Proceedings IEEE International Conference on Robotics and Automation, ICRA 01, May 21-26 2001, Seoul, Korea

Planning and Obstacle Avoidance for Mobile Robots

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ABSTRACT

A planning methodology for nonholonomic mobile manipulators that employs smooth and continuous functions such as polynomials is developed. The method decouples kinematically the manipulator from the platform by constructing admissible paths that drive it to a final configuration and is based on mapping the nonholonomic constraint to a space where it can be trivially satisfied. In addition, the method allows for direct control over the platform orientation. The developed transformation also maps Cartesian space obstacles to transformed ones and allows for obstacle avoidance by increasing the order of the polynomials that are used in planning trajectories. The additional parameters required are computed systematically. It is shown how the method can be extended for avoiding obstacles of any number.

1. INTRODUCTION

Mobile manipulator systems, consisting of a mobile platform equipped with manipulators, are of great importance mainly due to their ability to reach targets that are initially outside of the manipulator reach. Applications for such systems abound in mining, construction, forestry, planetary exploration and the military. A wide category of such systems employs wheeled mobile robots, which is the system under study in this paper.

A very important problem in mobile robots is motion planning which is concerned with obtaining open loop controls that steer the platform from an initial state to a final one, without violating the nonholonomic constraints. The idea of employing piecewise constant inputs to generate motions in the directions of iterated Lie brackets has been exploited by Lafferriere and Sussmann, [1]. Murray and Sastry used sinusoids at integrally related frequencies to steer systems in power or chained form, [2]. A survey of recent developments in control of nonholonomic systems can be found in [3].

None of the above methods takes explicitly under consideration obstacles in the workspace. Barraquand and Latombe used an exhaustive search based method that explores the system's configuration space by propagating step motions corresponding to some controls, [4]. Laumond et al. use families of linear segments and circular arcs to transform paths, calculated by a geometric planner ignoring the motion constraints, into feasible ones, [5]. However, the above geometric planner returns paths with discontinuous curvature profiles. Fleury et al. used *clothoids* and *anticlothoids* to replace a collision free trajectory, consisting of straight directed line segments, by a smooth, time optimal one, [6].

Moving mobile manipulator systems present many unique problems that are due to the coupling of holonomic manipulators with nonholonomic bases. Seraji presents a simple on line approach for motion control of mobile manipulators using augmented Jacobian matrices, [7]. The approach is kinematic and requires additional constraints to be met for the manipulator configuration. Perrier et al. represent the nonholonomy of the vehicle as a constraint displacement and the proposed method tries to make the global feasible displacement of the system correspond to the desired one [8]. Tanner and Kyriakopoulos presented a motion planner for a mobile manipulator in cluttered environment based on discontinuous feedback law under the influence of a special potential field [9]. A planning and control technique for mobile manipulator systems was presented, allowing them to follow simultaneously desired end-effector and platform trajectories without violating the nonholonomic constraints [10].

In this paper, we first develop a planning methodology for nonholonomic mobile manipulators that uses smooth and continuous functions such as polynomials. The method is based on a transformation from Cartesian space to a space where the nonholonomic constraint can be trivially satisfied. Moreover, because the orientation of the mobile base is explicitly controlled, the method results in a minimum number of cusps. The developed transformation also maps Cartesian space obstacles to the transformed space and allows obstacle avoidance take place by increasing the order of the polynomials that are employed. The additional parameters required are computed systematically. The method can be extended for use in planning motions in the presence of obstacles of any number and shape.

2. MOBILE MANIPULATOR KINEMATICS

A prerequisite for the successful deployment of mobile manipulator systems such as the one depicted in Fig. 1, is the availability of a planning methodology that can generate feasible paths for driving the end effector to the desired coordinates.

Hence, the central problem that must be tackled is to find admissible paths to steer the system from some initial configuration to a desired final one. The calculated paths must satisfy the nonholonomic constraint of the mobile platform, must be able to steer the system away from obstacles, which may exist in its workspace, and should be computationally inexpensive to compute.

To find these paths, the system is decoupled kinematically into two subsystems, the holonomic manipulator and the nonholonomic platform. An advantage of this decoupling is that it is very simple to extend the method to mobile systems with multiple manipulators on board. For simplicity reasons, we concentrate here to a mobile system, which consists of a two dof manipulator mounted on a differentially driven mobile platform, see Fig. 1. However the developed methodology can be applied equally well to systems with N dof manipulators, or to car-like mobile platforms.

