Modeling, Design and Control of a Portable Washing Machine during the Spinning Cycle

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Abstract. This paper presents a simplified threedimensional dynamic model of a horizontal-axis portable washing machine. This model is used to predict the verge of walking instability during the spinning cycle. Next, two novel methods of stabilization are presented. The design-based method reduces the instability and is cost effective. The control-based method eliminates instability and vibrations and is associated with active balancing. Both methods satisfy the current trend towards portable, lightweight full-feature washing machines.

I. INTRODUCTION

The manufacturing of washing machines has lately been an important issue for the appliance industry. Current environmental awareness demands the improvement of washer efficiency. To this end, the use of closed-loop control instead of traditional open-loop approaches is being adopted [1, 2]. In addition, although horizontal-axis washers have a higher manufacturing cost, [3], they are becoming more popular because it has been estimated that they consume less energy, water and detergent compared to the vertical-axis ones, [4].

The reduction of washer mass is of crucial importance not only for environmental, but also for financial reasons. Unfortunately, washing machines remain big and heavy, weighing usually over fifty kilograms. This is due to the unbalanced rotation of the laundry mass during spinning. The rotating clothes are not evenly dispersed in the drum, resulting in significant centrifugal imbalance forces, which tend to destabilize the washer. This problem, which has been traditionally solved by adding a large concrete mass to the system, can have three modes: Translational slip, rotational slip and tip. Depending on the spinning speed, the mass of the laundry and system geometry, each of these modes can become the most important destabilizing agent.

A simplified two-dimensional model has been used to prove the existence of a threshold speed for translational slip and tip, [5]. The same work also showed that vertical-axis washers are slightly more stable than horizontal-axis washers. However, the rotational slip problem has not been addressed.

In terms of stabilization techniques, research is focused on the use of suspension systems. More specifically, a lot of work is carried out in suspension system analysis, [6], and optimization, [7, 8, 9, 10, 11]. In addition, it has been determined that there is a relationship between walk performance and suspension design, [12]. Another study showed that this relationship can be explicitly formulated for impending walk, [13]. Naturally, all these techniques improve the washer's dynamic behavior, but are insufficient unless the machine's mass is over fifty kilograms. However, such a machine cannot be portable.

An interesting approach to stabilization has been proposed by Zuoxin, who introduced the idea of passive balancing by adding a ring containing liquid at the drum [14]. The idea takes advantage of the centrifugal forces and of the washer's suspension to counteract the imbalance. This solution compensates for vibration caused by imbalance but introduces problems associated with system resonances. Finally, Lemaire introduced an out-of-balance detection system, but did not propose any method of counteracting these forces, [15].

This paper analyzes the problem of rotational slip and shows how it is related to translational slip and the design of a washing machine. It is proved that the critical speed for impending translational slip is higher than that of rotational slip. A design-based method is suggested, which increases the vertical force and contributes to the machine's stability allowing the reduction of the washer's mass. Finally, an active method is proposed, which employs sensors, a micro-controller and stepping motors to minimize vibrations. It is also shown that an improved estimation of the drum angular position and velocity results in greatly reduced residual vibrations.

II. THREE-DIMENSIONAL MODELING

A model of a small washing machine is shown in Fig. 1.



Fig. 1. Schematic view of horizontal-axis washing machine.

In order to derive the steady-state equations of motion, the following assumptions are made: (i) The drum's rotation speed ω is constant and high enough so that the laundry mass, due to centrifugal forces, rotates with the same speed. In other words, there is an imbalance laundry mass, m_{cl} , which rotates at a radius r_{cl} , see Fig. 2. (ii) The drum is mounted to the cabinet body so that only one degree of freedom, (x-axis rotation), is of significant importance. (iii) Drum and cabinet body are

assumed to be rigid. (iv) The plane OPQR is a plane of geometric symmetry and the washer's center of mass is on this plane. (v) The machine remains stationary and in contact with the floor, i.e. it is secured against tip, and walk is impending. (vi) The Coulomb friction model with a constant friction coefficient f is adopted.

