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Pitch Control for Running Quadrupeds Using Leg Positioning in Flight

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Abstract— **Pitching is often present as an undesirable motion in quadruped running. In gaits such as pronking, pitching must be completely suppressed for the proper execution of the gait. In this work, a novel control method is presented for controlling the pitching motion of a running quadruped robot in the pronking gait. A model of the quadruped robot is used to analyze the pitching motion dynamics. This analysis shows that setting the difference of the angles with which the back and front legs strike the ground is sufficient for controlling the pitching motion in the pronking gait. An analytical expression is derived which provides the necessary difference of the leg touchdown angles. It is further shown that there exists a particular robot design that is superior when suppressing the robot pitching motion. Implementation of the control scheme is shown through simulations of the robot model.**

I. INTRODUCTION

Research into legged robots has been intense in recent years, from all aspects. Many hardware designs have been used, including robots using one, two four, six or more legs for locomotion [1,2,3,4,5]. A variety of gaits has been achieved with these robots, and the controllers applied are also diverse. Moreover, considerable work has been done regarding the analysis and understanding of the dynamics of these robots [1,6,7,8,9,10].

The work regarding the analysis of legged robot dynamics generally focuses on unveiling the mechanisms at work behind the locomotion that is observed in the physical robot. These mechanisms are still not completely understood, even in the case of simple systems. For example, locomotion may be achieved in robots without full knowledge of the exact interactions of the control inputs with the dynamics. An indication of this is that controllers often involve parameters which are found by trial and error experimentation.

The better the knowledge of the interaction of the control inputs with the dynamics, the more likely it becomes to achieve the same motion with less actuators [8], or with significantly less actuator effort [11]. There exists a significant amount of work exploring these mechanisms. A large portion of this work concentrates on the case of the Spring-Loaded Inverted Pendulum (SLIP) model, either to interpret the behavior of the one-legged robot, or as an abstraction of a more complex multi-legged robot. Some work also exists for the case of four legged robot models, which is the focus of the present work. It is usual to study the dynamics of quadruped robots in the plane of motion, for particular gaits. In this case, the problem is often reduced to studying a two-legged robot, where each modeled leg represents two of the robot physical legs [6,9,10].

An issue that is still not fully investigated is how the angles with which the legs strike the ground actually interact with the robot dynamics, and how they should be chosen to achieve a gait of desired characteristics. The understanding of mechanisms involving the leg touchdown angles is of particular importance, as the legs can be brought to their desired touchdown positions during the robot flight phase with little actuator effort. Therefore, using the touchdown angles as control inputs, may drastically reduce the overall actuator effort.

To some extent, the effect of the leg touchdown angles on the dynamics of legged robots has been addressed, and incorporated into robot control. For example, as early as Raibert's work on the hopping monopod, the effect of the leg touchdown angle on the robot motion has been investigated, and used in the robot control [1]. Since then, many studies, particularly using the SLIP model and its variants, have investigated how the leg touchdown angle may be used for controlling forward speed or the apex height of the robot. Some work has also addressed issues in the case of quadruped systems [1,6,12].

One avenue that has not yet been explored fully is how the touchdown angles interact with pitching dynamics, and to what extent they can be used to suppress pitching, in gaits where it is undesirable. A common example of such a gait is the pronking gait, in which the robot body ought to be always level and not to rotate. In this work, it is shown how the difference of the leg touchdown angles may be used for controlling the pitching motion in the case of quadruped pronking. Further, it is found that there exists a particular robot design that is favorable for this purpose. Finally, application of the control to the quadruped robot model is shown to be successful.

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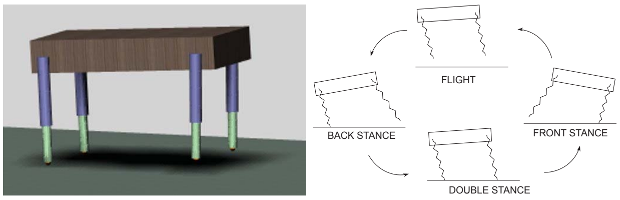
II. QUADRUPED ROBOT

A. Robot design and gait evolution

The quadruped robot studied has springy legs actuated at the hips, which is the only robot actuation, see Fig. 1a. This is the type of quadruped that the Scout II robot belongs to, [5], which has successfully run with a variety of gaits.



(a) (b)

Fig. 1 Quadruped robot and gait phases.