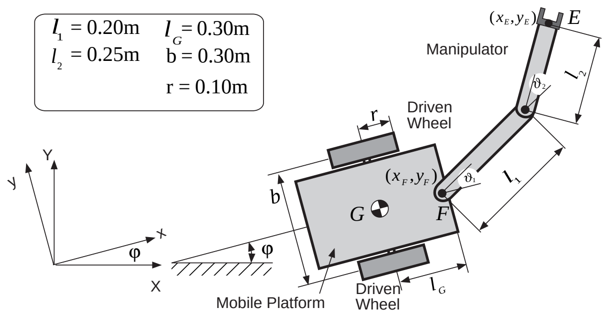


Fig. 1. Mobile manipulator system on a differentially driven platform.

2. 1. Holonomic Manipulator Subsystem

The Cartesian coordinates of the end effector E relative to the world frame are given by, see Fig. 1,

$$x_E = x_F + l_1 \cos(\varphi + \vartheta_1) + l_2 \cos(\varphi + \vartheta_1 + \vartheta_2) \qquad (1)$$

$$y_E = y_F + l_1 \sin(\varphi + \vartheta_1) + l_2 \sin(\varphi + \vartheta_1 + \vartheta_2) \qquad (2)$$

where (x_F, y_F) is the position of mounting point F of the mobile platform, φ is the platform orientation, ϑ_1 and ϑ_2 represent the joint angles and l_1 and l_2 denote the link lengths of the manipulator arm. Eqs. (1)-(2) show that the position of the end-effector depends on the position and the orientation of its base on the mobile platform. If the motion of this base is planned and available, it is trivial to compute trajectories for the end-effector.

The direct kinematics equations given by Eqs. (1) and (2) can be inverted. Thus, we can find ϑ_1 and ϑ_2 which correspond to a given end-effector position (x_E, y_E) when the platform position (x_F, y_F) and orientation φ are known, see [11].

The existence of a solution of the inverse kinematics problem requires that

$$|c\varphi| \le 1 \Rightarrow (x_E - x_F)^2 + (y_E - y_F)^2 \le (l_1 + l_2)^2 \qquad (3)$$

If the above inequality is not satisfied at some point, then the target is outside the manipulator reach and thus the mobile platform must move in order to bring the target into the manipulator's workspace.

2. 2. Nonholonomic Mobile Platform Subsystem

The mobile platform we study here consists of two independently driven wheels, see Fig. 1. However, the analysis that follows applies equally well to car-like mobile platforms, [11]. We assume that the speed at which the system moves is low and therefore the two driven wheels do not slip sideways. Hence, the velocity of the platform center of mass, υ_G, is normal to the wheel axis. This results to a constraint among velocities, which for the manipulator attachment point F, see Fig. 1, is,

$$\dot{x}_F \sin\varphi - \dot{y}_F \cos\varphi + \dot{\varphi} l_G = 0 \qquad (4)$$

Eq. (4) represents a nonholonomic constraint and as is well known, it cannot be integrated analytically to result in a constraint between the configuration variables of the platform, namely, x_G, y_G, and φ. Also, the configuration space of this system is three-dimensional (completely unrestricted) while the velocity space is two-dimensional.

The mobile platform control variables include the angular velocities of the left and the right independently driven wheels, ϑ_ℓ and ϑ_r respectively. The Cartesian output velocities of point F $(\dot{x}_F, \dot{y}_F, \dot{\varphi})$ are related to the control variables $(d\vartheta_\ell / dt, d\vartheta_r / dt)$

$$\begin{bmatrix} \dot{x}_F \\ \dot{y}_F \\ \dot{\varphi} \end{bmatrix} = \begin{bmatrix} \frac{r}{2} c\varphi + \frac{l_G r}{b} s\varphi & \frac{r}{2} c\varphi - \frac{l_G r}{b} s\varphi \\ \frac{r}{2} s\varphi - \frac{l_G r}{b} c\varphi & \frac{r}{2} s\varphi + \frac{l_G r}{b} c\varphi \\ -\frac{r}{b} & \frac{r}{b} \end{bmatrix} \begin{bmatrix} \dot{\vartheta}_\ell \\ \dot{\vartheta}_r \end{bmatrix} \qquad (5)$$

If one eliminates the two control variables in Eq. (5), Eq. (4) results. Eq. (5) shows that the output velocities are nonzero even when only one wheel is driven.