Taking into account these assumptions, the problem could be defined as follows: Find the critical speed ω_{sl} for which rotational slip of the cabinet is impending.

Starting from the drum, the forces exerted from the rotating laundry mass on the drum are, see Fig. 2:



Fig. 2. Laundry forces on the drum.

$$N_z = m_{cl} \cdot \omega^2 \cdot r_{cl} \cdot \cos a + m_{cl} \cdot g \tag{1}$$

$$N_{v} = m_{cl} \cdot \omega^{2} \cdot r_{cl} \cdot \sin a \tag{2}$$

where N_z , N_y are the total vertical and horizontal forces respectively and g is the acceleration of gravity. These forces are transmitted to the cabinet body. Hence, its free body diagram, viewed from above, is as shown in Fig. 3.



Fig. 3. Washing machine model.

In this figure, the laundry mass rotates on a plane normal to the plane of the paper. Forces N_z and N_y are projections of the rotating force vector. The center of mass M_w of the body cabinet is not on the laundry mass rotation plane, and this introduces three-dimensional considerations. It should be noted that, although friction forces F_{A_y} , F_{B_y} , F_{C_y} , F_{D_y} generally can have any direction in the x - y plane, they are now pointing in the y direction because walk is impending in this direction. Bearing in mind the above assumptions and summing forces in the z direction, the following equations are obtained:

$$F_{A_z} + F_{B_z} = \frac{M_w \cdot g \cdot x_1}{d} + \frac{N_z \cdot x_2}{d}$$
(3)

$$F_{C_{z}} + F_{D_{z}} = \frac{M_{w} \cdot g \cdot (d - x_{1})}{d} + \frac{N_{z} \cdot (d - x_{2})}{d}$$
(4)

Summing forces in the *y* direction yields:

$$F_{A_y} + F_{B_y} = \frac{N_y \cdot x_2}{d} \tag{5}$$

$$F_{C_{y}} + F_{D_{y}} = N_{y} \cdot (d - x_{2}) / d$$
(6)

From Eqs. (5) and (6), it is evident that when $x_2 > d/2$, then $F_{A_y} + F_{B_y}$ has to be greater than $F_{C_y} + F_{D_y}$ in order to counteract the moment that tends to rotate the system along the z axis. Also, from Eq. (3) and (4), when $x_1 < d/2$ then $F_{A_z} + F_{B_z}$ is less than $F_{C_z} + F_{D_z}$. This means that the maximum obtainable friction force is higher at points C and D, than at points A and B. Thus, it can be predicted that the machine tends to lose its stability primarily at the front side (according to Fig. 3), so impending walk occurs when:

$$f \cdot \left(F_{A_z} + F_{B_z}\right) = F_{A_y} + F_{B_y} \tag{7}$$

Substituting Eqs. (3) and (5) into Eq. (7) yields:

$$f \cdot \left(\frac{M_w \cdot g \cdot x_1}{d} + \frac{N_z \cdot x_2}{d}\right) = \frac{N_y \cdot x_2}{d}$$
(8)

Also, substituting Eqs. (1) and (2) into Eq. (8) results in the spin speed at which walk is impending.

$$\omega_{st} = \sqrt{\frac{f \cdot g \cdot \left(M_{w} \cdot x_{1} + m_{cl} \cdot x_{2}\right)}{m_{cl} \cdot r_{cl} \cdot x_{2} \cdot \left(\sin a - f \cdot \cos a\right)}}$$
(9)

Differentiating Eq. (9) with respect to angle a and setting the result equal to zero, yields the angle at which the minimum critical spin speed occurs:

$$a = \tan^{-1} \left(-\frac{1}{f} \right) \tag{10}$$

Substituting Eq. (10) into Eq. (9), and after simple manipulations, the *minimum critical spin speed* for impending rotational slip is obtained:

$$\boldsymbol{\omega}_{sl} = \sqrt{\frac{f \cdot g \cdot \left(M_w \cdot \frac{x_1}{x_2} + m_{cl}\right)}{m_{cl} \cdot r_{cl} \cdot \sqrt{1 + f^2}}}$$
(11)

