A Novel Energy Pumping Strategy for Robotic Swinging

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Abstract — In this work we show that an Acrobot can be made to behave as a robotic swing. This is achieved by controlling the first joint, provided that a given condition is satisfied. When this condition is not satisfied, the system undergoes through singular points. Even when this happens, we are again able to make the system behave as a swing by controlling the second joint and employing a new Energy Pumping strategy. This strategy presents important advantages compared to previously proposed strategies, as it is the only one that can start the system from rest and drive it to large heights. Moreover, it is fast and requires very small torques.

Index Terms — Robotic swing, energy pumping, gymnast robots, underactuated systems.

I. INTRODUCTION

The swing problem has attracted the interest of a number of researchers during the last thirty years. Indeed, the problem is very interesting as it involves increasing the energy of a multibody system using internal (chemical in the case of humans) energy or motions. However, most research focused on dynamic analysis rather than on methods that can result in robotic swinging using controls. Also, none of these studied the effects of singular points or verified if the proposed movements are feasible for a particular under-actuated system.

The work up-to-date can be classified in two broad categories. The first deals with swing analysis and reports alternative kinematic strategies without a plan to implement them with active control. They also focus on techniques that allow an increase of the width of oscillation of a system (for Energy Pumping) but do not deal with how to make the system swing with a given amplitude and keep this oscillation constant, see for example [1-7]. One of the earliest works considered the swing model as a simple pendulum with variable length. [11]. Several years later, a strategies for initiation and pumping the swing from a standing position was published following a qualitative approach [2]. Swinging from standing and sitting positions was studied and it was concluded that the swing is best characterized as a forced oscillator, [3], [4]. Two different kinds of swinging were compared in [5]. In another study, the question whether people act as self optimizing machines while they swing was investigated, [6]. These studies do not address the issue of robotic swinging, which is dealt with in [7], using a sitting swing strategy but relying only on linear controls based purely on common experience.

The second type of work deals with the Acrobot problem in which the goal is to bring the system (an underactuated inverted pendulum) to the up right position, [8-12]. In his pioneering work, M. Spong used partial feedback linearization to bring the Acrobot to the upright position, [9]. Later, researchers tried to achieve the same goal, but most controllers were based on energy methods (e.g. [10], [11]). Other works have used Lyapunov methods and were successful to bring the first Acrobot link to any desired position [12]. Bringing the Acrobot to the up right position with constraints to the second link has been studied, [13]. This kind of motion is close to the motion that gymnasts make on the high bar.

The aim of this paper is to study robotic swinging of an Acrobot-type robot using partial model based control. Here, the second link is restricted from making a full revolution. The encountered singular points due to the loss of angular momentum coupling are studied. Their dynamic nature is explained, as well as how they can be avoided using a new swinging strategy. A new energy pumping strategy is proposed that presents important advantages over existing strategies. This strategy can start the system from rest, is fast and requires low torques.

II. SYSTEM DYNAMICS

To study the robotic swing and the pumping of energy that occurs, (i.e. the transfer of energy from the actuated dot to the unactuated one), an Acrobot-type system is employed. The Acrobot is an under-actuated robotic system with two degrees of freedom, (dof), i.e. the angle of the first link, \( q_1 \), and the angle of the second link, \( q_2 \), see Fig. 1. Of those, only the second dof is actuated.

![Figure 1. Acrobot system and its parameters.](image-url)
Since the structure of this robotic system approximates a sitting person swinging, it was chosen as the system to be studied here.

The equations of motion may be derived using the Lagrangian of the system and are described by,

$$M(q)\ddot{q} + C(q, \dot{q})\dot{q} + G(q) = \tau$$  (1)

where \( q = [q_1, q_2]^T \), \( \tau = [\tau_1, \tau_2]^T \), and \( M \), \( C \), and \( G \) are given in Appendix A.