3. PATH PLANNING

A mobile system is especially useful when the manipulator task is outside the manipulator reach. Therefore, in this section we assume that this is the case, in other words that Eq. (3) is not satisfied for a given target.

In this section, we focus our attention in finding a path for the mobile platform, which connects its initial configuration $\left(x_F^{in}, y_F^{in}, \varphi^{in}\right)$ to the final one $\left(x_F^{fin}, y_F^{fin}, \varphi^{fin}\right)$. This problem is not trivial, because we must satisfy the nonholonomic constraint and achieve a change in a three dimensional configuration space with only two controls.

The nonholonomic constraint of the differentially-driven mobile platform given by Eq. (4) is scleronomic and thus it can be written in the Pfaffian form

$$P(x_F, y_F, \varphi)dx_F + Q(x_F, y_F, \varphi)dy_F + R(x_F, y_F, \varphi)d\varphi = 0 \qquad (6)$$

where,

$$P = \sin\varphi, \; Q = -\cos\varphi, \; R = l_G$$

Although Eq. (6) can be integrated numerically by specifying trajectories for two of its variables, e.g. x_F and y_F, and by calculating the third one e.g. φ, the final desired value for φ can not be met, since it will depend on the specific trajectories the other two variables follow to their final values.

Planning can be facilitated if this form is transformed to one in which only two differentials appear. This is indeed possible because it is known, see [12], that nonintegrable Pfaffian equations of the form of Eq. (6) can be written as,

$$du + v\,dw = 0 \tag{7}$$

where u, v, and w should be properly selected functions of the platform position and orientation, x_F, y_F, and φ. To find these functions, one must construct and solve the partial differential equations that u, v, and w must satisfy. The procedure is described in detail in [11]. In a nutshell, this leads to the following equations

$$u(x_F, y_F, \varphi) = x_F \cdot \sin\varphi - y_F \cdot \cos\varphi \tag{8}$$

$$v(x_F, y_F, \varphi) = l_G - x_F \cdot \cos\varphi - y_F \cdot \sin\varphi \tag{9}$$

$$w(x_F, y_F, \varphi) = \varphi \tag{10}$$

Eqs. (8)-(10) constitute a transformation $(x_F, y_F, \varphi) \rightarrow (u, v, w)$, which is defined at every point of the configuration space of the system. With this, Eq. (6) is transformed to the form given by Eq. (7).

Once this is achieved, our attention is directed to choosing appropriate trajectories for u, v, and w. Note that if these are chosen as follows

$$w = f(t) \tag{11}$$

$$u = g(w) \tag{12}$$

$$v = -\frac{du}{dw} = -g'(w) \tag{13}$$

where t is the time, then Eq. (7) is satisfied identically. Therefore, the planning problem reduces to the selection of functions f and g such that satisfy the initial and final configuration variables.

For example, because of Eq. (11), f can be any function of time whose value at the initial and final time is equal to the initial and final orientation φ. Such functions can be polynomials, splines, or any other continuous and smooth time functions. Also note that due to Eqs. (10) and (11), the method gives complete control over the platform orientation, and therefore, paths with undesired cusps or orientation changes can be avoided by choosing appropriate functions for w.

Next, function g is constructed similarly, using Eqs. (8)-(10) for computing the initial and final values for u. Finally, v is computed using Eq. (13), which requires a simple differentiation. Once u, v, and w have been found, the platform coordinates are computed by inverting Eqs. (8)-(10).

Example 1. To illustrate the methodology described above, we employ the mobile system shown in Fig. 1. The main task for the system is to have the end-effector reach a desired target point with coordinates (x_E, y_E), while the platform must reach a desired position and orientation.

In order to find a path for the platform using the method presented above, we select functions f and g in Eqs. (11)-(13) to be respectively fifth and third order polynomials

$$f(t) = a_5\,t^5 + a_4\,t^4 + a_3\,t^3 + a_2\,t^2 + a_1\,t + a_0$$

$$g(w) = b_3\,w^3 + b_2\,w^2 + b_1\,w + b_0$$

where the coefficients above are calculated such that f and g satisfy the initial and final conditions of the motion. These conditions include positions, velocities and for the platform orientation, accelerations too.