It is evident that this speed is increasing as the ratio x_1 / x_2 is increasing up to unity. If it becomes greater than unity, then an equation similar to Eq. (11) will predict loss of stability at the rear side (points *C* and *D*). When the ratio x_1 / x_2 is unity, the critical spin speed becomes:

$$\omega_{sl}' = \sqrt{\frac{f \cdot g \cdot \left(M_w + m_{cl}\right)}{m_{cl} \cdot r_{cl} \cdot \sqrt{1 + f^2}}}$$
(12)

and the machine will not rotate, but instead it will translate. Since $x_1 / x_2 \le 1$, a comparison of Eq. (11) to Eq. (12) yields:

$$\omega_{sl} \le \omega_{sl} \tag{13}$$

It is thus clear that the threshold spin speed for impending translational slip is greater than that for rotational slip and consequently reducing the effect of the latter should be of major priority.

III. DESIGN-BASED STABILIZATION METHOD

From the above analysis, it is evident that the machine's design plays an important role in its stability. More specifically, according to Eq. (11), the washer's center of mass should ideally be on the plane of rotation of the laundry mass. Naturally, since clothes' position cannot be detemined exactly, the drum should be designed so as to

guide the plane of rotation of the laundry center of mass to include the cabinet's center of mass. Also, since high coefficients of friction contribute to a washer's stability, the pad material should be selected carefully. However, the results of these methods offer limited reliability.

To illustrate this, a best-case scenario is employed assuming a 15 kg portable machine, and a laundry mass of 3 kg rotating at a radius of 0.1 m. The critical spin speed as function of the friction coefficient is plotted in Fig. 4 with the help of Eq. (12). Bearing in mind that clothes dry at spinning speeds above 300 rpm, it is evident that the task of stable spinning cannot be achieved. Therefore, apart from placing the cabinet center of mass on the predicted laundry center of mass rotation plane, alternative techniques must be implemented in order to achieve both stability and efficient drying.



Fig. 4. Critical speed as function of the friction coefficient.

One passive technique that can improve stability is to increase temporarily the vertical force exerted on the floor and therefore increase the stabilizing frictional forces. This can be done in two ways: (i) by taking advantage of the rinsing water and (ii) by creating a vacuum at the contact surface between the pads and the floor.

As far as the former method is concerned, it is estimated that during the rinsing cycle, a portable washing machine consumes about 30lt of water. This amount can be kept in a special container in the washing machine and disposed of after the spinning cycle. Assuming that the typical dimensions of a portable washing machine are: (i) 50 cm length and (ii) 50 cm width, the container's height should be 12cm, which is an acceptable increase in the machine's total height.

As for the latter method, this vacuum can be created with a system using suction cups and venturi vacuum generators. However, this technique increases the noise level, while extensive calculations show that it is power consuming, as it requires more than 250W. The use of passive suction cups, seems to be a wiser decision. Using such cups, an additional vertical force F_y is achieved:

$$F_{v} = p_{v} \cdot A \tag{14}$$

where p_v is the vacuum created by the suction cup and *A* is the total area of the cups.

By implementing the two normal force increasing techniques on the washing machine, Eq. (12) becomes:

$$\omega_{sl} = \sqrt{\frac{f \cdot \left[g \cdot (M_w + m_{cl} + m_r) + p_v \cdot A\right]}{r_{cl} \cdot m_{cl} \cdot \sqrt{1 + f^2}}}$$
(15)

where m_r is the rinsing water mass.

Using Eq. (15), Fig. 5 is plotted. This figure shows the critical spin speed of a typical portable machine as a function of the friction coefficient and also includes the machine's parameters. It has been assumed that four suction cups of diameter $d_c = 10 cm$ were employed. It is evident that, for friction coefficients between 0.4-0.6 the washing machine remains motionless while the laundry spins at 330-380 rpm, which is satisfactory.



Fig. 5. Critical speed after the implementation of the method.

