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On Configuration Planning for Free-floating Space Robots

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Abstract. Free–floating space manipulator systems, have spacecraft actuators turned off and exhibit nonholonomic behavior due to angular momentum conservation. A path planning methodology in joint space for planar free–floating space manipulator systems is developed that allows spacecraft attitude control using manipulator motions. The method is based on mapping the angular momentum to a space where it can be satisfied trivially. Smooth and continuous functions such as polynomials are employed and the system is driven to a desired configuration. Two cases are studied. In the first, the manipulator is mounted on an arbitrary point of the spacecraft and the corresponding transformation is found. Then, a second transformation is found for the particular case where the manipulator is mounted on the center of mass of the spacecraft. It is shown that the derived transformation allows for smooth configuration changes in finite time. Limitations in reaching arbitrary final systems configurations are discussed. The application of the methodology is illustrated using an example.

I. INTRODUCTION

Space exploration is a relatively new field in science and engineering. Robotic manipulators are already playing important roles in space missions because of their ability to act in environments which are inaccessible or too risky for humans. In the case of robotic systems in orbit, robotic manipulators are mounted on a thruster –equipped spacecraft, called free–flying space manipulator systems. If the spacecraft thrusters are not operating, as for example during capture operations, then these systems are called free–floating space manipulator systems. In free – flying systems, thruster jets can compensate for manipulator induced disturbances, but their extensive use limits the system's useful life span. In free–floating systems, dynamic coupling between the manipulator and the spacecraft exists, and manipulator motions induce disturbances to the system's spacecraft. In these cases, the spacecraft is permitted to translate and rotate in response to its manipulation motions. This mode of operation can be feasible when no external forces and torques act on the system and when the total momentum of the system is zero.

A free-floating space robot exhibits a nonholonomic behavior. The nonholonomy in its mechanical structure is due the nonintegrability of the angular momentum, [1]. This property complicates the planning and control of such systems, which have been studied by a number of researchers. Vafa and Dubowsky have developed a technique called Virtual Manipulator (V.M.) method [2]. The kinematic and momentum equations of free-floating space manipulator systems were developed using this technique, which was subsequently used for path planning of such systems. Inspired by astronaut motions, they proposed a planning technique which employed small cyclical motions in the manipulator's joint space to modify its spacecraft's attitude.

Papadopoulos and Dubowsky studied the *Dynamic Singularities* of free-floating space manipulator systems, which are not found in earth bound manipulators and depend on the dynamic properties of the system, [1,3]. At a dynamic singularity the manipulator is unable to move its end-effector in some inertial direction. These singularities must be considered in the design, planning, and control of free-floating systems because of their important effects system performance.

Nakamura and Mukherjee explored Lyapunov techniques to achieve simultaneous control of spacecraft's attitude and its manipulator joints, [4]. To limit the effects of a certain null space, the authors proposed a *bidirectional* approach, in which two desired paths were planned, one starting from the initial configuration and going forward and one starting from the final configuration and going backwards. The final path was made of these two paths, up to the point where they intersected. However, this method was not immune to null space problems and resulted non-smooth joint trajectories that required that the joints come to a stop at the switching point.

In another attempt to plan a space robotic system's motion, Papadopoulos proposed a method that allowed Cartesian motion of the manipulator from an initial point to a final point avoiding dynamic singularities, [5]. The method involved small Cartesian cyclical motions of the end-effector designed in such way as to change the attitude of the spacecraft to one that was known of avoiding dynamic singularities, [5], [6].

Recently, Franch et al. have employed flatness theory to plan trajectories for free-floating systems. However, their method requires selection of robot parameters so that the system is made controllable and linearizable by prolongations, [7].

In this paper, a path planning methodology in joint space for planar free–floating space manipulator systems is developed that allows control of α spacecraft's attitude using manipulator motions. The method is based on mapping the nonholonomic constraint to a space where it can be satisfied trivially. Smooth and continuous functions such as polynomials are employed, driving the system to a desired configuration. Two cases are studied. First, the general case where the manipulator is mounted on an arbitrary point of the spacecraft is studied and the corresponding transformation is found. In addition, a particular transformation is found for the case where the manipulator is mounted on the center of mass of a spacecraft. It is shown that the derived transformation allows for smooth configuration changes in finite time. Limitations on reaching arbitrary final systems configurations are discussed. An example illustrates the proposed methodology.

II Dynamics of Free-Floating Space Manipulators

A space manipulator system consists of a spacecraft and a manipulator mounted on it, as shown in Figure 1. When the system is operating in free-floating mode, the spacecraft's attitude control system is turned off. In this mode, no external forces and torques act on the system, and hence the spacecraft translates and rotates in response to manipulator movements.

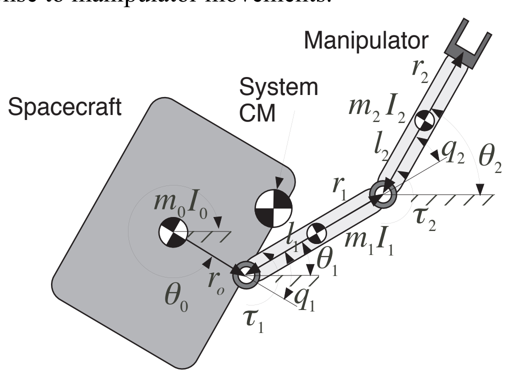

Fig. 1. A Free-Floating Space Manipulator System.

This section develops briefly the equations of motion of a rigid free-floating system. For simplicity, the manipulator is assumed to have revolute joints and an open chain kinematic configuration, so that, in a system with N-degree-of-freedom (dof) manipulator, there will be $N+6$ dof.

Under the assumptions of absence of external forces, the system Center of Mass (CM) does not move, and the system linear momentum, $P = M\dot{r}_{CM}$, is constant. With the further assumption of zero initial momentum, i.e. $\dot{r}_{CM}(0) = 0$, the system CM remains fixed in inertial space, i.e. $r_{CM} = const$, and the origin, O, can be chosen to be the system's CM.

