CONTROL SYSTEMS LABORATORY
DEPARTMENT OF MECHANICAL ENGINEERING
NATIONAL TECHNICAL UNIVERSITY OF ATHENS

On the Control of Quadrupedal Bounding with a Flexible Torso and a Tail

Konstantinos Koutsoukis, Konstantinos Machairas and
Evangelos Papadopoulos

Presentation Overview

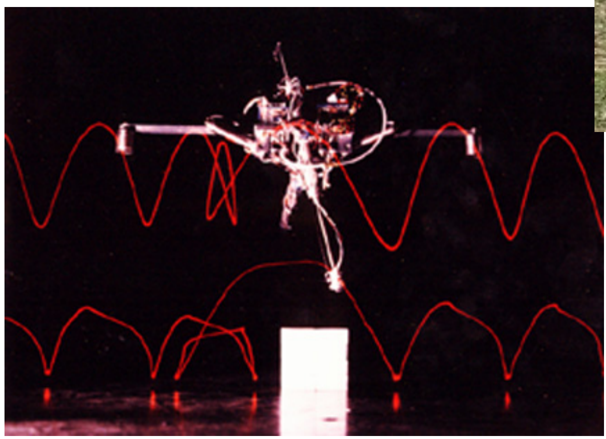
- Introduction – Quadrupeds with Flexible Torso & Tail
- Learning from Biology
- Attitude Control
- Coronal Plane Dynamics - Simulation
- Sagittal Plane Dynamics - Simulation
- 3D Model Attitude Control
- Simulation Results
- Conclusions

Introduction - Legged Robots

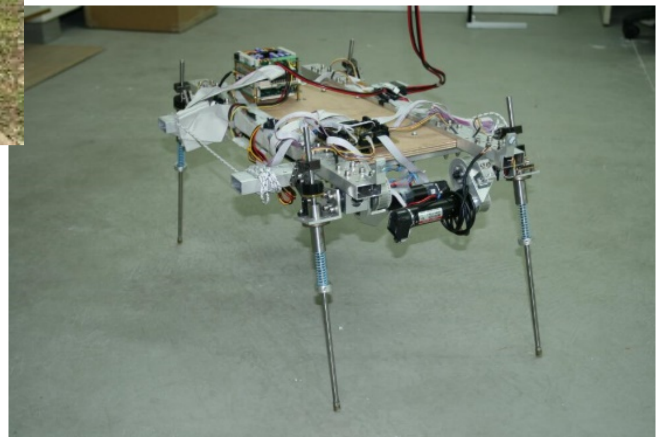
- Point contact with ground
- Move on terrain with discontinuities
- Obstacle avoidance