Alternatively, one can allow the oscillation of the washer along the y-axis by mounting wheels. Specifically, the use of wheels eliminate the effects of Coulomb friction forces. In such a case, the phase lag between lateral and vertical forces that is responsible for the washer's walk, [5], vanishes, and the amplitude of oscillation becomes $A = m_{cl}r_{cl} / (M_w + m_{cl} + m_r)$. Using the parameters mentioned in Fig. 5 yields A = 6.25mm. Thus, the washer is secured against unstable walk and the only restriction remaining is the minimum critical spin speed for impending tip, which is significantly higher than that of slip. The potential considerations introduced are concerned with washer sensitivity against the slope of the ground, the minimization of rinsing water sloshing and of the minor friction forces at the wheel bearings.

Apart from the fact that imbalance forces will still be transmitted through the machine's structural elements to its base, an important problem with design-based stabilization is its strong dependence on uncontrollable parameters. For bigger laundry loads or for greater radii of rotation than those expected, or even for less smooth floors, the critical spin speed may drop significantly. It is expected that this dependency on system parameters can be reduced significantly with the use of an imbalance detection scheme.

More precisely, one can detect either the cabinet's motion by using an accelerometer or the imbalance by monitoring the variations in the machine's electric motor speed or current, [15]. As soon as the problem is detected, the motor drive can reduce the spin speed to a safe value at which the clothes can be redistributed inside the drum, removing the imbalance, or to a walk-safe speed if the imbalance cannot be corrected.

IV. CONTROL-BASED METHOD

The thrust of this method is to counteract vibrations at the place of their generation, the drum. More specifically, the cause of vibration is the rotating centrifugal force $F_{cl} = m_{cl}\omega^2 r_{cl}$ due to the unbalanced laundry mass. Thus, if we can manage to position a mass, which will hereafter be called the *balancing mass* m_b , exactly at opposite ends from the unbalanced laundry mass and make it rotate at the same speed ω at a radius r_b , we practically create another centrifugal force $F_b = m_b \omega^2 r_b$, which counteracts

the laundry imbalance. In such a case, if it holds that:

$$m_b \omega^2 r_b = m_{cl} \omega^2 r_{cl} \Longrightarrow$$
$$m_b \cdot r_b = m_{cl} \cdot r_{cl} \qquad (16)$$

then the total imbalance disappears completely. Since the value of m_b cannot be user selectable, it must be a constant. Hence, in order to have Eq. (16) satisfied, one must control the radius r_b and the position of the balancing mass with respect to the laundry. Next, two techniques that can achieve this are discussed.

A. Active Balancing using one balancing mass.

The balancing mass is able to move along the radial direction, so as to control the radius r_b and at the same time, it can rotate with respect to the drum in order to position itself at opposite ends from the laundry mass, see Fig. 6. This means that the system requires two actuators, one for each kind of movement.



Fig. 6. Schematic view of a one-mass balancing system.

Neglecting the potential dynamic balancing problems, we consider the two-dimensional problem described by imbalance vectors in Fig. 7. This figure shows the laundry imbalance $m_{cl} \cdot r_{cl}$, the effect of the balancing mass, $m_b \cdot r_b$, and the total imbalance $m \cdot r$.



Fig. 7. Two-dimensional force modeling.

Using inexpensive angle sensors, such as magnetic encoders or phototransistors and vibration sensors, the total imbalance $m \cdot r$ can be calculated both in magnitude and location. The use of open-loop position controllable actuators such as stepping motors, also allows us to know the position of the balancing mass. Therefore, the location and magnitude of the laundry imbalance is:

$$|m_{cl} \cdot \mathbf{r}_{cl}| = \sqrt{(m \cdot r)^2 + (m_b \cdot r_b)^2 - 2 \cdot m_b \cdot r_b \cdot m \cdot r \cdot \cos\theta} \quad (17)$$

$$\psi = 180^{\circ} - \tan^{-1} \frac{m \cdot r \cdot \sin \theta}{m_b \cdot r_b - m \cdot r \cdot \cos \theta}$$
(18)