The N equations of motion for a free-floating system can be found using a Lagrangian approach, and have the form, [3]

$$\boldsymbol{H}(\boldsymbol{q})\ddot{\boldsymbol{q}} + \boldsymbol{C}(\boldsymbol{q},\dot{\boldsymbol{q}})\dot{\boldsymbol{q}} = \boldsymbol{\tau} \tag{1}$$

where $\boldsymbol{H}(\boldsymbol{q})$ is an $N \times N$ positive definite symmetric matrix, called the reduced system inertia matrix, and $\boldsymbol{C}(\boldsymbol{q},\dot{\boldsymbol{q}})\dot{\boldsymbol{q}}$ contains the nonlinear Coriolis and centrifugal terms. The $N \times 1$ column vectors $\boldsymbol{q}$, $\dot{\boldsymbol{q}}$ represent manipulator joint angles and velocities, and the $N \times 1$ vector τ is the manipulator joint vector equal to $[\tau_1, \tau_2, ..., \tau_N]^T$. In these N equations of motion, Eq. (1), the spacecraft attitude and position variables do not enter because the system kinetic energy does not depend on spacecraft attitude or position nor on its linear or angular velocity when the initial angular momentum is zero and the system is free of external forces and torques. However, the attitude of the spacecraft enters in Eq. (2), which represents the conservation of angular momentum and can be computed with its help as, [1],

$${}^{0}\omega_{0} = -\,{}^{0}\boldsymbol{D}^{-1}\,{}^{0}\boldsymbol{D}_{q}\,\dot{\boldsymbol{q}} \tag{2}$$

where ${}^{0}\omega_{0}$ is the spacecraft angular velocity expressed in the spacecraft 0^{th} frame, and ${}^{0}\boldsymbol{D}$, ${}^{0}\boldsymbol{D}_q$ are inertia-type matrices of appropriate dimensions.

For simplicity, in this paper we focus on a free-floating robotic system consisting of a two degree-of-freedom (dof) manipulator mounted on a spacecraft. The spacecraft is constrained to move in the plane perpendicular to the axis of the manipulator rotation (Planar Case). Considering the planar free-floating space manipulator and assuming zero initial angular momentum, the conservation of angular momentum, Eq. (2), is written as

$$D\dot{\theta}_0 + (D_1 + D_2)\dot{q}_1 + D_2\,\dot{q}_2 = 0 \tag{3}$$

where q_1, q_2, are manipulator relative joint angles and θ_0 is the spacecraft's attitude. The coefficients D, D_1 and D_2 in Eq. (3) depend on system parameters and on angles q_1, q_2. This equation can be written also as

$$D_0\,\dot{\theta}_0 + D_1\,\dot{\theta}_1 + D_2\,\dot{\theta}_2 = 0 \tag{4}$$

where θ_1, θ_2 are manipulator absolute joint angles, see Fig. 1. The terms D_0, D_1 and D_2 in Eq. (4) depend on system parameters and angles θ_0, θ_1, θ_2, and are given in Appendix A.

The angular momentum, given by Eq. (3) or (4), cannot be integrated to analytically yield the spacecraft orientation θ_0 as a function of the system's configuration. However, if the manipulator joint angle trajectories is known as a function of time, then Eqs. (3) or (4) can be integrated numerically to yield the trajectory for the spacecraft orientation. This nonintegrability property introduces nonholonomic characteristics to free-floating systems, and results from the dynamic structure of the system. In other words, it is not due to system kinematics, as is the case with the nonholonomic constraints in mobile manipulators.

III Nonholonomic Path Planning

In this paper, we focus our attention in finding a path for a free-floating space manipulator system, which connects its initial configuration described by $(\theta_0^{in},\theta_1^{in},\theta_2^{in})$ to the final one, described by $(\theta_0^{fin},\theta_1^{fin},\theta_2^{fin})$. It is desired that the spacecraft orientation is controlled actuating the two manipulator joints only. It is well known that this problem is not trivial, since one must satisfy the nonholonomic constraint and achieve a change in a three-dimensional configuration space with two controls only. Next, a planning methodology is described that allows for a systematic approach in the planning of systems subject to nonholonomic constraints of the form of Eq. (3) or (4). Here, Eq. (4) is used because in this case it is much easier to find the desired transformation.

The constraint given by Eq. (4) is scleronomic and can be written in the Pfaffian form,

$$P(\theta_0,\theta_1,\theta_2)d\theta_0+Q(\theta_0,\theta_1,\theta_2)d\theta_1+R(\theta_0,\theta_1,\theta_2)d\theta_2=0 \quad (5)$$

where,

$$P(\theta_0,\theta_1,\theta_2)=I_0+\frac{m_0(m_1+m_2)}{m_0+m_1+m_2}r_0^2 + \frac{m_0 r_0}{m_0+m_1+m_2}\left[l_1(m_1+m_2)+r_1 m_2\right]\cos(\theta_0-\theta_1) + \frac{m_0 m_2}{m_0+m_1+m_2}r_0 l_2\cos(\theta_0-\theta_2) \quad (6a)$$

$$Q(\theta_0,\theta_1,\theta_2)=I_1+\frac{m_0 m_1}{m_0+m_1+m_2}l_1^2 + \frac{m_1 m_2}{m_0+m_1+m_2}r_1^2+\frac{m_0 m_2}{m_0+m_1+m_2}(l_1+r_1)^2 + \frac{m_0 r_0}{m_0+m_1+m_2}\left[l_1(m_1+m_2)+r_1 m_2\right]\cos(\theta_0-\theta_1) + \frac{l_2 m_2}{m_0+m_1+m_2}\left[m_1 r_1+m_0(l_1+r_1)\right]\cos(\theta_1-\theta_2) \quad (6b)$$

$$R(\theta_0,\theta_1,\theta_2)=I_2+\frac{m_2(m_0+m_1)}{m_0+m_1+m_2}l_2^2 + \frac{m_0 m_2}{m_0+m_1+m_2}r_0 l_2\cos(\theta_0-\theta_2) + \frac{m_2 l_2}{m_0+m_1+m_2}\left[m_1 r_1+m_0(l_1+r_1)\right]\cos(\theta_1-\theta_2) \quad (6c)$$

where the geometric and mass properties in Eq. (6) are defined in Figure 1.

Note that Eq. (5) contains three differentials. Planning can be facilitated if this form is transformed to one in which two differentials appear. This is indeed possible because it is known that nonintegrable Pfaffian equations of the form of Eq. (5) can be written as, [8],

$$du+vdw=0 \quad (7)$$

where *u, v, w* are properly selected functions of $\theta_0,\theta_1,\theta_2$.