BigDog

RHex



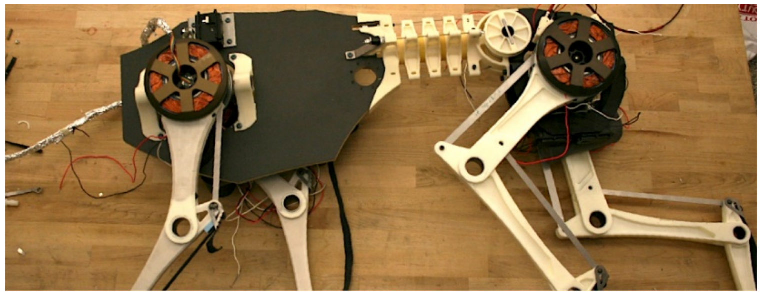
Raibert's monopod

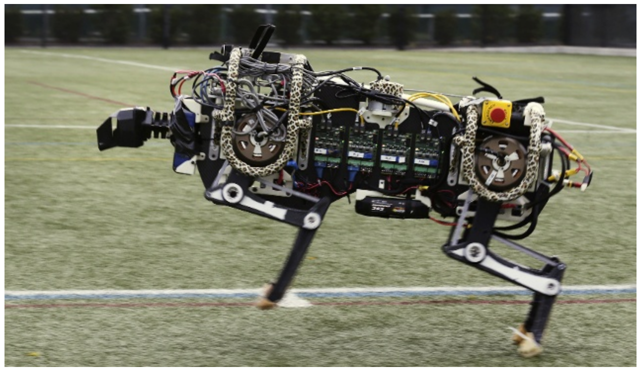


NTUA's Quadruped

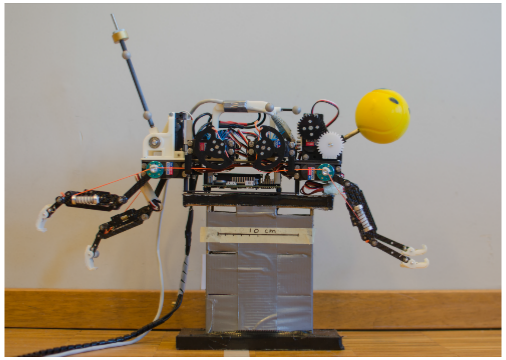
Introduction - High Performance Quadrupeds

- Boston Dynamics: Cheetah (13 m/s) - Wildcat
- MIT: Cheetah (6 m/s)
- Slower than their natural counterpart (29 m/s = 104 km/h)