Therefore, the radial actuator has to displace the balancing mass by:

$$\Delta x = \frac{\sqrt{(mr)^2 + (m_b r_b)^2 - 2m_b r_b mr \cos \theta - m_b r_b}}{m_b}$$
(19)

while the angular actuator should cause a rotation with respect to the drum equal to an angle:

$$\Delta a = \psi - 180^\circ = -\tan^{-1} \frac{m \cdot r \cdot \sin \theta}{m_b \cdot r_b - m \cdot r \cdot \cos \theta}$$
(20)

However, dynamic balancing can be a limitation of this method, unless a complex design based on an additional mass inhibits the creation of rotating moments.

B. Active Balancing using two balancing masses

In this method, two balancing masses move along the rim of the drum. The rotation plane of the balancing masses can be easily chosen to be wherever judged suitable, always targeting at the reduction of the induced moments. Fig. 8 shows a schematic view of such a system.



Fig. 8. Schematic view of a two-mass balancing system.

Again, two actuators are required in order to move the balancing masses along the drum periphery. Assuming equal masses and considering the vector imbalance diagram of Fig. 9, the following equilibrium equations result:



Fig. 9. Two-dimensional force modeling.

$$m_{cl} \cdot r_{cl} \cdot \cos \psi + m_b \cdot r_b \cdot \cos \phi + m_b \cdot r_b = m \cdot r \cdot \cos \theta \ (21)$$

 $m_{cl} \cdot r_{cl} \cdot \sin \psi + m_b \cdot r_b \cdot \sin \phi = m \cdot r \cdot \sin \theta \qquad (22)$ Solving Eqs. (21) and (22) yields:

$$\psi = \tan^{-1} \frac{m \cdot r \cdot \sin \theta - m_b \cdot r_b \cdot \sin \phi}{m \cdot r \cdot \cos \theta - m_b \cdot r_b - m_b \cdot r_b \cdot \cos \phi}$$
(23)

$$|m_{cl} \cdot \mathbf{r}_{cl}| = \frac{m \cdot r \cdot \sin \theta - m_b \cdot r_b \cdot \sin \phi}{\sin \psi}$$
(24)

Then, the desired angle ϕ' between the balancing masses that removes the drum imbalance is equal to:

$$\phi' = \cos^{-1} \left[\frac{1}{2} \cdot \left(\frac{m_{cl} \cdot r_{cl}}{m_b \cdot r_b} \right)^2 - 1 \right]$$
(25)

In a word, after acquiring the data from the sensors, the micro-controller carries out the above calculations and commands the actuators to move the two masses by:

$$\Delta \alpha_{1} = \psi - \left(180^{\circ} + \frac{\phi'}{2}\right) \tag{26}$$

$$\Delta \alpha_2 = \left(\psi - \phi\right) - \left(180^\circ + \frac{\phi'}{2}\right) \tag{27}$$

It is clear that this method presents the advantage of a simpler design and thus of a lower cost than the one-mass balancing method.

C. Implementation Requirements

From a theoretical standpoint, the active balancing method proves to be very reliable. However, prior to an actual implementation, several issues must be considered.

First, the position and the magnitude of the imbalance and at the same time the actual drum rotation speed must be known. As discussed previously, vibration sensors for the imbalance magnitude and angle sensors for its position are required.

As for the rotational speed, one can use the angle sensors that transmit pulses at Δt intervals. Then, the angular speed can be estimated by a simple calculation:

$$\omega = \frac{\gamma}{\Delta t} \tag{28}$$

where γ is the angle between two consecutive pulses.

Secondly, one must consider the cost of position control of the balancing masses, whether stepping motors or other controllable actuators are used.

Schematically, the implementation of active balancing is shown in Fig. 10. As shown in this figure, a micro-controller controls both the drive of the drum motor and the stepping actuators, while it reads the vibration level and the angular position of the drum.



Fig. 10. Active balancing block diagram.