As mentioned, the aim of this work is to control the pitching of the robot during the pronking gait. In pronking, pitching should be absent for the gait to be properly performed. In the ideal pronk with no pitching, all legs are always in phase. This means that all the legs strike the ground at the same instant, and leave the ground at the same instant once the stance phase is complete. However, in the non-ideal case, where for some reason there is pitching of the robot body, the back or front legs may strike the ground first. In this case the pronk actually reduces to the bounding gait.

Therefore, observing the robot in the plane of the forward motion, see Fig. 1b, there may generally be up to four motion phases in the non-ideal case. In ideal pronking, as explained above, only the double stance phase and flight phase exist, see Fig. 1b. Next, a planar model of the robot is laid out for use in control design.

B. Simplified model for control design

The planar model is shown in Fig. 2. The body has its center of mass (CoM) at its geometrical center, and is supported on two springy legs. The legs each have total mass m_l, inertia I_l, and are actuated by torques τ_f, τ_b at the two hips. Each model leg has twice the mass, inertia and spring stiffness of a robot leg and includes viscous friction, of viscous coefficient b. Table 1 shows quantities used.

The influence of the leg mass on stance phase dynamics is negligible, and for the double stance phase a Lagrangian approach is used for the model dynamics, using body Cartesian coordinates, x, y, and pitch, θ, as generalized variables. Leg angles γ_b, γ_f, and lengths l_b, l_f, appear in the dynamics as functions of x, y, θ, see Fig. 2, and are substituted for compactness, so the dynamics is:

$$\begin{aligned}&m_b\ddot{x}+k(L-l_b)\sin\gamma_b-b\cdot\dot{l}_b\sin\gamma_b+\tau_b\cos\gamma_b/l_b\\&+k(L-l_f)\sin\gamma_f-b\cdot\dot{l}_f\sin\gamma_f+\tau_f\cos\gamma_f/l_f=0\end{aligned}\quad(1)$$

$$\begin{aligned}&m_b\ddot{y}+mg-k(L-l_b)\cos\gamma_b+b\cdot\dot{l}_b\cos\gamma_b+\tau_b\sin\gamma_b/l_b\\&-k(L-l_f)\cos\gamma_f+b\cdot\dot{l}_f\cos\gamma_f+\tau_f\sin\gamma_f/l_f=0\end{aligned}\quad(2)$$

$$\begin{aligned}&I_b\ddot{\theta}-bd\cos(\gamma_b-\theta)\dot{l}_b+bd\cos(\gamma_f-\theta)\dot{l}_f\\&-(d\sin(\gamma_b-\theta)-l_b)\tau_b/l_b+(d\sin(\gamma_f-\theta)+l_f)\tau_f/l_f\\&+d\cdot k\cos(\gamma_b-\theta)(L-l_b)-d\cdot k\cos(\gamma_f-\theta)(L-l_f)=0\end{aligned}\quad(3)$$

TABLE 1

VARIABLES AND INDICES USED IN THE WORK

x	CoM horizontal position	L	leg rest length
y	CoM vertical position	b	viscous friction coefficient
θ	body pitch angle	g	acceleration of gravity
l	leg length	m_l	leg mass
γ	leg absolute angle	I_l	leg inertia
γ_{sum}	sum of leg absolute angles	$\gamma_{b,td}$	back leg touchdown angle
γ_{dif}	difference of leg absolute angles	$\gamma_{f,td}$	front leg touchdown angle
k	leg spring stiffness	τ	hip torque
m_b	body mass	T_{st}	stance duration
m	total robot mass	f	*as index*: front leg
I_b	body inertia	b	*as index*: back leg
d	hip joint to CoM distance	td	*as index*: value at touchdown

The double stance dynamics above also yields the dynamics for the remaining stance phases by removing non pertinent terms. In flight, the system CoM performs a ballistic motion. Also, the angular momentum of the system of the body and two legs, with respect to the system CoM, is conserved:

$$H_O=D_1\dot{\gamma}_b+D_2\dot{\gamma}_f+D_3\dot{\theta}=const.\quad(4)$$

where D_1, D_2, D_3 are given by:

$$\begin{aligned}&D_1=(I_l m^2+l_1^2 m_1 m(m-m_1)-l_1^2 m_l^2 m\cos(\gamma_b-\gamma_f)\\&\quad-dl_1 m^2 m_l\sin(\gamma_b-\theta))\big/m^2\\&D_2=D_1+(l_1^2 m_l^2 m\cos(\gamma_b-\gamma_f)+dl_1 m^2 m_l\sin(\gamma_f-\theta))\big/m^2\\&D_3=I_b+2d^2 m_l-dl_1 m_l\sin(\gamma_b-\theta)+dl_1 m_l\sin(\gamma_f-\theta))\end{aligned}\quad(5)$$

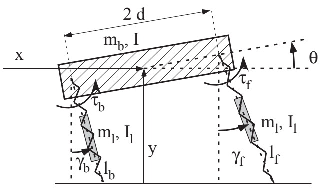


Fig. 2 Planar quadruped robot model.