For the simulation, the total time was chosen equal to 6s. Note that choosing a different total time will make the system move faster or slower, but will have no effect on the Cartesian path of the mobile platform.

Fig. 2 depicts snapshots of the motion of the mobile manipulator system while Fig. 3 depicts platform and manipulator joint velocities that correspond to the calculated path.

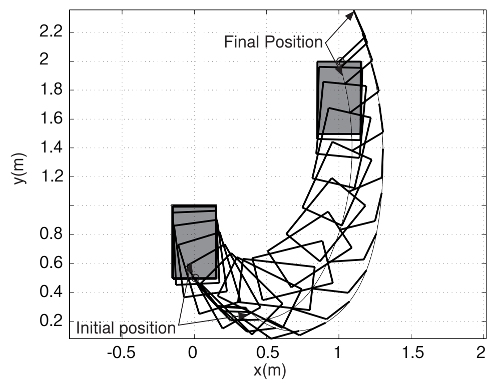


Fig. 2. Motion animation of the differential-drive mobile manipulator.

Figs. 2 and 3 show that all trajectories are smooth and that the mobile system starts and stops smoothly, moving from the initial configuration to the desired final one.

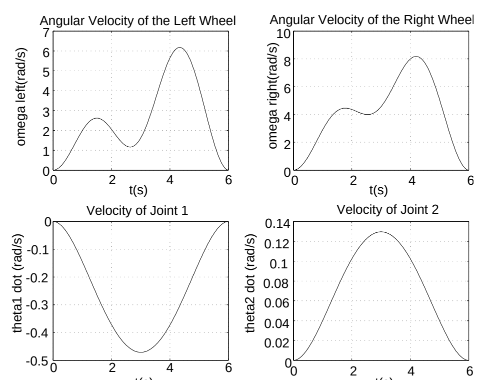


Fig. 3. Input velocities of the differentially-driven mobile manipulator.

4. OBSTACLE MAPPING

The planning methodology developed will be more useful if it can allow the construction of paths that avoid obstacles in the vicinity of the system. In this section we examine this problem, under the assumption that the location of obstacles in the workspace of the system is known and fixed.

Eqs. (8)-(10) map obstacles in the Cartesian x-y space into the u-v-w space. Mapping from a two dimensional to a three dimensional space adds one dimension which, in this case, corresponds to the orientation of the platform. Therefore, an obstacle is mapped to a family of obstacles whose members are identified by some value of the orientation of the platform.

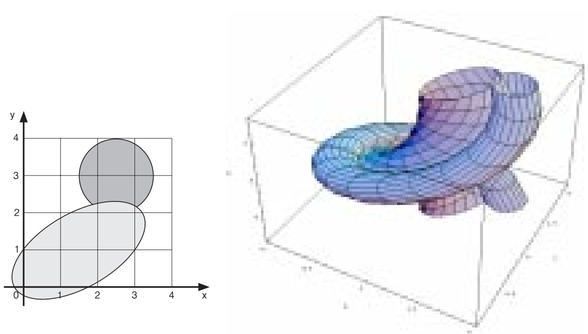
The developed transformation has some important properties, which simplify the problem significantly. Notice that Eqs. (8)-(10) can be written as

$$\begin{bmatrix} u \\ v \\ w \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos(\pi/2-\varphi) & -\sin(\pi/2-\varphi) & 0 & 0 \\ \sin(\pi/2-\varphi) & \cos(\pi/2-\varphi) & 0 & -l_G \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_F \\ y_F \\ \varphi \\ 1 \end{bmatrix}$$

$$\Rightarrow \mathbf{u} = \mathbf{T}_1\, \mathbf{T}_2\, \mathbf{x} \qquad (14)$$

where matrix $\mathbf{T}_1$ corresponds to a reflection and matrix $\mathbf{T}_2$ to a rotation by $\pi/2-\varphi$ and a translation by $-l_G$. Obviously the determinants of the above matrices are always nonzero. Therefore, Eqs. (8)-(10) constitute a global diffeomorphism in the configuration space.