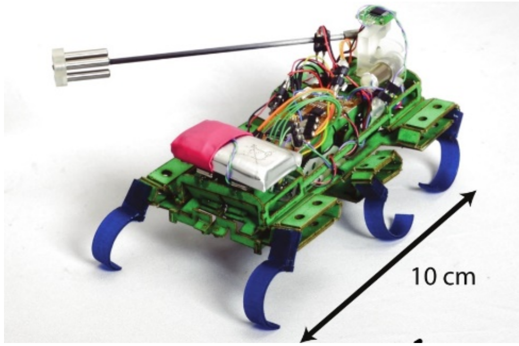




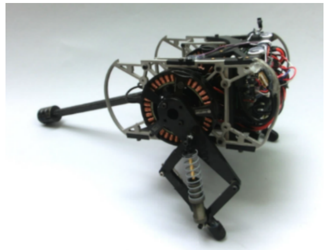
Introduction - Robots with Tails



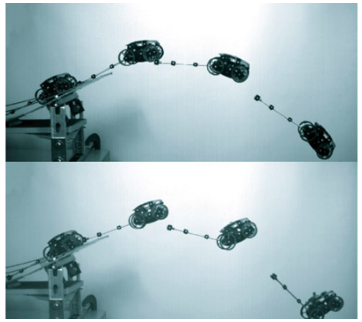
Cheetah-Cub
EPFL - Pitch



TAYLRoACH, University of California, Berkeley - Yaw

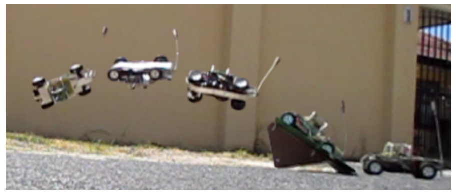


Penn Jerboa, Upenn - Pitch



Tailbot, University of California, Berkeley - Pitch

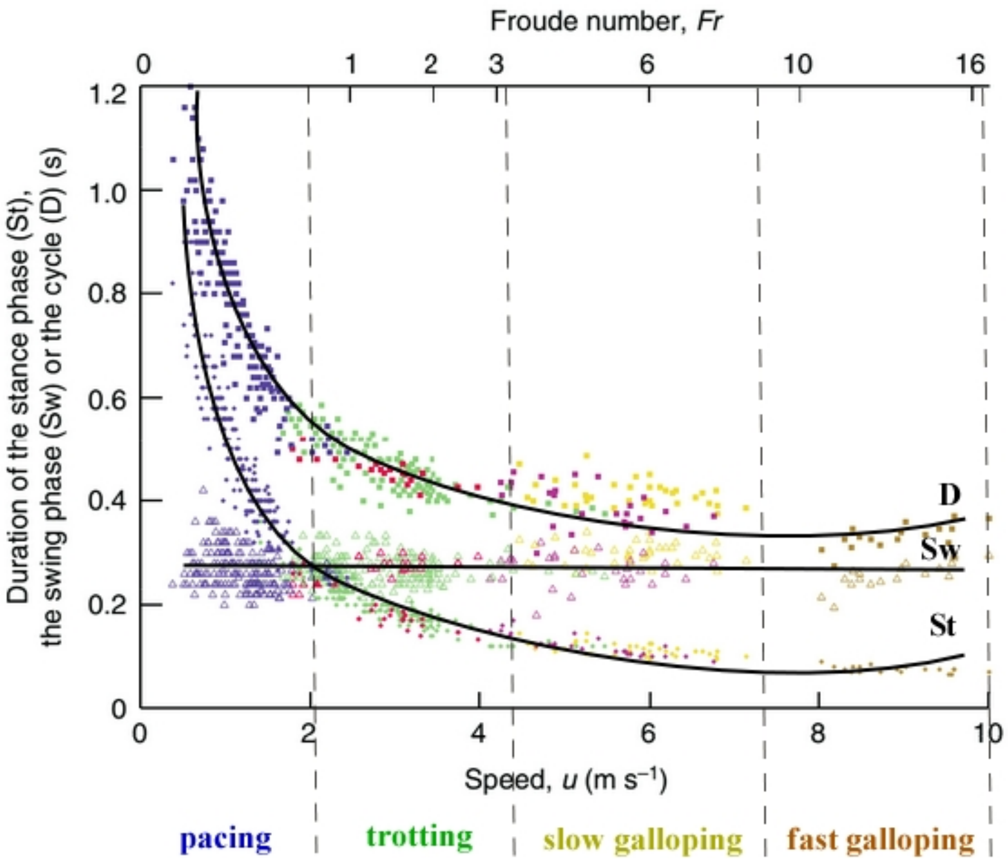
XRL, Upenn - Pitch



FlipBot, University of Cape Town - Roll, Pitch

- Numerous legged robots have been designed, but only a minority of them include appendages for angular momentum management, such as tails, or reaction wheels.

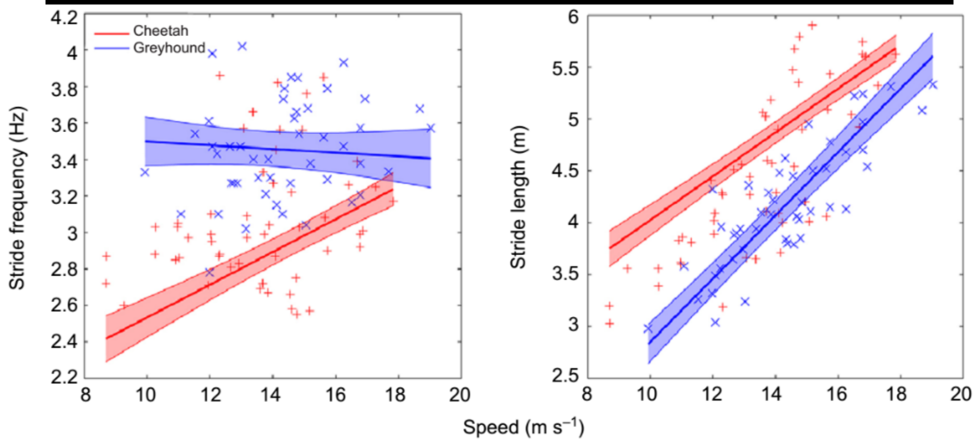
Learning from Biology - Increasing Speed



Gait duration and speed

- Data from Belgian Shepards
- Stance duration (St) drops with speed
- Swing duration (Sw) constant
- Over 5 m/s, stride duration (D) cannot be reduced
- High speed galloping (v>8m/s) achieved with extensive torso/ spine motion