In this paper, we are interested in designing a controller capable of initiating a swinging motion of the system and converging to a swinging oscillation with given amplitude. Since the robotic swing is underactuated, one can directly control one of the two degrees of freedom only. As it will be presented later, a dual strategy is chosen to swing the robotic system. If we understand the swing system well enough to produce a close to ideal energy pumping strategy, then the convenient second degree of freedom \( q_2 \) is used as our controlled variable. However, if no such strategy is available, then the first degree of freedom \( q_1 \) is controlled under the requirement to oscillate such that the entire system behaves like a swing. In both cases, we use partial model-based control with nonlinear terms cancellation.

III. ROBOTIC SWING WITH CONTROL ON THE FIRST JOINT

Swinging when \( q_1 \) is controlled is facilitated by the coupling terms in (1). Due to this fact, no special strategy is needed to initiate swinging, and this is clearly an advantage. Since the first joint is not actuated, its motion must be generated by the actuator acting on the second joint.

Another advantage is that employing control on \( q_1 \) and studying the resulting response of \( q_2 \), allows one to develop a new strategy for swinging and Energy Pumping. However, a disadvantage of using control on \( q_1 \) is the appearance of singular points. When these occur, the system is unable to pump energy smoothly, and as a consequence, the required torque gets large accelerating the second link. Next, we develop the swinging strategy with control on \( q_1 \) and start with planning.

1) Planning

Here \( q_1 \) is controlled. We are interested in initiating swinging, and upon reaching a desired amplitude, to be able to hold the motion so that the system at the steady state swings. A simple strategy is to require that \( q_1 \) changes as a sinusoidal function with continuously increasing width of oscillation,

$$q_{1d}(t) = q_{1offset} + C_1 \cdot \sin(\omega t)$$  (2)

where,

$$C_1 = \begin{cases} k \cdot t & \text{for } t_f > t \geq 0 \\ k \cdot t_f & \text{for } t \geq t_f \end{cases}$$  (3)

where \( k \) is the constant rate at which the oscillation amplitude increases, \( t_f \) is the time at which the steady state oscillation occurs, \( C_{1f} \) is the width of oscillation at the steady state, and \( \omega \) is the lower natural frequency of the system, computed around the stable equilibrium point of the robot; this enables the system to swing, requiring small torque. This strategy works even if the input frequency is not equal to the lower natural frequency of the system. However, in such a case, a higher torque will be needed.

In (2), \( q_{offset} \) determines the angle around which the oscillation occurs. For swinging and the conventions in Fig. 1, \( q_{offset} \) is 270°. Fig. 2 shows the oscillation of the first joint around its offset value. The response can be divided in two different states,

(a) The transient state, where Energy Pumping occurs and the width of oscillation continuously increases, and,

(b) The steady state, where Energy Pumping does not occur and the width of oscillation remains constant.

![Figure 2. The transient and steady state response for the first joint angle.](image)

Although (2) and (3) are very simple, they still allow one to define both the \( t_f \), the width of oscillation \( C_1 \) and the speed at which this is reached, \( k \). Obviously, these parameters have an effect on required actuator torques and size.

2) Model Based Control

Here, our aim is to force the system to follow the trajectory described by (2). This can be done using a partial Model Based Control technique with nonlinear term cancellation.

To do this, a second order differential equation, with respect to \( q_1 \), that contains the input torque is needed. This can be obtained using the equations of motion, (1). By eliminating \( q_2 \) from (1), we come up to the following:

$$\ddot{q}_1 \cdot A_1 + B_1 = \tau_2$$  (4)

where:

$$A_1 = \frac{g_1(q_1, q_2)}{-(p_2 + p_3 \cdot \cos q_2)}$$  (5)

$$B_1 = \frac{f_1(q_1, q_2, q_1, \dot{q}_2)}{-(p_2 + p_3 \cdot \cos q_2)}$$  (6)

where functions \( g_1, f_1, p_2 \) and \( p_3 \), are given in Appendix A.

The following controller makes sure that \( q_1 \) will reach its desired value in prescribed time,

$$\tau_2 = \left( \ddot{q}_{1d} + k_\tau \cdot (q_{1d} - q_1) + k_d \cdot (\dot{q}_{1d} - \dot{q}_1) \right) \cdot A_1 + B_1$$  (7)

Equation (7) constitutes a Model Based Control with nonlinear term cancellation. Assuming knowledge of system
parameters, terms $A_i$ and $B_i$ cancel the nonlinear terms in (4), while the terms in the parentheses constitute a PD feedback controller that can regulate the system response using the control gains $k_p, k_d$.