It is well known that transformations such as rotations, translations and reflections, preserve both the length and the shape of an object. Since the map is a function of the orientation φ, a single object in the Cartesian space is mapped to a family of such objects that correspond to the range of φ that is considered. For example, ellipses in the Cartesian x-y space are transformed to a family of ellipses in the u-v-w space having the same axes length as the original ellipse and orientation which depends on the current orientation φ of the platform. The same properties hold for circles, see Fig. 4.



(a) (b)

Fig. 4. (a) Obstacles in the Cartesian and (b) in the transformed u-v-w space.

5. OBSTACLE AVOIDANCE

The planning method developed above did not take into account obstacles, which may exist in the workspace: If the planned path encounters an obstacle, collision will occur. It is obvious that to avoid an obstacle, we need to introduce in our planning scheme additional freedom.

A simple way to achieve this is to introduce additional coefficients in the polynomial $u(w)$ possibly depending on the number of the obstacles and their positions in the workspace. These additional coefficients should not affect the satisfaction of the initial and final conditions but should allow us to shape the trajectory in the u-v-w space and therefore, to avoid collisions with the transformed obstacles. Hence in this way, the problem of avoiding Cartesian obstacles is reduced to the problem of finding appropriate values for the additional coefficients. Due to the nature of the involved equations this is a much simpler problem, that results in a simple *analytical* solution without the use of intensive numerical searches.

As an example of the use of the method we first study the case of a single obstacle. The section ends by providing extensions for the case of multiple obstacles, which are assumed to be either circular or elliptic for simplicity.

5.1 Single Obstacle.

We assume the presence of a single obstacle and introduce an additional coefficient b_4 in the third order polynomial $u(w)$. Because the nonholonomic constraint must be satisfied during system's motion, the following equations for u and v must hold

$$u(w) = b_4\, w^4 + \sum_{i=0}^{3} b_i\, w^i \qquad (15)$$

$$v(w) = -4\, b_4\, w^3 - \sum_{i=0}^{3} i\, b_i\, w^{i-1} \qquad (16)$$

Introducing the initial and final conditions for u and w, which correspond to the initial and final positions of the vehicle, we obtain the following linear system with respect to the unknown coefficients b_i, $i = 0,...,3$,

$$\sum_{i=0}^{3} b_i\, w_{in}^i = u(w_{in}) - b_4\, w_{in}^4 \qquad (17)$$

$$\sum_{i=0}^{3} b_i\, w_{fin}^i = u(w_{fin}) - b_4\, w_{fin}^4 \qquad (18)$$

$$\sum_{i=0}^{3} i\, b_i\, w_{in}^{i-1} = -v(w_{in}) - 4\, b_4\, w_{in}^3 \qquad (19)$$

$$\sum_{i=0}^{3} i\, b_i\, w_{fin}^{i-1} = -v(w_{fin}) - 4\, b_4\, w_{fin}^3 \qquad (20)$$

Solving the above system we find b_i, $i = 0,...,3$, as functions of b_4. Therefore, Eqs. (15) and (16) along with the solution of Eqs. (17)-(20) yield the polynomials u and v which satisfy the constraint and the initial and final conditions as functions of the additional coefficient b_4. The problem now is to find a range of values of b_4 which lead to admissible paths that avoid the obstacle.

Circular Obstacles: For an obstacle enclosed in a circle of radius R and centered at (x_0, y_0), the distance between the center of the obstacle and the front point of the platform must be greater than the radius R of the obstacle increased by a characteristic length for increased safety margin. Making use of the fact that a circular obstacle is mapped in the u-v-w space onto a circle of the same radius, then for each $w = \varphi$, the following inequality must hold

$$\left(u(w)-u_0(w)\right)^2+\left(v(w)-v_0(w)\right)^2>\left(R+l_{cr}\right)^2 \quad (21)$$

for all w in $\lfloor w_{in}, w_{fin} \rfloor$, where $u(w)$ and $v(w)$ are the transformed coordinates of the front point of the vehicle and $u_0(w)$ and $v_0(w)$ are the transformed coordinates of the center of the obstacle for the corresponding orientation $w=\varphi$.

Substituting Eqs. (15) and (16) into Eq. (21) and after some algebraic manipulations, we obtain the following expression

$$\alpha b_4^2+\beta b_4+\gamma>0 \quad (22)$$

Eq. (22) is a very practical representation of the criterion for obstacle avoidance. The coefficients α, β and γ are known functions of w and of the initial and final conditions, $u_{in}, v_{in}, w_{in}, u_{fin}, v_{fin}, w_{fin}$.