Learning from Biology – Flexible Torso



Stride frequency

Stride length

Flexible torso benefits

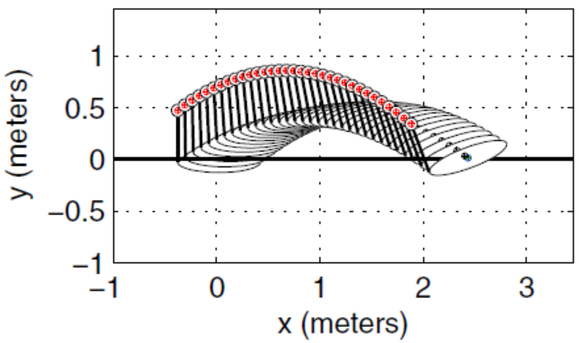
- Increased stride length by placing legs forward
- Energy storage at spine muscles tendons => increased energy efficiency
- Low stride frequency
  - the cheetah reaches the same forward velocity with lower frequency motion than the racing greyhound
- Stability of motion

Learning from Biology - Animal Tails

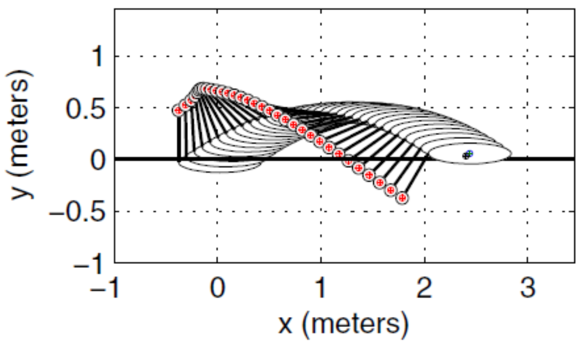
- Many legged animals have long tails which aid in balance and maneuverability at high speeds.
- Tail motion effective for adjustments to unexpected perturbations, when the legs are otherwise occupied.

Attitude Control

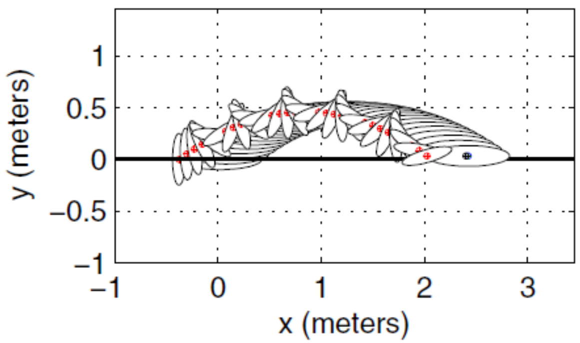
- Quadruped robots are *highly underactuated* machines.
- Tasks such as high speed galloping, jumping over obstacles, or gait transitions, require *precise control* of their attitude.
- Attitude control is achieved indirectly through the motion of the legs
  - This technique assigns more control tasks to the legs forcing them to trade-offs that may lead to low performance.
- To mitigate this challenge, dedicated appendages with large moment of inertia (MoI) can be used.