The method of finding the proper functions *u, v, w* is analytically presented in [9]. It is briefly described here for completeness. Eq. (5) can be transformed into Eq. (7), if the following equations hold

$$P=\frac{\partial u}{\partial\theta_0}+v\frac{\partial w}{\partial\theta_0},\quad Q=\frac{\partial u}{\partial\theta_1}+v\frac{\partial w}{\partial\theta_1},\quad R=\frac{\partial u}{\partial\theta_2}+v\frac{\partial w}{\partial\theta_2} \quad (8)$$

To find the unknown functions u, v, and w, the differential equations that they must satisfy are constructed. To this end, we define the following auxiliary functions,

$$P'=\frac{\partial Q}{\partial\theta_2}-\frac{\partial R}{\partial\theta_1},\quad Q'=\frac{\partial R}{\partial\theta_0}-\frac{\partial P}{\partial\theta_2},\quad R'=\frac{\partial P}{\partial\theta_1}-\frac{\partial Q}{\partial\theta_0} \quad (9)$$

Substitution of Eq. (8) into Eq. (9) and subsequent multiplication of P' by $\partial w/\partial\theta_0$, of Q' by $\partial w/\partial\theta_1$, and of R' by $\partial w/\partial\theta_2$ respectively, and addition of the results yields the following differential equation for *w*

$$P'\frac{\partial w}{\partial\theta_0}+Q'\frac{\partial w}{\partial\theta_1}+R'\frac{\partial w}{\partial\theta_2}=0 \quad (10)$$

Similarly, multiplying P' by $\partial v/\partial\theta_0$, Q' by $\partial v/\partial\theta_1$, and R' by $\partial v/\partial\theta_2$ respectively, and adding the results, yields the following differential equation for *v*

$$P'\frac{\partial v}{\partial\theta_0}+Q'\frac{\partial v}{\partial\theta_1}+R'\frac{\partial v}{\partial\theta_2}=0 \quad (11)$$

Therefore, both w and v satisfy the same first order partial differential equation, i.e. any solution to Eq. (10) is also a solution to Eq. (11).

Finally, multiplying P' by $P-\partial u/\partial\theta_0$, Q' by $Q-\partial u/\partial\theta_1$ and R' by $R-\partial u/\partial\theta_2$, adding the results, and using Eqs. (8), and (10) yields

$$P'\frac{\partial u}{\partial\theta_0}+Q'\frac{\partial u}{\partial\theta_1}+R'\frac{\partial u}{\partial\theta_2}=PP'+QQ'+RR'\neq 0 \quad (12)$$

The right hand side in the above equation does not vanish, because the condition of integrability is not satisfied. If the system were holonomic, then u would have satisfied the same differential equation as v and w.

Next, the partial differential equation, Eq. (10), is solved to yield w. The general solution to it is any function of the two independent integrals

$$a(\theta_0,\theta_1,\theta_2)=k_1 \tag{13a}$$

$$\beta(\theta_0,\theta_1,\theta_2)=k_2 \tag{13b}$$

of the subsidiary system, [10],

$$\frac{d\theta_0}{P}=\frac{d\theta_1}{Q}=\frac{d\theta_2}{R} \tag{14}$$

In Eqs. (13), k_1 and k_2 are arbitrary real numbers.

For simplicity let w equal to one of these integrals,

$$w=a(\theta_0,\theta_1,\theta_2)=k_1 \tag{15}$$

Eq. (5), in view of Eqs (8) and (15), yields

$$\begin{aligned} &P\,d\theta_0+Q\,d\theta_1+R\,d\theta_2= \\ &\frac{\partial u}{\partial\theta_0}d\theta_0+\frac{\partial u}{\partial\theta_1}d\theta_1+\frac{\partial u}{\partial\theta_2}d\theta_2=du=0 \end{aligned} \tag{16}$$

i.e. for any particular value k_1 of w, Eq. (5) is a perfect differential. Next, a solution for u is obtained by integration of Eq. (16) under the constraint imposed by Eq. (15). Eq. (16) is used to find u by expressing any variable and its differential, say θ_2 and $d\theta_2$, with respect to the other variables and k_1. Substitution of these into Eq. (16) results in a perfect differential $dh(\theta_0,\theta_1,k_1)$. Integration of this differential results in the function $h(\theta_0,\theta_1,k_1)$. Replacing k_1 using Eq. (13a), results in the function $u(\theta_0,\theta_1,\theta_2)$. Hence, expressions for u and w have been found. The expression for v is found using any of Eqs. (8).

This transformation is very helpful for planning purposes. Indeed if we chose functions f and g as

$$w=f(t) \tag{17a}$$

$$u=g(w) \tag{17b}$$

$$v=-\frac{du}{dw}=-g'(w) \tag{17c}$$

then Eq (7) is satisfied identically. Therefore, the planning problem reduces to choosing functions f and g such that they satisfy the initial and final configuration variables. Such functions can be polynomials, splines, or any other continuous and smooth functions. For example, one possibility is to choose function f as a fifth order polynomial, so that the system initial and final configuration, velocity and acceleration can be specified, and function g as a third order polynomial, so that initial and final system configurations can be specified.

Next, this methodology is applied to free-floating space manipulator systems in the general case, where the manipulator is mounted on an arbitrary spacecraft point and in the particular case, where the manipulator is mounted on the spacecraft's center of mass.

A. Manipulator Mounted on an Arbitrary Point

Applying the method described in the previous section, the two solutions of the subsidiary system, Eq. (14), are (Appendix C)

$$\begin{aligned} a(\theta_0,\theta_1,\theta_2)=&\frac{m_0 r_0(m_0+m_1+m_2)}{m_0 l_1+r_1(m_0+m_1)}\cos\theta_0 \\ &+(m_0+m_1+m_2)\cos\theta_1+ \\ &+\frac{(m_0+m_1+m_2)m_2 l_2}{m_2 r_1+l_1(m_1+m_2)}\cos\theta_2=k_1 \end{aligned} \tag{18a}$$

$$\begin{aligned} \beta(\theta_0,\theta_1,\theta_2)=&\frac{m_2 l_2}{m_0+m_1+m_2}\left[m_1 r_1+m_0(l_1+r_1)\right]\cos(\theta_1-\theta_2) \\ &+\frac{m_0 m_2}{m_0+m_1+m_2}r_0 l_2\cos(\theta_0-\theta_2) \\ &+\frac{m_0 r_0}{m_0+m_1+m_2}\left[l_1(m_1+m_2)+r_1 m_2\right]\cos(\theta_0-\theta_1)=k_2 \end{aligned} \tag{18b}$$

Note that the integral $a(\theta_0,\theta_1,\theta_2)$ is expressed in absolute manipulator angles while the integral $\beta(\theta_0,\theta_1,\theta_2)$ is expressed in relative manipulator angles. The former is much easier to use because it facilitates the expression of any variable and its differential in terms of the other two variables and their differentials. Thus we choose the function w be equal to integral $a(\theta_0,\theta_1,\theta_2)$.