3) Singular points
Looking carefully at (5), (6) and (7) one can easily see that the denominator may become equal to zero. This point to the existence of algorithmic singular points. These have no relationship to kinematics, and cannot be computed using the Jacobian of the system. Their location depends on system physical parameters. In addition, generally these points appear only during the transient state.

In order to investigate the effects of these points, we set as $K$ the denominator in question and study it further.

$$K = p_2 + p_3 \cos(q_2)$$

(8)

As it was mentioned before, when a system comes from a singular point, then a denominator is becoming equal to zero and the controller fails. To obtain a clear physical meaning of what happens at such points, we find the system angular momentum with respect to the first joint. This is given by,

$$H = (p_1 + p_2 + 2p_3 \cos(q_3)) \dot{q}_1 + (p_2 + p_3 \cos(q_3)) \dot{q}_2$$

(9)

The angular momentum is constituted of two terms. The first term is the contribution of the first link and the second is the contribution of the second one. Since at singular points $K$ is zero, it can be seen that at such points the second link has no effect on system angular momentum, and the coupling, which is important for energy pumping, is lost.

Singular point existence causes problems to system behavior. At such instances, the response of $q_1$ is not smooth any more, and the torque $\tau_2$ locally increases drastically, trying to reduce the tracking error in $q_1$. Since no coupling exists at these points, the torque rapidly accelerates the second link, making it to undergo full rotations. In such cases, pumping of energy is erratic and no proper swinging can result. Despite this, swinging may occur, but this may take unpredictable time.

The important question that arises is whether it is possible to design a controller capable of swinging without requiring large torques and without unacceptably high accelerations of the second link. To this end, we examine when the term $K$ can be nonzero.

$$K = p_2 + p_3 \cos(q_2) > 0 \Rightarrow p_2 > p_3$$

(10)

Substituting the terms $p_2$, and $p_3$, (10) becomes.

$$m_2 l_2^2 + I_2 > m_2 l_2$$

(11)

Using the expression for $I_2$ given in Appendix A, (11) becomes:

$$I_2 > \frac{3}{2} l_2$$

(12)

If (12) holds, then coupling between the two links never fails and pumping can occur without infinite torques and second link accelerations.

4) New Energy Pumping strategy
Up to this point, swinging and energy pumping is possible only if (12) is in effect. An important question is whether it is possible to develop a new Energy Pumping strategy, which could provide sufficient pumping, without going through singular points, and even if (12) is not in effect.

Notice that singular points appear due to the exploitation of the coupling between the two links and drive the first joint using the actuator for the second joint. Therefore, to avoid the singularities, it is natural to explore the possibility of driving the second joint directly. Based on this observation, our aim is to find a new strategy of Energy Pumping that can be used to pump energy in systems in which (12) does not hold.

A new strategy can be developed influenced by the study of the response of $q_1$. In order to do this, we study simulation results obtained using a system in which (12) does not hold and therefore the second link is not always coupled dynamically to the first one. This is motivated by the fact that despite the non smooth response of the system, after long time, the system tends to stabilize in some smooth swinging. This is shown in Figure 3, where the system has an erratic behavior for about 26 s, but swings after that time.

We define the following variables,

$$q_{1\text{new}}(t) = q_1(t + a \cdot t_j), a > 1$$

$$q_{2\text{new}}(t) = q_2(t + a \cdot t_j), a > 1$$

that describe the system response during smooth swinging.

![Smooth Swinging](image_url)

**Figure 3.** The response of the second angle reaches eventually a steady state and smooth swinging.

To learn from $q_{1\text{new}}$ and $q_{2\text{new}}$ orbits, we record the smooth swinging response part and analyze it with the help of Fourier analysis. Applying an FFT algorithm on the steady state part of the response of $q_2$, see Figure 4, one can notice: (a) the appearance of peaks at higher harmonics of the input frequency, and (b) that the energy of the first harmonic is by far the highest. This observation allows us to neglect the higher harmonics and keep the first one only. This points to the direction that $q_1$ oscillates with relatively
large amplitude, when \( q_2 \) is a pure sinusoidal function with a single frequency, close to the lowest natural frequency, and has a constant difference in phase from \( q_1 \).