Since Eq. (22) is a distance (norm) criterion, and since b_i, $i=0,...,3$, are linear functions of b_4, the resulting inequalities will always include second order polynomials. This fact simplifies significantly the problem of finding the appropriate values of b_4 for which Eq. (21) is true. Indeed, the coefficient α is always a positive number, so b_4 must be chosen to lie outside of the roots $b_4^a(w)$, $b_4^b(w)$ of the second order polynomial of Eq. (22). Also, it can be shown that the smallest b_4 corresponds to a minimum path disturbance from the path without obstacles, therefore this fact can be used for choosing a b_4 from its admissible range.

Notice that the criterion for obstacle avoidance, Eq. (21), was written for the front point and thus this inequality guarantees that only this particular platform point will not intersect the obstacle. However, one can take into account additional points of interest e.g. the corners of the vehicle, the joints of the manipulator and other control points. The computational cost of doing this is small since all involved equations are explicitly known.

Elliptic Obstacles: Consider an ellipse with center at (x_0, y_0) and with principal axes lengths R_a and R_b, which is rotated by an angle ψ. Using the results in Sect. 3, the criterion for obstacle avoidance for each $w=\varphi$ becomes

$$R_b^2\left[\left(u(w)-u_0\right)\cos\psi'-\left(v(w)-v_0\right)\sin\psi'\right]^2+ R_a^2\left[\left(u(w)-u_0\right)\sin\psi'+\left(v(w)-v_0\right)\cos\psi'\right]^2-R_a^2R_b^2>0 \quad (23)$$

Substituting Eqs. (15) and (16) into Eq. (23) and after some algebraic manipulations, we obtain again the quadratic expression

$$\alpha' b_4^2+\beta' b_4+\gamma'>0 \quad (24)$$

Again $\alpha'>0\ \forall w\in(w_{in}, w_{fin})$, and therefore the same method for selecting b_4 as in the case of circular obstacles can be applied.

5.2. Multiple Obstacles

The above method can be easily extended to the case in which we have N obstacles enclosed in circular and elliptic shapes. Then, several inequalities of the form of Eqs. (22) and (24) must be satisfied, each representing a distance from each of the obstacles. Depending on obstacle arrangement, one or several additional coefficients may be needed.

The criterion that the additional coefficient must meet in the presence of N obstacles is

$$\alpha_{i4} b_4^2+\beta_{i4} b_4+\gamma_{i4}>0 \quad (25)$$

for $i=1,...,N$. It can be shown that $\alpha_{4i}>0$ $\forall w\in(w_{in}, w_{fin})$, and therefore, employing the methodology presented above, we can find a coefficient b_4, if it exists, such that all the above inequalities are satisfied. This is demonstrated by an example.

Example 2. Assume that three obstacles exist: one elliptic obstacle, with center at $\left(x_1, y_1\right)=\left(1.2m, 0.9m\right)$, length of principal axes $R_a=0.8m$ and $R_b=0.2m$, and angle of rotation $\psi=30^0$, and two circular ones with centers at $\left(x_2, y_2\right)=\left(-0.2m, 1.6m\right)$, $\left(x_3, y_3\right)=\left(0.5m, 1.55m\right)$ and radii $R_2=0.24m$ and $R_3=0.1m$ respectively, see Fig. 5.

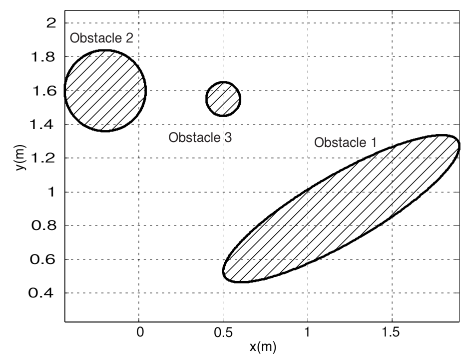


Fig. 5. Distribution of multiple obstacles in the workspace.

The roots of the second order polynomials in b_4, involved in Eqs. (22) and (24), are plotted for each obstacle, resulting in Fig. 6. This figure also illustrates the possible forms of the root plots.

