia $\mathbf{I}_1^{c1} = \text{diag}(0, 0, I_1)$, and $\mathbf{I}_2^{c2} = \text{diag}(0, 0, I_2)$. Since fast motions will take place, dynamic inertial loads are likely to be as important as quasi-static loads. We therefore need to solve for the equation of motion of joint 2.

(b) Using the iterative Newton-Euler method, show that the equation of motion for joint 2 is as follows for an unloaded end-effector:

$$f_{2,a} = m_2 l_1 \ddot{\theta} + m_2 \ddot{d}_2 - m_2 d_2^* \dot{\theta}^2 + m_2 c_1 \mid g \mid$$

(c) If the robot is exerting a quasi-static force (${}^0F_x$, ${}^0F_y$) on an object what is the new form of the equation for $f_{2,a}$?