Given that the difference between the leg angles is very small, the angular momentum can be written as:

$$H_0=(I+2d^2 m_l)\dot{\theta}+(I_l m^2+l_1^2 m_1 m(m-2m_1))(\dot{\gamma}_b+\dot{\gamma}_f)/m^2\quad(6)$$

where l_1 is the leg CoM to hip distance. The simplified form of (6) has the advantage of predicting body pitch orientation as it may be integrated.

III. DIFFERENCE OF LEG TOUCHDOWN ANGLES AS A CONTROL INPUT

The difference of the leg touchdown angles, $\gamma_{dif,td}$, is given by:

$$\gamma_{dif,td}=\gamma_{b,td}-\gamma_{f,td}\quad(7)$$

where $\gamma_{b,td}$, $\gamma_{f,td}$ denote the touchdown angles of the back and front legs respectively.

In order to use $\gamma_{dif,td}$ to control the pitching motion of the robot, it is necessary for it to appear in the pitching dynamics. Further, to be able to examine the pitching motion independent of the forward and vertical motions of the robot, the effect of $\gamma_{dif,td}$ on the dynamics of the forward and vertical dynamics should be very weak. In this section, it will be shown, for the case of the pronking gait, how the control input $\gamma_{dif,td}$ may appear in the pitching dynamics, as well as that it has only a very limited influence on the forward and vertical motions.

In ideal pronking, the only stance phase is the double stance. As mentioned in Section II, the slightest pitching in the pronking motion will introduce the back and front stance phases, see Fig. 1b. However, given that our goal is to suppress the pitching motion, the duration of these extra phases is considered negligible when compared to the double stance phase duration, and our model has the dynamics of (1) to (3), during the stance phase. This dynamics may take on a simpler form by assuming the leg angles and pitch angle to be small, writing out the leg lengths as functions of y, θ, γ_b, γ_f, see Fig. 2, and approximating the leg lengths by the leg rest length L for the terms that include torques:

$$m_b\ddot{x} + k(L-y)\sin\gamma_{sum} = -(\tau_b + \tau_f)/L \qquad (8)$$

$$m_b\ddot{y} + 2ky + 2b\dot{y} + mg - k\cdot L(\cos\gamma_b + \cos\gamma_f) = 0 \qquad (9)$$

$$I_b\ddot{\theta} + 2d^2k\theta + 2bd^2\dot{\theta} + k\cdot Ld(\cos\gamma_b - \cos\gamma_f) + \tau_b + \tau_f = 0 \qquad (10)$$

For a gait of speed $\dot{x}$, and during stance, the evolutions of the leg angles can be taken as [1]:

$$\gamma_i = \gamma_{i,td} - \dot{x}t/L \qquad (11)$$

where $i = b, f$, $\gamma_{i,td}$ is the leg touchdown angle, and time t counts from each leg touchdown. Using trigonometry and (11), the double stance dynamics then become:

$$m_b\ddot{x} + k(L-y)\sin\left(\gamma_{sum,td} - \frac{2\dot{x}t}{L}\right) = -\frac{\tau_b + \tau_f}{L} \qquad (12)$$

$$m_b\ddot{y} + 2(b\dot{y} + ky) = -m_b g + 2kL\cos\left(\frac{\gamma_{sum,td}}{2} - \frac{\dot{x}t}{L}\right)\cos\left(\frac{\gamma_{dif,td}}{2}\right) \qquad (13)$$

$$\begin{aligned} &I_b\ddot{\theta} + 2d^2b\dot{\theta} + 2d^2k\theta = \\ &2kLd\sin\left(\frac{\gamma_{sum,td}}{2} - \frac{\dot{x}t}{L}\right)\sin\left(\frac{\gamma_{dif,td}}{2}\right) - \tau_b - \tau_f \end{aligned} \qquad (14)$$

where $\gamma_{sum,td}$ is the sum of the leg touchdown angles, $\gamma_{dif,td}$ is the difference of the leg touchdown angles.