The phase difference in question can be found by studying the response of \( q_2 \), with \( q_1 \) controlled and (12) not in effect. In cases where the actual phase difference deviates from this value, then the system might still be capable of Energy Pumping but will require a higher torque. This however can only be achieved provided the input frequency is close to the natural frequency.

We can now proceed with the development of a new strategy for Energy Pumping, i.e. we determine how the second link \( q_2 \) should move so that the unactuated first angle \( q_1 \) increases its width of oscillation. Based on the previous observations, Energy Pumping can occur if the second angle is driven by

\[
q_{2d}(t) = C_2 \sin(\omega t + \varphi)
\]  

(14)

where \( \omega \) is the lowest system natural frequency, and \( \varphi \) is a phase difference between \( q_1 \) and \( q_2 \).

The advantage of this pumping strategy over others is that it can start with zero initial conditions and result in large oscillation amplitudes. Although this strategy does not maintain constant amplitude of oscillation, and therefore it is not a strategy for swinging, it is still a new strategy for effective Energy Pumping and can be used to increase the width of oscillation of a robotic swing.

IV. ROBOTIC SWING WITH CONTROL ON THE SECOND JOINT

The advantage of using control on \( q_2 \) is that it is very easy to be controlled since \( q_2 \) is the actuated degree of freedom. As mentioned earlier, the disadvantage is it requires a good swing strategy. This is discussed next.

1) Planning

The system must be able to swing at desired amplitude. Therefore, during the transient response, a pumping strategy is needed. When the desired level of swinging is reached, pumping must stop. This is achieved by the following command for \( q_2 \),

\[
q_{2d}(t) = \begin{cases} 
C_2 \cdot \sin(\omega t + \varphi) & \text{if } C_2 < C_{1f} \\
q^* & \text{if } C_2 \geq C_{1f}
\end{cases}
\]

(15)

where \( q^* \) is the value of \( q_2 \) at the moment when \( q_1 \) reaches the desirable amplitude for the first time and the first link angular speed is null (for smoother switching). Upon examination of (15), one can easily see that to increase the width of oscillation, the Energy Pumping strategy developed earlier is used. To evaluate the performance of swinging, an amplitude error is defined as

\[
e_{q1}(t) = |q_1(t_f) - q_{1f}| \leq \frac{\pi}{2} (10)
\]

which indicates the distance of the amplitude at zero velocity after the stable equilibrium point from the desired one. Once the correct amplitude is achieved, the second joint is locked and the system behaves as a simple pendulum. With this strategy, either the transient settling time or the oscillation amplitude can be set. Parameter \( C_{1f} \), which determines the maximum width of oscillation that the system can reach, is found by trial and error. In general, high values result in reduced oscillation amplitude accuracy.

2) Model based control

With a methodology similar to that in Section III, one can design a control law to force the system follow the desired trajectory. Following some manipulation of (1), we get,

\[
\dot{q}_2 \cdot A_2 + B_2 = \tau_2
\]

(17)

where \( A_2, B_2 \) are given in Appendix A and are functions of the states and velocities. To guarantee tracking for \( q_2 \), a partial model based control law with nonlinear term cancellation is designed that yields the torque \( \tau_2 \) as,

\[
\tau_2 = \left( q_{2d} + k_d \cdot (q_{2d} - q_2) + k_p \cdot (q_{2d} - q_1) \right) \cdot A_2 + B_2
\]

(18)

V. SIMULATION RESULTS

In this section, we first assume a system in which (12) applies and by controlling the first joint, (controller in Section III), we make it swing and realize pumping of energy. Next, we study a system for which (12) does not hold, and in which singular points exist. Using phase information from this system, we apply the controller of Section IV on the second joint and show that this results in smooth swinging and energy pumping.