The nonholonomic constraint described by Eq. (5) can be written in the form given by Eq. (7) if the following transformation is used

$$\begin{aligned} w(\theta_0,\theta_1,\theta_2)=&\frac{m_0 r_0(m_0+m_1+m_2)}{m_0 l_1+r_1(m_0+m_1)}\cos\theta_0+ \\ &+(m_0+m_1+m_2)\cos\theta_1+ \\ &+\frac{(m_0+m_1+m_2)m_2 l_2}{m_2 r_1+l_1(m_1+m_2)}\cos\theta_2 \end{aligned} \tag{19a}$$

$$\begin{aligned} u(\theta_0,\theta_1,\theta_2)=&\alpha_1\cdot\theta_0+\alpha_2\cdot\theta_2+\alpha_3\cdot\arcsin(\cos\theta_1)+ \\ &+\alpha_4\cdot\arcsin(c_0+c_1\cdot\cos\theta_0-\cos\theta_1+c_2\cdot\cos\theta_2)+ \\ &+(\alpha_5\cdot\cos\theta_0+\alpha_6\cdot\cos\theta_1+\alpha_7\cdot\cos\theta_2)\cdot \\ &\sqrt{1-(c_0+c_1\cdot\cos\theta_0-\cos\theta_1+c_2\cdot\cos\theta_2)^2}+ \\ &+\alpha_8\cdot\sin(2\theta_0)+\alpha_9\cdot\sin(2\theta_1)+ \\ &+\alpha_{10}\cdot\sin(2\theta_2)+ \\ &+\alpha_{11}\cdot\sin(\theta_0+\theta_1)+ \\ &+\alpha_{12}\cdot\sin(\theta_0+\theta_2)+ \\ &+\alpha_{13}\cdot\sin(\theta_1+\theta_2) \end{aligned} \tag{19b}$$

$$
\begin{aligned}
&v(\theta_0,\theta_1,\theta_2)= \\
&\frac{b_0+b_1\cdot\cos\theta_0+b_2\cdot\cos^2\theta_0+b_3\cdot\cos\theta_1}{\sqrt{1-(c_0+c_1\cdot\cos\theta_0-\cos\theta_1+c_2\cdot\cos\theta_2)^2}}+ \\
&+\frac{b_4\cdot\cos\theta_0\cdot\cos\theta_1+b_5\cdot\cos^2\theta_1+b_6\cdot\cos\theta_2}{\sqrt{1-(c_0+c_1\cdot\cos\theta_0-\cos\theta_1+c_2\cdot\cos\theta_2)^2}}+ \\
&+\frac{b_7\cos\theta_0\cos\theta_2+b_8\cos\theta_1\cos\theta_2+b_9\cos^2\theta_2}{\sqrt{1-(c_0+c_1\cdot\cos\theta_0-\cos\theta_1+c_2\cdot\cos\theta_2)^2}}+ \\
&+b_{10}\cdot\sqrt{1-(c_0+c_1\cdot\cos\theta_0-\cos\theta_1+c_2\cdot\cos\theta_2)^2}+ \\
&+b_{11}\cdot\sin\theta_0+b_{12}\cdot\sin\theta_1+b_{13}\cdot\sin\theta_2
\end{aligned}
\tag{19c}
$$

where the coefficients a_i, b_j in Eqs (19b)-(19c) depend on system parameters and are given in Appendix B.

Note that from the system dynamics point of view, the spacecraft is equivalent to the last manipulator body. Therefore, the transformation must be symmetric with respect to angles θ_0 and θ_2 or lengths r_0 and l_2, etc. Careful study of the transformation given by Eqs. (19) confirms that indeed, this is the case here.

The transformation given by Eqs. (19a)-(19c) exists if and only if the following inequality is satisfied

$$-1\le c_0+c_1\cdot\cos\theta_0-\cos\theta_1+c_2\cdot\cos\theta_2\le 1 \tag{20}$$

This inequality obviously introduces a constraint among the absolute angles of the system. This constraint must always hold, therefore it does not permit the selection of arbitrary initial and final system configurations.

The transformation defined by Eqs. (19a)-(19c), consists of nonlinear terms and its inverse cannot be found easily to yield analytically variables $\theta_0,\theta_1,\theta_2$ as functions of u,v,w. This is an issue of current research.

B. Manipulator Mounted on Spacecraft CM

When the parameter r_0 is equal to zero, the system manipulator is mounted on the spacecraft center of mass. Then, the coefficients of the nonholonomic constraint given by Eqs (6a)-(6c), take the simpler form

$$P(\theta_0,\theta_1,\theta_2)=\alpha_0 \tag{21a}$$

$$Q(\theta_0,\theta_1,\theta_2)=\alpha_1+\alpha_3\cdot\cos(\theta_1-\theta_2) \tag{21b}$$

$$R(\theta_0,\theta_1,\theta_2)=\alpha_2+\alpha_3\cdot\cos(\theta_1-\theta_2) \tag{21c}$$

where the coefficients α_i are given by

$$\alpha_0=\mathrm{I}_0 \tag{22a}$$

$$\alpha_1=\mathrm{I}_1+\frac{l_1^2\,m_0\,m_1}{m_0+m_1+m_2}+\frac{m_1\,m_2\,r_1^2}{m_0+m_1+m_2}+\frac{m_0\,m_2\,(l_1+r_1)^2}{m_0+m_1+m_2} \tag{22b}$$

$$\alpha_2=\mathrm{I}_2+\frac{m_2\,(m_0+m_1)\,l_2^2}{m_0+m_1+m_2} \tag{22c}$$

$$\alpha_3=\frac{l_2\,m_1\,m_2\,r_1}{m_0+m_1+m_2}+\frac{l_2\,m_0\,m_2\,(l_1+r_1)}{m_0+m_1+m_2} \tag{22d}$$

In this case, the two solutions of the subsidiary system, Eq. (14), are

$$a(\theta_0,\theta_1,\theta_2)=\theta_1=k_1 \tag{23}$$

$$\beta(\theta_0,\theta_1,\theta_2)=\theta_2=k_2 \tag{24}$$

Choosing the function w be equal to the integral $a(\theta_0,\theta_1,\theta_2)$, the nonholonomic constraint given by Eq. (5) can be written in the form given by Eq. (7) if

$$u(\theta_0,\theta_1,\theta_2)=\alpha_0\cdot\theta_0+\alpha_2\cdot\theta_2-\alpha_3\cdot\sin(\theta_1-\theta_2) \tag{25a}$$

$$v(\theta_0,\theta_1,\theta_2)=\alpha_1+2\cdot\alpha_3\cdot\cos(\theta_1-\theta_2) \tag{25b}$$

$$w(\theta_0,\theta_1,\theta_2)=\theta_1 \tag{25c}$$

Assuming the planning has been achieved in the $u-v-w$ space, one still needs to know the joint trajectories and rates for achieving the planned motion. To this end, the inverse transformation from $u-v-w$ to $\theta_0,\theta_1,\theta_2$ must be found.