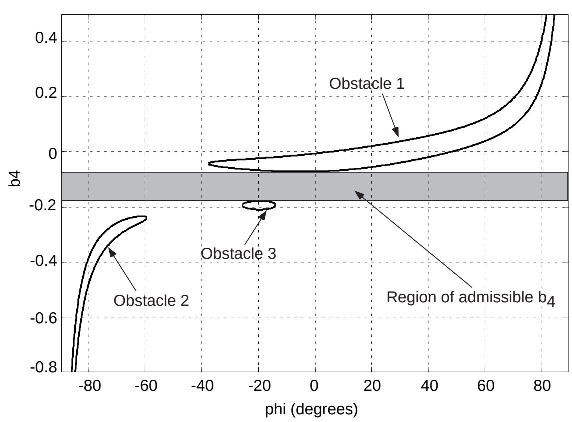


Fig. 6. Region of admissible b_4 for the three obstacles shown in Fig. 5.

The selection of the value of the additional coefficient is now an easy task. For example, by selecting $b_4=-0.12$, the obstacles are avoided for all orientations in the employed range. Fig. 7 depicts the modified path which avoids all obstacles. The corresponding system inputs are shown in Fig. 8. As shown in Figs. 7 and 8, the resulting path is smooth with a continuous curvature profile, and requires no excessive input velocities.

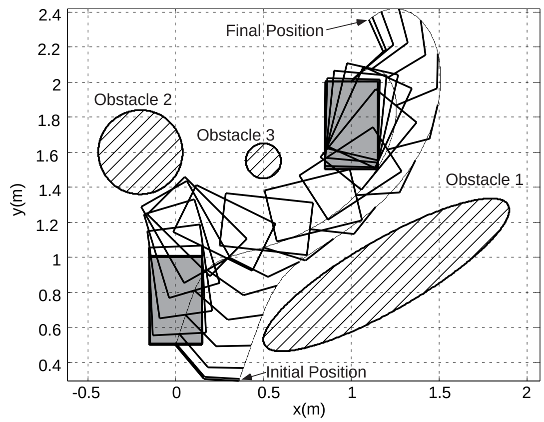


Fig. 7. Modified path that avoids the obstacles.

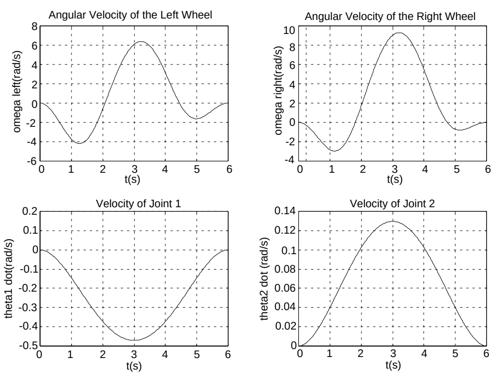


Fig. 8. Input velocities for the path in Fig. 7.

If the obstacle distribution is such that one coefficient cannot lead to a modified path that avoids all obstacles, then more coefficients should be used, depending on the distribution of the obstacles. In this case, N inequalities will result representing the distance from each obstacle, although they will be of the form

$$\sum_{j=4}^{k}\left(\alpha_{ij}\, b_j^2 + \beta_{ij}\, b_j + \gamma_{ij}\right) > 0 \tag{26}$$

for $i = 1,\ldots,N$, where k is the number of additional coefficients. Eq. (26) shows that each inequality is a sum of second order polynomials and thus the selection of the b_j's can be done for each of the second order polynomials, which are involved in each inequality, by using the known polynomial properties.

6. CONCLUSIONS

In this paper, a planning methodology was developed for nonholonomic mobile manipulators that uses smooth and continuous functions such as polynomials. A transformation that maps the nonholonomic constraint to a space where it can be trivially satisfied was used. The resulting paths and trajectories are smooth due to the nature of the map and to the use of smooth polynomials. In addition, the method allows for direct control over the platform orientation. The properties of the proposed transformation can be utilized in the case obstacles exist in the workspace of the robot: It maps Cartesian space obstacles to families of transformed ones. It was shown that obstacle avoidance can be achieved by increasing the order of the polynomials that are used in planning trajectories. Enclosing general obstacles in simple shapes such as ellipses or circles facilitated computation of the additional parameters required.

7. ACKNOWLEDGMENTS

Support for this work by IRIS III, Canadian Centres of Excellence, (LAW), is gratefully acknowledged.

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