In (14) it is evident that the difference of the leg touchdown angles, $\gamma_{dif,td}$, is a strong control input to the pitching dynamics. Further, from (12), (13) it is apparent that, other than the pitching dynamics, only the vertical dynamics in (13) have any explicit dependence on the control input $\gamma_{dif,td}$. In this case, $\gamma_{dif,td}$ appears as the argument of a cosine. As seen in Section II, in the ideal pronking gait the difference of the leg touchdown angles, $\gamma_{dif,td}$, is zero, and it is expected that in the controlled motion $\gamma_{dif,td}$ will assume some deviation around the zero value. As a result, the influence of $\gamma_{dif,td}$ on the vertical dynamics is very small. The forward motion is also independent of $\gamma_{dif,td}$, as it relates only to the vertical dynamics, as seen in (12).

Overall, $\gamma_{dif,td}$ does not significantly affect the forward and vertical dynamics. However, as is evident from (14), the difference of the touchdown angles has a strong influence on the pitching dynamics. Therefore, $\gamma_{dif,td}$ is a suitable control input for suppressing the pitching motion in the pronking gait.

Here, it should be mentioned that, using $\gamma_{dif,td}$ to control the pitching motion of the robot, leaves available the sum of the leg touchdown angles, $\gamma_{sum,td}$, and the hip torques, τ_b, τ_f, for controlling the forward and vertical motions of the robot. It is significant to the analysis, later on, that the hip toques are made to apply constant torques of some value during the entire stance phase. Also, it has been found that a suitable combination of applied hip torque in stance and sum of touchdown angles may be found to control the robot's forward and vertical motion. This is not developed further here, as it deviates from the object of controlling the pitching motion of the robot.

IV. CONTROL DESIGN

In this section it is shown how the difference of the leg touchdown angles, $\gamma_{dif,td}$, can be used to control the pitching motion in the pronking gait.

A. Solving pitching dynamics

The pitching dynamics for the double stance phase are given in (14). Recalling that the hip torques applied during stance are constant, as explained in Section III, the pitching dynamics may be simply integrated, yielding the evolution of the body pitch angle and the body pitch velocity during the stance phase, given the initial conditions at the touchdown, see Fig. 3a. Here, the evolution of the pitch velocity of the body is given:

$$\begin{aligned} \dot{\theta} = &-\frac{1}{I_b}\exp\left(\frac{-bd^2t}{I_b}\right)\Big(bd^2\left(c_1\cos(\omega t) - c_2\sin(\omega t)\right) \\ &+\sqrt{-b^2d^4 + 2d^2I_bk}\left(c_2\cos(\omega t) + c_1\sin(\omega t)\right)\Big) \\ &-\frac{\left(2bd^2L\dot{x}\right)2dkL^2\dot{x}\sin\left(\gamma_{dif,td}/2\right)}{-4d^2I_bkL^2\dot{x}^2 + I_b^2\dot{x}^4 + 4d^4(k^2L^4 + b^2L^2\dot{x}^2)} \\ &\left(\frac{2d^2kL^2 - I_b\dot{x}^2}{2bd^2L\dot{x}}\cos\left(\frac{\gamma_{sum,td}}{2} - \frac{\dot{x}t}{L}\right) - \sin\left(\frac{\gamma_{sum,td}}{2} - \frac{\dot{x}t}{L}\right)\right) \end{aligned} \qquad (15)$$

where the constant ω is given by:

$$\omega = \sqrt{-b^2d^4 + 2d^2I_bk}\Big/I_b \qquad (16)$$

and the quantities c_1, c_2, are given in the Appendix.

Replacing time t in (15) with the duration of the stance phase, the pitch velocity of the robot body at the moment of liftoff is computed. Then, collecting terms which include the difference of the leg touchdown angles $\gamma_{dif,td}$, it is possible to write the pitch velocity of the body at liftoff in the form:

$$\dot{\theta}_{lo} = a_0 + a_1 \sin\left(\gamma_{dif,td}/2\right) \tag{17}$$

where quantities a_0, a_1, are given in the Appendix.

Requiring that the pitch velocity of the body has some desired value $\dot{\theta}_{lo,des}$ at the moment of liftoff, it is possible to solve (17) for the necessary $\gamma_{dif,td}$ to achieve this desired pitch velocity:

$$\gamma_{dif,td} = 2\arcsin\left((-a_0 + \dot{\theta}_{lo,des})/a_1\right) \tag{18}$$

Equation (18) provides the necessary $\gamma_{dif,td}$ for the pitch velocity of the body at liftoff, see Fig. 3c, to have the desired value $\dot{\theta}_{lo,des}$.