After some algebraic manipulations, the inverse transformation can be found and is given by,

$$\theta_0=\frac{1}{\alpha_0}(u-\alpha_2\,w)-\frac{1}{\alpha_0}\left[\alpha_2\arccos\left(\frac{v-\alpha_1}{2\alpha_3}\right)+\alpha_3\sqrt{1-\left(\frac{v-\alpha_1}{2\alpha_3}\right)^2}\,\right] \tag{26a}$$

$$\theta_1=w \tag{26b}$$

$$\theta_2=w+\arccos\left(\frac{v-\alpha_1}{2\alpha_3}\right) \tag{26c}$$

It is easy to see that the forward transformation given by Eqs. (25a)-(25c) is defined for any system configuration $(\theta_0,\theta_1,\theta_2)$. On the other hand, the inverse transform given by Eqs. (26a)-(26c) is defined if and only if the following inequality is satisfied

$$-1\le\frac{v(\theta_0,\theta_1,\theta_2)-\alpha_1}{2\alpha_3}\le 1 \tag{27}$$

It is obvious that to satisfy the constraints described by Eq. (27), additional freedom must be introduced in the planning scheme. A simple way to achieve this is to introduce additional coefficients in the polynomial $u(w)$. These additional coefficients should not affect the satisfaction of the initial and final conditions but should allow one to shape the path in the $u-v-w$ space so as to satisfy Eq. (27).

To satisfy Ineq. (27), more than one additional coefficients b_i are needed. However, in such a case, the order of the function v increases and makes an analytical approach very difficult. Therefore, we give more freedom by assuming that the final spacecraft orientation θ_0^{fin} is free and we study which orientations are possible from the given initial configuration. With these remarks, we let the function $u(w)$ have the form

$$u(w)=b_4 w^4 + b_3 w^3 + b_2 w^2 + b_1 w + b_0 \qquad (28)$$

Because of Eq. (17c), $v(w)$ is given by

$$v(w)=-4b_4 w^3 - 3b_3 w^2 - 2b_2 w - b_1 \qquad (29)$$

Using the initial and final system configuration and the transformation given by Eqs. (25), the initial and final conditions for u,v,w are found and the following linear system is obtained with respect to the unknown coefficients b_i, $i = 0,1,2,3$:

$$\sum_{i=0}^{3} b_i w_{in}^i = u_{in} - b_4 w_{in}^4 \qquad (30c)$$

$$\sum_{i=0}^{3} b_i w_{fin}^i = u_{fin} - b_4 w_{fin}^4 \qquad (30b)$$

$$\sum_{i=0}^{3} i\, b_i w_{in}^{i-1} = -v_{in} - 4 b_4 w_{in}^3 \qquad (30c)$$

$$\sum_{i=0}^{3} i\, b_i w_{fin}^{i-1} = -v_{fin} - 4 b_4 w_{fin}^3 \qquad (30d)$$

The above system can be solved to yield the b_i, $i = 0,1,2,3$, as linear functions of b_4 and θ_0^{fin}. Replacing these coefficients in Eq. (29), the polynomial v is written as a function of the additional coefficient b_4 and the unknown spacecraft final orientation θ_0^{fin}. The problem reduces to finding a range of values of b_4 and a range of orientations θ_0^{fin} which leads to paths that satisfy Eq. (27) for all $w \in [w_{in}\ w_{fin}]$. These ranges can be found, studying the following function

$$h(w, b_4, \theta_0^{fin}) = \frac{v(w, b_4, \theta_0^{fin}) - \alpha_1}{2\alpha_3} \qquad (31)$$

This function is a third-order polynomial with respect to w. Due to Eq. (25b), once the initial and final θ_1 and θ_2 are given, the initial and final values of the function h are set and known. Also, these two values always are in the range of $[-1, 1]$ for $w \in [w_{in}\ w_{fin}]$, because the forward transformation given by Eq. (25) is always defined. Note that the values of h along the path are still given by Eq. (31).

To satisfy Ineq. (27), $h(w, b_4, \theta_0^{fin})$ must be in the range $[-1, 1]$. This function either has no extremes for $w \in [w_{in}\ w_{fin}]$, and therefore Ineq. (27) is always satisfied, or has extremes whose values must be in the range $[-1, 1]$. Obviously, some limitations in the reachable configurations may result because of this reason. The application of the planning methodology is illustrated next with an example.

IV. EXAMPLE

To illustrate the methodology described above, the free-floating space manipulator shown in Figure 1 is employed. The system parameters are shown in Table 1.

Table 1. System Parameters.

Body	$l_i\,(m)$	$r_i\,(m)$	$m_i\,(Kg)$	$I_i\,(Kg\cdot m^2)$
0	.5	0	40	1.667
1	.5	.5	4	0.333
2	.5	.5	3	0.250

The main task for the system is to move from some initial configuration $(\theta_0^{in}, \theta_1^{in}, \theta_2^{in})$ to a final one $(\theta_0^{fin}, \theta_1^{fin}, \theta_2^{fin})$ at a given time and using manipulator actuators only. For the simulation, the total move time is chosen equal to 10 s. The initial configuration is $(\theta_0^{in}, \theta_1^{in}, \theta_2^{in}) = (10^o, 40^o, 45^o)$ and the final desired one is $(\theta_0^{fin}, \theta_1^{fin}, \theta_2^{fin}) = (\theta_0^{fin}, 60^o, 70^o)$. In this case, the initial and final values of the function h are 0.996 and 0.984 respectively. Since these values are close to +1, it is advantageous to have a minimum for h, because this increases the range of possible final configurations available.