A. Model-based Control on \( q_1 \)

Table I displays the parameters of a system in which condition (12) holds. Therefore, no singular points are expected while controlling the first joint. The system starts with null initial conditions, the settling time is chosen to be 30 s, and the final width of oscillation of the first link, \( C_{1f} = 60^\circ \). Then, (3) yields,

\[
k = 0.03489 \ \text{rad/s}
\]

(19)

<table>
<thead>
<tr>
<th>( m_1 ) [kg]</th>
<th>( m_2 ) [kg]</th>
<th>( l_1 ) [m]</th>
<th>( l_2 ) [m]</th>
<th>( \omega ) [rad/s]</th>
</tr>
</thead>
<tbody>
<tr>
<td>10.00</td>
<td>20.00</td>
<td>0.50</td>
<td>1.00</td>
<td>4.91</td>
</tr>
</tbody>
</table>

TABLE I. PARAMETERS OF A SYSTEM FOR WHICH (12) HOLDS.
Figure 5 displays the joint angle responses and the applied torque on the second joint. The system response has a smooth and stable behavior. Angle $q_1$ follows the desired trajectory, and the error $e = q^{\text{d}}_1 - q_1$, (not shown), is practically zero. Also, as shown from the response of $q_2$, the second link does not accelerate continuously and does not complete full rotations. Since no singularities exist, the input torque at the second joint is small and smooth.

Next, the same controller is used to initiate swinging, but here (12) does not hold. The system parameters are shown in Table II. The remaining conditions are as before. Figure 6 shows the system response and the applied torque. During the transient state, angle $q_1$ follows the desired trajectory with some small error, which disappears at the steady state. Nevertheless, the second link is accelerated by very large torques that try to compensate for the loss of coupling. The result is that the link undergoes full rotations and no swinging is achieved.

Following the erratic transient phase, the system achieves a swinging response, see Figure 7. Fitting a sinusoidal functions on the response of the two angles, results in correlation coefficient very close to “1”. From these, the difference in phase is found to be:

$$\varphi = 1.073 \text{ rad} = 61.5^\circ$$ \hspace{1cm} (20)

B. Model-based Control on $q_2$

We apply the strategy that was developed in Section IV, using a system whose parameters are given in Table II. The desired trajectory for $q_2$ is given by (14) and the phase difference is given by (20). The system starts from null initial conditions and $C'_{f} = 60^\circ$. The parameter $C_2$ is set to 0.98 so that pumping is fast. Figure 8 displays the obtained system response. The response is smooth as desired. The system starts from null initial conditions and reaches the desired width of oscillation very quickly and with small amplitude errors. In addition the required torque is smooth and small in magnitude.

Here we emphasize that, to the best of our knowledge, the developed Energy Pumping strategy is the only which can start the system from zero initial conditions, and can lead to high swinging amplitudes in a controlled fashion.

As was shown above, with this Energy Pumping strategy, the system can swing smoothly. The developed method allows one to require energy pumping up to a specific settling time or energy pumping up to a given level. Setting both the height (amplitude) that the system will reach...
VI. CONCLUSIONS

In this work we showed that an Acrobat can be controlled to behave as a robotic swing. This can be achieved by controlling the first joint angle \( q_1 \) when a particular condition is satisfied. In this case, we can set the desirable oscillation amplitude, as well as the time in which this must be achieved. It was shown that this strategy may be subject to singular points depending on system parameters. These singular points result in large input torques, and high second link accelerations. When this condition is not satisfied, then again we are able to swing the system controlling \( q_2 \) and employing a new Energy Pumping strategy. This new strategy requires that the second link should have as an orbit a sinusoidal function with frequency equal to the lower natural frequency of the system. The difference of phase between the two orbits depends system parameters and is constant. This strategy presents important advantages compared to others since it is the only one that can begin with null initial conditions and make the system reach high swinging amplitudes. In addition, it is very fast and requires very small torques.