The issue now is to find a correct value for $\dot{\theta}_{lo,des}$, so that the aim of suppressing the pitching motion during the pronking gait is achieved. Initially, one thought may be to set the $\dot{\theta}_{lo,des}$ to zero. This would indeed be a suitable choice, if the legs of the robot were massless. However, they are not, and therefore, even if the desired liftoff velocity was controlled to be zero, the positioning of the robot legs at the beginning of the flight phase would provoke some body pitching, due to the conservation of angular momentum during flight. The greater the inertia of the robot legs, the greater the pitching velocity the body acquires would be. Therefore, a better option for the value of the desired body pitch velocity, $\dot{\theta}_{lo,des}$, at liftoff is to select such a value that *after* leg positioning the body pitch velocity, $\dot{\theta}_{alp}$, will be zero, see Fig. 3.

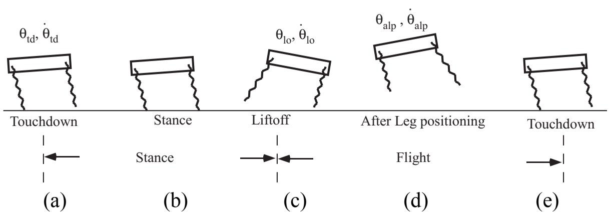


Fig. 3 Sequence of events during the quadruped gait.

To find this value, the conservation of angular momentum during flight is used, as expressed in (4). Solving (4) for the body pitch velocity after the legs are positioned, we acquire:

$$\dot{\theta}_{alp} = H_O/D_3 = (D_1\dot{\gamma}_{b,lo} + D_2\dot{\gamma}_{f,lo} + D_3\dot{\theta}_{lo,des})/D_3 \tag{19}$$

Requiring that the pitch velocity, after the legs are positioned, is zero, the desired pitch velocity of the robot body at liftoff can be found as:

$$\dot{\theta}_{lo,des} = -D_1\dot{\gamma}_{b,lo} - D_2\dot{\gamma}_{f,lo}/D_3 \tag{20}$$

In (20), the two leg rotation velocities $\dot{\gamma}_{lo}$ may be given by (11), assuming some forward speed $\dot{x}$:

$$\dot{\gamma}_{b,lo} = \dot{\gamma}_{f,lo} = -\dot{x}/L \tag{21}$$

B. *Analysis of $\gamma_{dif,td}$ for the steady state motion*

Observing the expressions of the quantities a_0, a_1, involved in (18), and given in the Appendix, it can be seen that only a_0 has a dependence on the initial conditions of the pitching motion at touchdown. The quantity a_1 has no such dependence. In the steady state motion of the robot, when the pitching motion of the body has been suppressed, it is considered that:

$$\theta_{td,ss} = 0, \quad \dot{\theta}_{td,ss} = 0 \tag{22}$$

where *ss* denotes the steady state motion.

Then, considering the steady state, and with reference to the Appendix, the quantity a_0 in steady state becomes:

$$a_{0,ss} = \frac{-\exp\left(-bd^2T_{st}/I_b\right)}{\sqrt{-b^2d^4 + 2d^2I_bk}}(\tau_b + \tau_f)\sin\left(\omega T_{st}\right) \tag{23}$$

The value of a_1 remains unchanged, as it has no dependence on the initial conditions. For the moment, let the robot legs be massless, in which case the desired pitch velocity at liftoff, $\dot{\theta}_{lo,des}$, is zero, as found by (20). In this case, and in the steady state, the required difference of the leg touchdown angles to *maintain* the absence of any pitching can be found from (18) to be:

$$\gamma_{dif,td,ss} = 2\arcsin\left(-a_{0,ss}/a_{1,ss}\right) \tag{24}$$

Ideally, the legs are in phase in the pronking gait, and so it is desired for the $\gamma_{dif,td,ss}$ in steady state, given in (24), to be zero. Observing (24), the $\gamma_{dif,td,ss}$ required in the steady state motion is zero when the quantity $a_{0,ss}$ becomes zero. Examining (23), it can be seen that $a_{0,ss}$ becomes zero for a special selection of the robot design parameters. Specifically, the sine in (23) becomes zero when its argument, ωT_{st}, has a particular value:

$$\omega T_{st} = n \cdot \pi \tag{25}$$

for $n = 0,1,2,3...$

It has been found that for $n = 0,2,$ and over, that the design implications are less realistic. On the other hand, when taking n to be equal to 2, the design constraint is interesting. For n equals 1, and using (16) to provide ω, it is possible to solve (25) for the body inertia I_b:

$$I_b = (d^2kT_{st}^{\ 2} - \sqrt{-4b^2d^4\pi^2T_{st}^{\ 2} + d^4k^2T_{st}^{\ 4}})/(4\pi^2) \tag{26}$$