Since h is a third order polynomial, its derivative is a second order and therefore h can have up to two extremes. In this example, one of these is a minimum and is described by point $A(w_1, h(w_1))$, and the other is a maximum, described by $B(w_2, h(w_2))$. To ensure that only the minimum in h appears in the range $[w_{in}, w_{fin}]$, the following conditions must hold,

$$w_1 \in [w_{in}, w_{fin}] \text{ and } w_2 \notin [w_{in}, w_{fin}] \qquad (32)$$

Under these conditions, Ineq. (27) reduces to,

$$h(w_1) \geq -1 \qquad (33)$$

The above conditions are functions of b_4 and θ_0^{fin}. The validity of these conditions is checked for candidate final orientations in the range $\theta_0^{fin} \in [-360^o, 360^o]$ and $b_4 \in (-100,0) \cup (0,100)$. The resulting reachable range for the final spacecraft orientation is $\theta_0^{fin} \in (-75^o, -35^o)$. If the final orientation is chosen to be $\theta_0^{fin} = -45^o$, then the corresponding range of b_4 that allow this configuration change is $b_4 \in (-108,0) \cup (0,110)$.

Figure 2 shows the motion of the free-floating space manipulator system for a final spacecraft orientation equal to $\theta_0^{fin} = -45^o$ and for $b_4 = 1$. The trajectories of the configuration variables are shown in Figure 3. The rates of the relative joint rates and the spacecraft orientation is shown in Figure 4. It can be seen that all trajectories are smooth throughout the motion, and that the system starts and stops smoothly at zero velocities, as expected. This is an important characteristic of the method employed and is due to the use of smooth functions, such as polynomials.

The joint torques that correspond to the configuration change in Fig. 2 are shown in Figures 5 and 6. These torques are computed using Eq. (1) and the elements of the reduced inertia matrix, given in Appendix A. As shown in Figures 5 and 6, the required torques are relatively small and smooth. The implication of this is that joint motors can apply such torques with ease and therefore the resulting configuration maneuver is feasible. These torques can be made arbitrarily small, if the duration of the maneuver is increased.

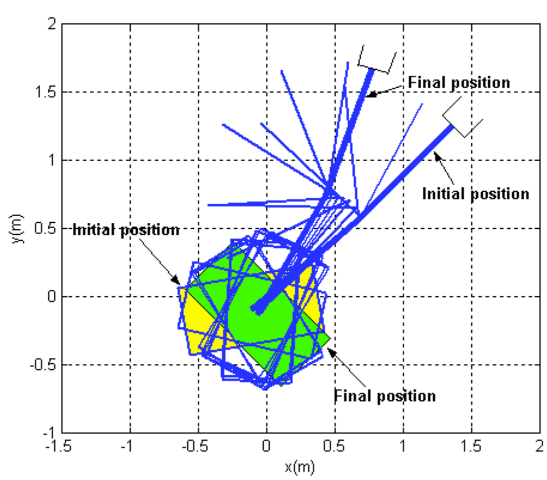

Fig. 2. Motion Animation of space manipulator.

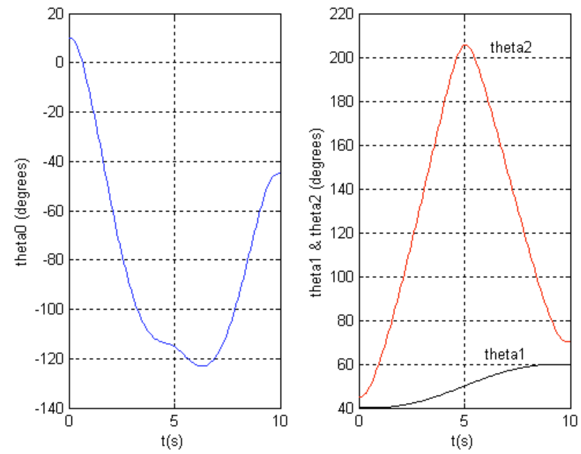

Fig. 3. Trajectories of system absolute angles that correspond to the snapshots in Fig. 2.

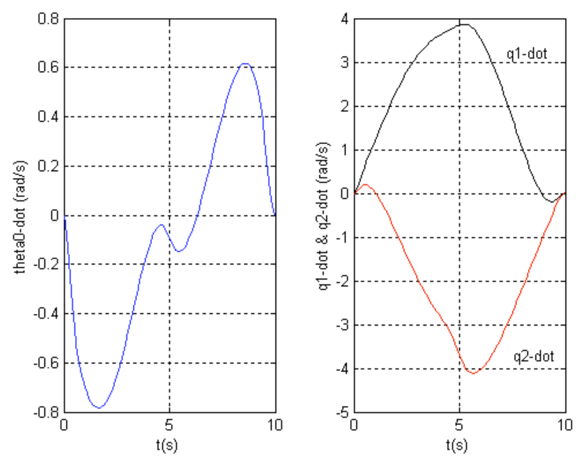

Fig. 4. Rates of relative joint angles and of the spacecraft orientation that correspond to the snapshots in Fig. 2.

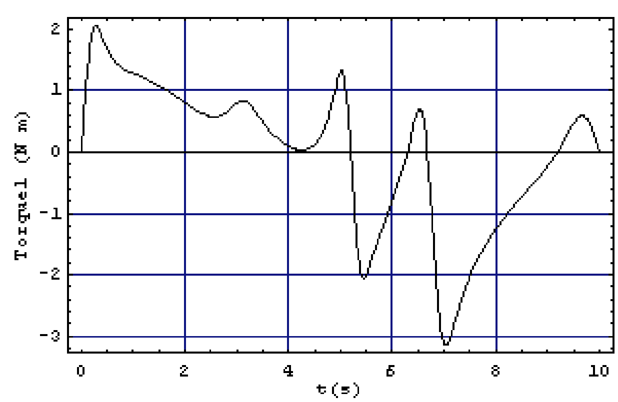

Fig. 5. Torque applied on the manipulator forearm.

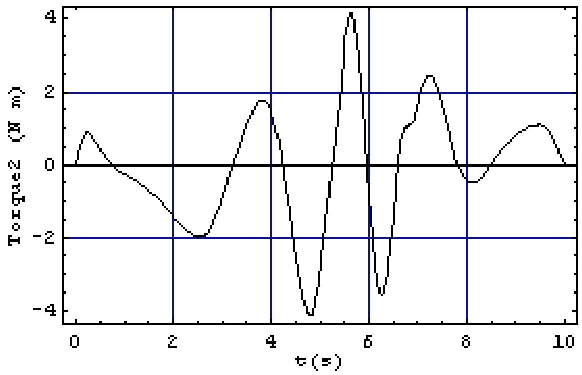

Fig. 6. Torque applied on the manipulator upper arm.