Above all, design issues associated with safety and reliability have to be overcome. Such issues include the transmission of power to the motors, and the protection from water and detergents. These issues have to be well thought of to result in a high reliability product.

D. Simulation Results

To demonstrate the effectiveness of active balancing, we consider the two-dimensional case, neglecting dynamic balancing issues. In this case, the two active balancing techniques described above have almost identical responses. We consider the one-mass active balancing system using an inexpensive one-count-per-rotation angle sensor monitoring the drum rotation speed. The complex transient phenomenon during the start-up cycle is not simulated, i.e. it is assumed that the clothes are always stuck on the drum. The parameters employed were estimated from a small washing machine shown in Fig. 11 and are depicted in Table 1.

A ramp speed profile for the drum is generated, as shown in Fig 12. The speed becomes constant when it reaches the drying speed at 300 rpm.



Fig. 11. Instrumented portable washing machine.

Table 1. Machine and Laundry Parameters	
Laundry mass (kg)	3
Clothes' radius of rotation (m)	0.1
Washer's mass (kg)	15
Friction coefficient	0.5
Drum radius (m)	0.2
Balancing mass (kg)	2
Initial angle between m_b and m_{cl} (deg)	135°
Balancing mass angular velocity (rad s ⁻¹)	π
Balancing mass radial velocity (m s ⁻¹)	0.1
Motor step (deg)	1.8°
Sample rate (Hz)	1000



Simple speed ramp profile. Fig. 12.

To quantify the behavior during spinning, a "slip margin" M_{sl} is defined which is a measure of how close to impending walk the washing machine is. This is defined to be the residue of the absolute value of the maximum friction force minus the absolute value of the lateral force due to the imbalance, or:

$$M_{sl} = \left| f \left(M_w g + N_z \right) \right| - \left| N_y \right|$$
(29)

When the slip margin is zero, walk is impending.

In Fig. 13, the slip margin is plotted against time. The active balancing system is initiated 10 seconds after the beginning of drum rotation. As shown in this figure, active balancing minimizes indeed the vibrations. It can also be seen that the small imbalance left causes the progressive intensification of vibrations as the drum's speed increases further. This can be corrected with a second initiation of the active balancing system when a threshold slip margin is reached.

The fact that the imbalance does not vanish completely is due to three reasons: (i) the stepping motor step (ii) the sample rate and, (iii) the limited accuracy of the angle sensor. As far as the first two features are concerned, it is obvious that this small error becomes even smaller when the motor step decreases and the sample rate increases.



Fig. 13. Slip margin for simple speed profile.

The effect of the low resolution angular sensor on the response can be seen with the following example. Consider the speed profile shown in Fig.14, where a segment of constant angular velocity has been added.



Fig. 14. More complex speed profile.

Again the active balancing system is initiated at time $t = 10 \ s$. The slip margin for this scenario is presented in Fig. 15, which shows a great reduction in the vibration.



Fig. 15. Improved slip margin.

This reduction is due to the improved drum speed estimation by the angle sensor obtained during the constant speed segment introduced by the motor drive, see Fig. 14. Consequently, the use of more accurate devices is preferable, unless the speed remains constant when the balancing system is initiated.

V. CONCLUSIONS

This paper focused on modeling, design and control of horizontal-axis washing machines, with emphasis on lightweight, portable appliances. It has been concluded that rotational slip can be a major problem if the washer center of mass is not on the plane of rotation of the laundry mass. The threshold speed for impending walk is then less than that of translational slip. Next, a designbased stabilization method was introduced. This technique builds on the results of the previous analysis and employs suction cups and the water from the rinsing cycle to increase the washer's vertical force and its stability against walking. However, vibrations to the base may not vanish.

An innovative method of minimizing vibrations and thus reliably stabilizing the washer was also presented. This technique produces excellent results, although it may increase the machine's production cost. It was also shown that an improved estimation of the drum angular position and velocity results in greatly reduced residual vibrations. Finally, it is noted that the passive and active methods of stabilization are not exclusive and therefore, they could be employed in parallel, improving thus the washer's spinning response.

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