In fact, solving (25) for I_b provides two solutions, but that of (26) is of interest. A further interesting result is derived by using the common assumption for the duration of the stance phase, [1]:

$$T_{st} = \pi\, m/(2k) \tag{27}$$

Note that, in (27), double the leg stiffness is substituted for the leg stiffness in the expression in [1], as both the back and front legs are in contact with the ground at the same time. Using the stance duration given in (27) and substituting in (26), the result is:

$$I_b = \left(1 + \sqrt{1 - 2b^2/mk}\right)md^2/2 \tag{28}$$

As can be seen from (28), requiring the necessary difference of the leg touchdown angles to be 0 in steady state, imposes a design constraint on the robot. As the viscous friction forces in the leg are small, when compared to the spring forces, (28) dictates that the best robot design is that for which the inertia of the robot body is very close to md^2. Essentially, this means that the dimensionless inertia of the robot $j = I_b/(md^2)$ will be close to 1. Concerning the implementation of this design, it can generally be achieved without adding or removing mass from the robot, simply by changing the inertia of the robot body by rearranging the mass of the robot body, or changing the distance between the hips.

V. RESULTS

In this section the pitching control method is applied to the model of the quadruped robot. First, a robot is simulated with a leg spring stiffness of k =14 kN/m, a body mass of m_b =13 kg, and a leg rest length of L =0.25 m. The robot legs each have a non-trivial mass of 0.61 kg, and the viscous friction in the leg has a coefficient of b =20 N·s/m. The mass distribution on the body of the robot and the distance between the hips are such that the requirement of the design guideline in (28) is met. This gives a body inertia of I_b =0.81 kg·m^2, for half a hip spacing of d =0.25 m. The robot executes a pronking gait in a large range of speeds, as shown below.

The pitching motion is controlled by setting the difference of the leg touchdown angles, $\gamma_{dif,td}$, as in (18). It is worth noting that the maximum acceptable value for $\gamma_{dif,td}$ has been set to a deviation of 10 degs from the zero value, so that the assumptions of Section III are satisfied. Also, the sum of the leg touchdown angles and the hip torques applied during stance are chosen so as to control the forward and vertical motion of the robot, as explained in Section III. The robot response is shown in Fig. 4.

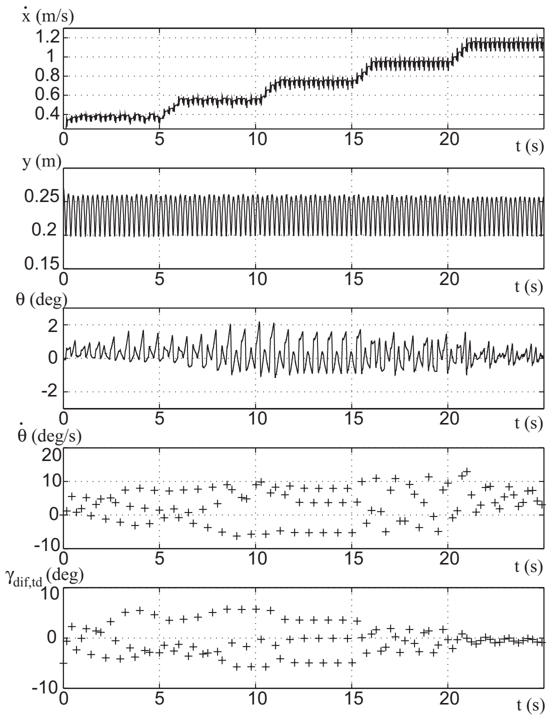


Fig. 4 Response of the robot to the control on the pitching motion, depicting the forward speed, the body height, the pitch angle, the pitch velocity at the apex of each flight phase, and the required $\gamma_{dif,td}$.

As can be seen, the pitching of the robot is successfully controlled, despite the robot running with speeds from as low as 0.35 m/s to speeds as high as about 1.3 m/s. The body height is a regulated quantity and may be chosen to be more or less than that used here. The pitch angle of the robot body typically stays within just 2 degs of the horizontal position. Note that typical bounding gaits may involve pitching of 10 degs or more.