V. Conclusions

A path planning methodology in joint space for planar free–floating space manipulator systems was developed. These systems are nonholonomic because of the angular momentum conservation. The spacecraft moves in response to manipulator motions, while the orientation of the spacecraft can be controlled by actuating the joint angles, only. The method was based on mapping the nonholonomic constraint to a space where it can be satisfied trivially. Smooth and continuous functions such as polynomials were employed and the system was driven to the desired configuration. Two cases were studied. First, the transformation was made for the general case where the manipulator is mounted on an arbitrary point of the spacecraft. Then a second transformation was found for the particular case where the manipulator is mounted on the center of mass of the spacecraft. It was shown that the derived transformation allow for smooth configuration changes in finite time. Limitations in reaching arbitrary final systems configurations were discussed.

Acknowledgments

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Appendix A

The D-terms in Eq. (3) are given by:

$$D_j = \sum_{i=0}^{2} {}^{0}d_{ij} \quad (j = 0,1,2)$$

$$D = D_0 + D_1 + D_2$$

$${}^{0}d_{00} = I_0 + \frac{m_0 (m_1 + m_2)}{m_0 + m_1 + m_2} r_0^2$$

$${}^{0}d_{10} = \frac{m_0 r_0}{m_0 + m_1 + m_2} \left[l_1 (m_1 + m_2) + r_1 m_2 \right] \cos(q_1) = {}^{0}d_{01}$$

$${}^{0}d_{20} = \frac{m_0 m_2}{m_0 + m_1 + m_2} r_0 l_2 \cos(q_1 + q_2) = {}^{0}d_{02}$$

$${}^{0}d_{11} = I_1 + \frac{m_0 m_1}{m_0 + m_1 + m_2} l_1^2 + \frac{m_1 m_2}{m_0 + m_1 + m_2} r_1^2 + \frac{m_0 m_2}{m_0 + m_1 + m_2} (l_1 + r_1)^2 \qquad \text{(A1)}$$

$$ {}^{0}d_{21}=\frac{m_2 l_2}{m_0+m_1+m_2}\left[m_1 r_1 + m_0\left(l_1+r_1\right)\right]\cos\left(q_2\right) = {}^{0}d_{12} $$

$$ {}^{0}d_{22}=I_2+\frac{m_2\left(m_0+m_1\right)}{m_0+m_1+m_2}l_2^2 $$

Replacing the relative angles by the absolute angles, Eqs. (A1), yield the D terms in Eq. (4).

The reduced system inertia matrix, defined in Eq. (1), for the planar case, has the form

$$ \boldsymbol{H}\left(\boldsymbol{q}\right)=\begin{bmatrix} {}^{0}d_{11}+2\,{}^{0}d_{12}+{}^{0}d_{22}-\frac{\left(D_1+D_2\right)^2}{D} & {}^{0}d_{12}+{}^{0}d_{22}-\frac{D_2\left(D_1+D_2\right)}{D} \\ {}^{0}d_{12}+{}^{0}d_{22}-\frac{D_2\left(D_1+D_2\right)}{D} & {}^{0}d_{22}-\frac{D_2^2}{D} \end{bmatrix} \tag{A2} $$

APPENDIX B

The coefficients in Eq. (19b) are given by

$$ \alpha_1=\frac{m_0 m_1 r_0^2 r_1+I_0\left(l_1 m_0+\left(m_0+m_1\right)r_1\right)}{l_1 m_0+\left(m_0+m_1\right)r_1} $$

$$ \alpha_2=\frac{l_1 l_2^2 m_1 m_2+I_2\left(r_1 m_2+\left(m_1+m_2\right)l_1\right)}{r_1 m_2+\left(m_1+m_2\right)l_1} $$

$$ \alpha_3=-I_1+l_1 m_1 r_1 $$

$$ \alpha_4=-I_1+l_1 m_1 r_1 $$

$$ \alpha_5=-\frac{m_0 r_0\left(r_1 m_2+\left(m_1+m_2\right)l_1\right)}{m_0+m_1+m_2} $$

$$ \alpha_6=-\frac{\left(l_1 m_0+\left(m_0+m_1\right)r_1\right)\left(r_1 m_2+\left(m_1+m_2\right)l_1\right)}{m_0+m_1+m_2} $$

$$ \alpha_7=-\frac{l_2 m_2\left(l_1 m_0+\left(m_0+m_1\right)r_1\right)}{m_0+m_1+m_2} $$

$$ \alpha_8=\frac{m_0^2 r_0^2\left(r_1 m_2+\left(m_1+m_2\right)l_1\right)}{\left(m_0+m_1+m_2\right)\left(l_1 m_0+\left(m_0+m_1\right)r_1\right)} $$

$$ \alpha_9=\frac{\left(l_1 m_0+\left(m_0+m_1\right)r_1\right)\left(r_1 m_2+\left(m_1+m_2\right)l_1\right)}{m_0+m_1+m_2} $$

$$ \alpha_{10}=\frac{l_2^2 m_2^2\left(l_1 m_0+\left(m_0+m_1\right)r_1\right)}{\left(m_0+m_1+m_2\right)\left(r_1 m_2+\left(m_1+m_2\right)l_1\right)} $$

$$ \alpha_{11}=\frac{m_0 r_0\left(r_1 m_2+\left(m_1+m_2\right)l_1\right)}{m_0+m_1+m_2} $$

$$ \alpha_{12}=\frac{l_2 m_0 m_2 r_0}{m_0+m_1+m_2} $$

$$ \alpha_{13}=\frac{l_2 m_2\left(l_1 m_0+\left(m_0+m_1\right)r_1\right)}{m_0+m_1+m_2} $$

The coefficients in Eq. (19c) are:

$$ b_0=\frac{-I_1+l_1 m_1 r_1}{m_0+m_1+m_2} $$

$$ b_1=\frac{m_0 r_0\left[l_1 m_0\left(l_2 m_2+\left(m_1+m_2\right)r_0\right)+m_2 r_1\left(m_0 r_0+\left(m_0+m_1\right)l_2\right)\right]}{\left(m_0+m_1+m_2\right)^2\left(l_1 m_0+\left(m_0+m_1\right)r_1\right)} $$

$$ b_2=-\frac{m_0^2 r_0^2\left(r_1 m_2+\left(m_1+m_2\right)l_1\right)}{\left(m_0+m_1+m_2\right)^2\left(l_1 m_0+\left(m_0+m_1\right)r_1\right)} $$

$$ b_3=\frac{l_1 m_0\left(l_2 m_2+\left(m_1+m_2\right)r_0\right)+m_2 r_1\left(m_0 r_0+\left(m_0+m_1\right)l_2\right)}{\left(m_0+m_1+m_2\right)^2} $$

$$ b_4=-\frac{2 m_0 r_0\left(r_1 m_2+\left(m_1+m_2\right)l_1\right)}{\left(m_0+m_1+m_2\right)^2} $$