REFERENCES


APPENDIX A

The matrices and vectors \( \mathbf{M}, \mathbf{C}, \) and \( \mathbf{G} \), are given by,

\[
\mathbf{M} = \begin{bmatrix}
 p_1 + p_2 + 2 p_3 \cos(q_2) & p_2 + p_3 \cos(q_2) \\
p_2 + p_3 \cos(q_2) & p_3 \\
\end{bmatrix}
\]

\[
\mathbf{C} = \begin{bmatrix}
 -p_3 \sin(q_2) \dot{q}_2 & -p_3 \sin(q_2) \dot{q}_2 - p_3 \sin(q_2) \dot{q}_1 \\
p_3 \sin(q_2) \dot{q}_1 & 0 \\
\end{bmatrix}
\]

\[
\mathbf{G} = \begin{bmatrix}
p_3 g \cos(q_1) + p_3 g \cos(q_1 + q_2) \\
p_3 g \cos(q_1 + q_2) \\
\end{bmatrix}
\]

with parameters:

\[
p_1 = m_1 l_1^2 + m_2 l_1 + I_1,
p_2 = m_2 l_1^2 + I_1,
p_3 = m_2 l_2^2 + I_2,
p_4 = m_2 l_2 + p _3 \cos(q_2),
\]

\[
q_1 = \frac{1}{12} m_1 (l_1)^2, \quad I_2 = \frac{1}{12} m_2 (l_2)^2
\]

The functions \( g_1 \) and \( f_1 \) are given by:

\[
g_1 = p_2 \cdot (p_1 + p_2 + 2 \cdot p_3 \cdot \cos(q_2)) - (p_2 + p_3 \cdot \cos(q_2))^2
\]

\[
f_1 = -p_3 \cdot q_1^2 \cdot \sin(q_2) \cdot (p_2 + p_3 \cdot \cos(q_2)) - p_3 \cdot q_2^2 \cdot \sin(q_2) \cdot p_2 - 2 \cdot p_3 \cdot p_2 \cdot q_2 \cdot \sin(q_2) -
\]

\[
-p_3 \cdot g \cdot \cos(q_1 + q_2) \cdot (p_2 + p_3 \cdot \cos(q_2)) +
\]

\[
p_3 \cdot g \cdot \cos(q_1 + q_2) \cdot p_2 + p_3 \cdot g \cdot \cos(q_1 + q_2) \cdot p_2
\]

The functions \( A_2 \) and \( B_2 \) are given by:

\[
A_2 = p_2 - \frac{(p_1 + p_2 + 2 \cdot p_3 \cdot \cos(q_2))^2}{(p_1 + p_2 + 2 \cdot p_3 \cdot \cos(q_2))}
\]

\[
B_2 = q_1^2 \cdot (p_1 + p_2 + 2 \cdot p_3 \cdot \cos(q_2)) \cdot p_3 \cdot \sin(q_2) +
\]

\[
+ p_3 \cdot q_1^2 \cdot (p_1 + p_2 + 2 \cdot p_3 \cdot \cos(q_2)) \cdot p_3 \cdot \sin(q_2) +
\]

\[
+ 2 \cdot p_3 \cdot q_2 \cdot \sin(q_2) \cdot (p_1 + p_2 + 2 \cdot p_3 \cdot \cos(q_2)) +
\]

\[
+ (p_1 + p_2 + 2 \cdot p_3 \cdot \cos(q_2)) \cdot p_3 \cdot \cos(q_2)
\]

\[
- (p_1 + p_2 + 2 \cdot p_3 \cdot \cos(q_2)) \cdot p_3 \cdot \cos(q_2)
\]

\[
+ p_3 \cdot g \cdot \cos(q_1 + q_2) \cdot p_2 \cdot \cos(q_2) +
\]

\[
+ p_3 \cdot g \cdot \cos(q_1 + q_2) \cdot p_2 \cdot \cos(q_2)
\]

\[
+ (p_1 + p_2 + 2 \cdot p_3 \cdot \cos(q_2)) \cdot p_3 \cdot \cos(q_2)
\]

\[
+ 2 \cdot p_3 \cdot q_2 \cdot \sin(q_2) \cdot (p_1 + p_2 + 2 \cdot p_3 \cdot \cos(q_2)) +
\]

\[
+ (p_1 + p_2 + 2 \cdot p_3 \cdot \cos(q_2)) \cdot p_3 \cdot \cos(q_2)
\]