To ensure that the control method is valid for a wide range of robots, many simulations have been run. To provide another example, a robot with a much softer leg than before is now studied. The stiffness of the leg spring is now set to k =7 kN/m, while maintaining the values of the remaining parameters. The results are shown in Fig. 5, for the same range of speeds as before. In Fig. 5, the forward speed and body height have a similar form to Fig. 4, and are omitted. Again, it can be seen that the pitching is very limited, while the control input $\gamma_{dif,td}$ takes on small values around the zero value.

In Fig. 6, the actuator torque and individual touchdown angles are shown for the second example, to illustrate that these quantities take realistic values during the motion.

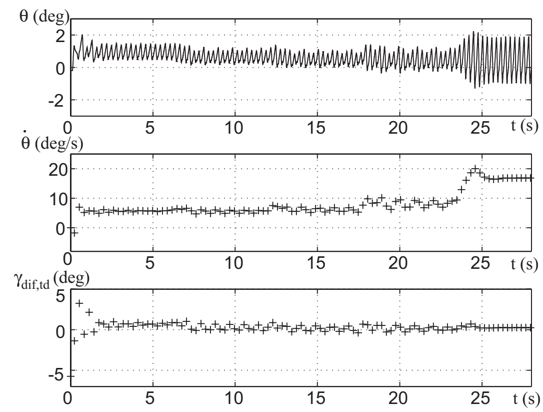


Fig. 5 Response to the pitching control method. The pitch angle, the pitch velocity at each flight apex, and the required $\gamma_{dif,td}$ are shown.

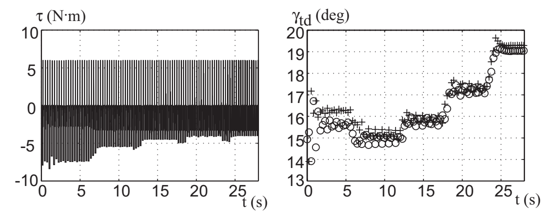


Fig. 6 Actuator applied torque and individual leg touchdown angles, where (+) denotes the back leg and (o) denotes the front leg.

Finally, a case of error in robot physical parameter estimation is considered, to further evaluate the control behavior. Specifically, the simulation of Fig. 5 is run again, but now with a 10% overestimation in key parameters of the robot, such as the body mass, both leg springs and the viscous friction coefficient in the legs. The robot response is shown in Fig. 7. As can be seen, despite the large estimation errors, the response remains stable and the pitching motion is successfully controlled. As is expected, in this case pitching is more exaggerated.

In both cases it can be seen that $\gamma_{dif,td}$ is not always exactly zero in the steady state parts of the motion where the speed is constant, as predicted by the results in Subsection B of Section IV, despite the fact that in both examples the robot fulfilled the design guideline of (28). There are two main reasons for this, the first of which is the fact that (27) provides the stance duration only approximately. Secondly, to arrive at the design guideline expressed in (28), the robot legs have been assumed to be massless, which in both the above examples they are not. Despite this, $\gamma_{dif,td}$ still remains small and successfully limits the pitching motion.

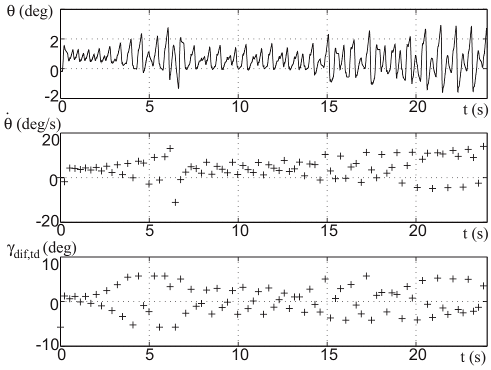


Fig. 7 Robot response in case of 10% key parameter estimation error.

Overall, it can be seen that the control method is successful in limiting the pitching motion of the body, typically to a range of 2 degs. Without the pitching control, execution of the pronking gait would not be possible and the stability of the pitching motion would not be ensured, which generally results in failure of the complete robot motion.

CONCLUSIONS

In this work, it was shown how the pitching of a quadruped robot executing a pronking gait was controlled only by defining the difference of the leg touchdown angles each cycle. The analysis of the pitching motion also leads to an interesting design rule. Specifically it was found that it is advantageous for the robot to fulfill a particular design constraint. Simulations of the robot model showed the control to be successful for a variety of robot parameters and a wide range of forward speeds.