$$ b_5=-\frac{\left(l_1 m_0+\left(m_0+m_1\right)r_1\right)\left(r_1 m_2+\left(m_1+m_2\right)l_1\right)}{\left(m_0+m_1+m_2\right)^2} $$

$$ b_6=\frac{l_2 m_2\left[l_1 m_0\left(l_2 m_2+\left(m_1+m_2\right)r_0\right)+m_2 r_1\left(m_0 r_0+\left(m_0+m_1\right)l_2\right)\right]}{\left(m_0+m_1+m_2\right)^2\left(l_1 m_0+\left(m_0+m_1\right)r_1\right)} $$

$$ b_7=-\frac{2 l_2 m_0 m_2 r_0}{\left(m_0+m_1+m_2\right)^2} $$

$$ b_8=-\frac{2 l_2 m_2\left(l_1 m_0+\left(m_0+m_1\right)r_1\right)}{\left(m_0+m_1+m_2\right)^2} $$

$$ b_9=-\frac{l_2^2 m_2^2\left(l_1 m_0+\left(m_0+m_1\right)r_1\right)}{\left(m_0+m_1+m_2\right)^2\left(r_1 m_2+\left(m_1+m_2\right)l_1\right)} $$

$$ b_{10}=\frac{\left(l_1 m_0+\left(m_0+m_1\right)r_1\right)\left(r_1 m_2+\left(m_1+m_2\right)l_1\right)}{\left(m_0+m_1+m_2\right)^2} $$

$$ b_{11}=-\frac{2 m_0 r_0\left(r_1 m_2+\left(m_1+m_2\right)l_1\right)}{\left(m_0+m_1+m_2\right)^2} $$

$$ b_{12}=-\frac{2\left(l_1 m_0+\left(m_0+m_1\right)r_1\right)\left(r_1 m_2+\left(m_1+m_2\right)l_1\right)}{\left(m_0+m_1+m_2\right)^2} $$

$$ b_{13}=-\frac{2 l_2 m_2\left(l_1 m_0+\left(m_0+m_1\right)r_1\right)}{\left(m_0+m_1+m_2\right)^2} $$

The coefficients in Eqs. (19) are:

$$c_0=\frac{m_0 r_0}{l_1 m_0+(m_0+m_1)r_1}+\frac{l_2 m_2}{r_1 m_2+(m_1+m_2)l_1}$$

$$c_1=-\frac{m_0 r_0}{l_1 m_0+(m_0+m_1)r_1}$$

$$c_2=-\frac{l_2 m_2}{r_1 m_2+(m_1+m_2)l_1}$$

APPENDIX C

In this appendix, the computation of the two independent integrals of the subsidiary system, Eq. (14), in the case of the manipulator mounted on an arbitrary spacecraft's point is described in more detail. Eq. (14) takes the form

$$\frac{d\theta_0}{A_0\sin(\theta_1-\theta_2)}=\frac{d\theta_1}{B_0\sin(\theta-\theta_2)}=\frac{d\theta_2}{C_0\sin(\theta-\theta_1)} \quad \text{(C1)}$$

where

$$A_0=2\left[\frac{m_1 m_2}{m_0+m_1+m_2}r_1 l_2+\frac{m_0 m_2}{m_0+m_1+m_2}l_2\left(l_1+r_1\right)\right]$$

$$B_0=-2\frac{m_0 m_2}{m_0+m_1+m_2}r_0 l_2$$

$$C_0=2\frac{m_0 r_0}{m_0+m_1+m_2}\left[l_1\left(m_1+m_2\right)+r_1 m_2\right]$$

It is known that

$$\sin(\alpha-b)=\sin\alpha\cos b-\cos\alpha\sin b \quad \text{(C2)}$$

Multiplying numerator and denominator of the first, second and the third fraction of Eq. (C1), with $\sin\theta_0$, $-\sin\theta_1$ and $\sin\theta_2$ respectively, and using the following fractions property,

$$\frac{\alpha}{\beta}=\frac{\gamma}{\delta}=\ldots=\frac{\kappa}{\lambda}=\frac{\alpha+\gamma+\ldots+\kappa}{\beta+\delta+\ldots+\lambda} \quad \text{(C3)}$$

yields

$$\frac{\frac{1}{A_0}\sin\theta_0\, d\theta_0}{\sin\theta_0\left(\sin\theta_1\cos\theta_2-\cos\theta_1\sin\theta_2\right)}=\frac{\frac{1}{A_0}\sin\theta_0\, d\theta_0-\frac{1}{B_0}\sin\theta_1\, d\theta_1+\frac{1}{C_0}\sin\theta_2\, d\theta_2}{0} \quad \text{(C4)}$$

From Eq. (C4), the following results

$$\frac{1}{A_0}\cos\theta_0-\frac{1}{B_0}\cos\theta_1+\frac{1}{C_0}\cos\theta_2=k_1 \quad \text{(C5)}$$

Eq. (C5) is the first independent integral of subsidiary system given by Eq. (14). Next, the second integral is computed. Eq. (C1) can be written as

$$\begin{aligned}\frac{d\theta_0-d\theta_1}{A_0\sin(\theta_1-\theta_2)-B_0\sin(\theta_0-\theta_2)}&=\\=\frac{d\theta_0-d\theta_2}{A_0\sin(\theta_1-\theta_2)-C_0\sin(\theta_0-\theta_1)}&\\=\frac{d\theta_1-d\theta_2}{B_0\sin(\theta_0-\theta_2)-C_0\sin(\theta_0-\theta_1)}&\end{aligned} \quad \text{(C6)}$$

Let,

$$x=\theta_0-\theta_1 \quad \text{(C7a)}$$

$$y=\theta_0-\theta_2 \quad \text{(C7b)}$$

$$z=\theta_1-\theta_2 \quad \text{(C7c)}$$

Eq. (C6) takes the form

$$\frac{dx}{A_0\sin z-B_0\sin y}=\frac{dy}{A_0\sin z-C_0\sin x}=\frac{dz}{B_0\sin y-C_0\sin x} \quad \text{(C8)}$$

Multiplying numerator and denominator of the first, second and third fraction of Eq. (C8), with $C_0\sin x$, $B_0\sin y$, and $A_0\sin z$ respectively, and using property (C3), we have

$$C_0\sin x\, dx-B_0\sin y\, dy+A_0\sin z\, dz=0$$

or

$$A_0\cos z-B_0\cos y+C_0\cos x=k_2 \quad \text{(C9)}$$

Eq. (C9) is the second integral of subsidiary system given by Eq. (14).