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APPENDIX A

The expressions for c_1, c_2 in (15) are:

$$\begin{aligned} c_1 = &\Big((2d^2k\theta_{td} + \tau_b + \tau_f)(4d^4k^2L^4 + \\ &4d^2(b^2d^2 - I_bk)L^2\dot{x}^2 + I_b{}^2\dot{x}^4) + 4d^3k^2L^3 \cdot \\ &\sin\left(\gamma_{dif,td}/2\right)(-2bd^2L\dot{x}\cos\left(\gamma_{sum,td}/2\right) + \\ &(-2d^2kL^2 + I_b\dot{x}^2)\sin\left(\gamma_{sum,td}/2\right))\Big)\Big/ \\ &\Big(2d^2k(4d^4k^2L^4 + 4d^2(b^2d^2 - I_bk)L^2\dot{x}^2 + I_b{}^2\dot{x}^4)\Big) \end{aligned} \tag{A1}$$

$$\begin{aligned} c_2 = &\frac{1}{2kI_b\omega}\Big((-2ibk\dot{\theta}_{td} - b(2d^2k\theta_{td} + \tau_b + \tau_f)) \\ &(2d^2k(4d^4k^2L^4 + 4d^2(b^2d^2 - I_bk)L^2\dot{x}^2 + I_b{}^2\dot{x}^4)) + \\ &4dk^2L^2\sin(gdif/2) \\ &(\dot{x}(2d^2(b^2d^2 \text{ - } ibk)L^2 + I_b{}^2\dot{x}^2)\cos\left(\gamma_{sum}/2\right) + \\ &bd^2L(2d^2kL^2 + I_b\dot{x}^2)\sin(\gamma_{sum}/2))\Big)/ \\ &(2d^2k(4d^4k^2L^4 + 4d^2(b^2d^2 - I_bk)L^2\dot{x}^2 + I_b{}^2\dot{x}^4)) \end{aligned} \tag{A2}$$

The expressions for a_0, a_1 in (17) are:

$$\begin{aligned} a_0 = &\frac{1}{\sqrt{-b^2d^4 + 2d^2I_bk}}\exp\left(-bd^2T_{st}/I_b\right)\cdot \\ &\left(\cos\left(\omega T_{st}\right)\sqrt{-b^2d^4 + 2d^2I_bk}\,\dot{\theta}_{td} - \right. \\ &\left.(bd^2\dot{\theta}_{td} + 2d^2k\theta_{td} + \tau_b + \tau_f)\sin\left(\omega T_{st}\right)\right) \end{aligned} \tag{A3}$$

$$\begin{aligned} a_1 = &\frac{2dkL(L + s_{pl})\exp(-bd^2T_{st}/I_b)}{(\omega I_b(4d^4k^2L^4 + 4d^2(b^2d^2 - I_bk)L^2\dot{x}^2 + I_b{}^2\dot{x}^4))}\cdot \\ &\Big(\sin\left(\gamma_{sum,td}/2\right)\Big(-2bd^2L\omega I_b\dot{x}^2\cos\left(\omega T_{st}\right) + \\ &2bd^2\exp(bd^2T_{st}/I_b)L\omega I_b\dot{x}^2\cos\left(T_{st}\dot{x}/L\right) + \\ &2d^2L(2d^2k^2L^2 + (b^2d^2 - I_bk)\dot{x}^2)\sin\left(\omega T_{st}\right) - \\ &2d^2\exp(bd^2T_{st}/I_b)kL^2\omega I_b\dot{x}\sin\left(T_{st}\dot{x}/L\right) + \\ &\exp(bd^2T_{st}/I_b)I_b\omega I_b\dot{x}^3\sin\left(T_{st}\dot{x}/L\right)\Big) - \\ &\dot{x}\cos\left(\gamma_{sum,td}/2\right)\Big(\omega I_b(-2d^2kL^2 + I_b\dot{x}^2)\cos\left(\omega T_{st}\right) + \\ &\exp((bd^2T_{st})/I_b)\omega I_b(2d^2kL^2 - I_b\dot{x}^2)\cos\left(T_{st}\dot{x}/L\right) + \\ &bd^2\left((-2d^2kL^2 - I_b\dot{x}^2)\sin\left(\omega T_{st}\right) + \right. \\ &\left.2\exp(bd^2T_{st}/I_b)L\omega I_b\dot{x}\sin(T_{st}\dot{x}/L)\right)\Big)\Big) \end{aligned} \tag{A4}$$

where the constant ω is given in (16).