Unacceptable

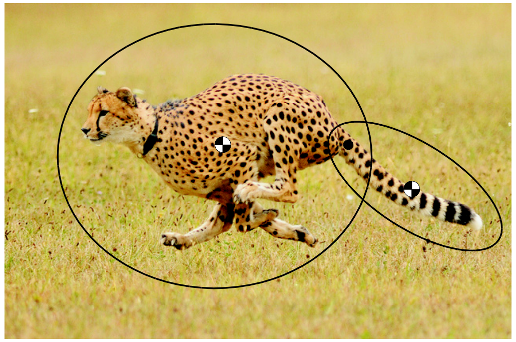


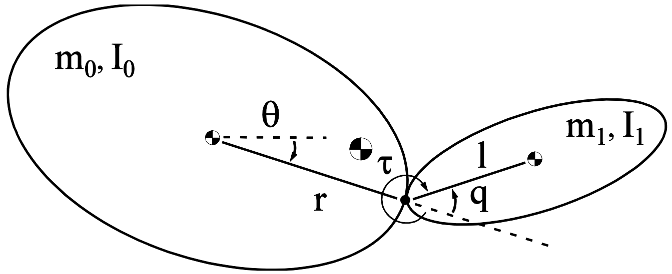
Tail



Wheel

General 2D Dynamic Model in Aerial Phase





- Equations of Motion (EoM) in Aerial Phase

$$(I_0+I_1+\mu(l^2+r^2+2rl\cos q))\ddot{\theta}+(I_1+\mu(l^2+rl\cos q))\ddot{q}-\mu rl\sin q\dot{q}^2-2\mu rl\sin q\dot{q}\dot{\theta}=0$$

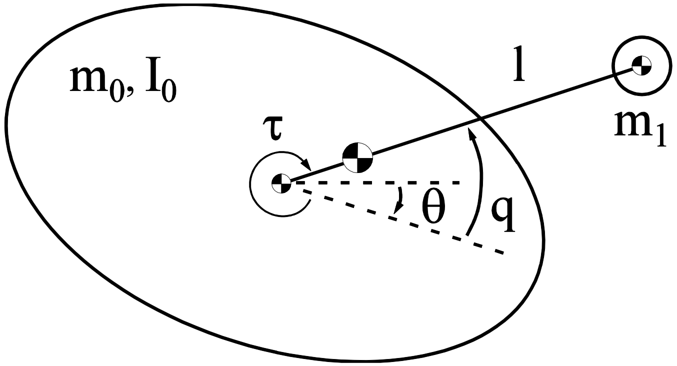
$$(I_1+\mu l^2+\mu rl\cos q)\ddot{\theta}+(I_1+\mu l^2)\ddot{q}+\mu rl\sin q\dot{\theta}^2=\tau$$

- Conservation of Angular Momentum

$$(I_0+I_1+\mu(l^2+r^2+2rl\cos q))\dot{\theta}+(I_1+\mu(l^2+rl\cos q))\dot{q}=h_0$$

- The analysis holds for body maneuvers in roll, pitch, and yaw, assuming these motions are decoupled.
- Next, we examine maneuvers on the *coronal plane* trying to control the *roll* angle of the body.

Dynamics and Control on the Coronal Plane



- For I_1=0, r=0, the EoM decouple, and the tail angle q becomes an *ignorable* coordinate.

$$I_0\ddot{\theta}=-\tau, \quad \frac{I_0\mu l^2}{I_0+\mu l^2}\ddot{q}=\tau$$

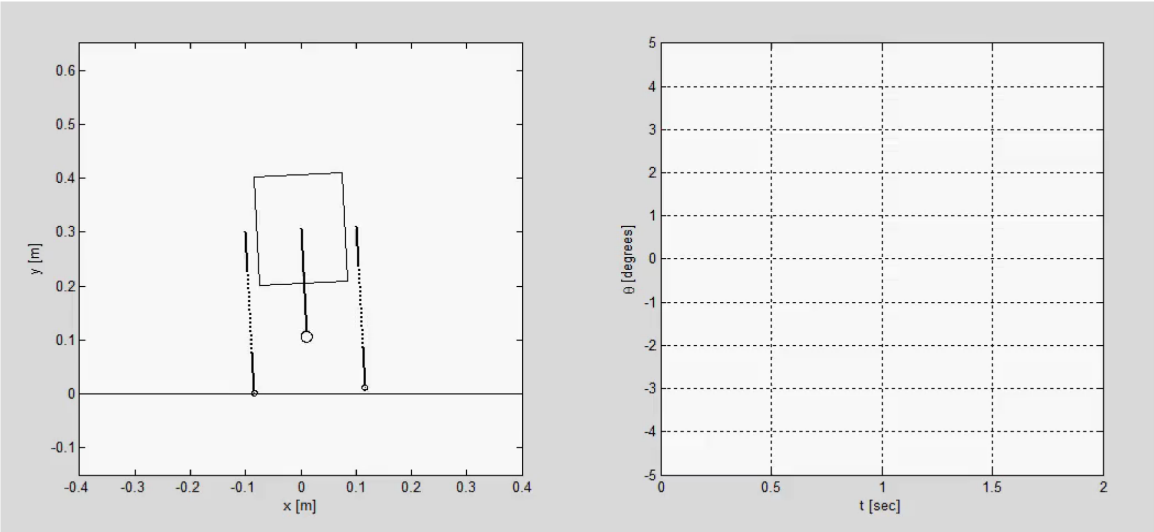
- As observed in the first equation, we can control roll angle θ, with a simple controller of the form:

$$\tau=-I_0(k_v\dot{e}_\theta+k_p e_\theta)$$

- The PD gains k_p, k_v depend on the time available to complete the maneuver, t_s.

$$k_P=36/t_s^2, \; k_V=12/t_s$$

Adding Legs and Ground Forces to the Model



Parameters

Body: I_0=0.3kgm^2, m_0=20.8kg

Hip to Hip distance: 0.1m

Leg: l=0.32m, k=3523N/m

Tail: m_1=0.5kg, l=0.2m.

Initial Conditions

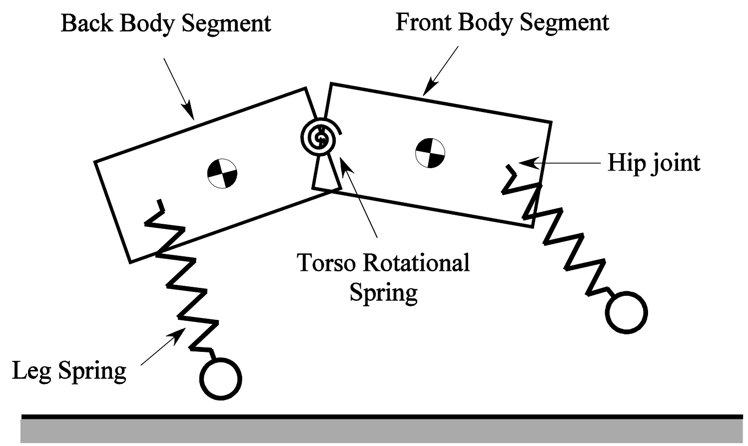
$\theta_0=3^o$, y_0=0.35m

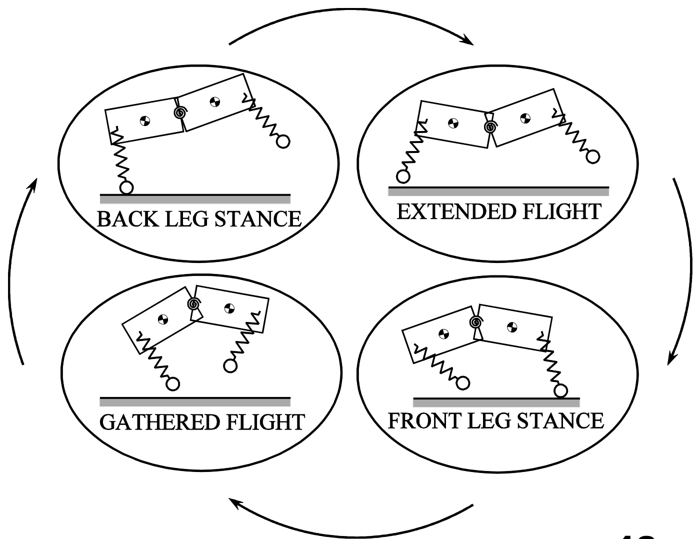
- Initial conditions and parameters obtained from a 3D experiment in ADAMS that started with *roll angle* $\theta_0=3^o$.
- The controller designed previously is used in aerial phases to bring the body angle to zero. The tail manages to stabilize the body angle.

Modeling and Dynamics: Sagittal Plane

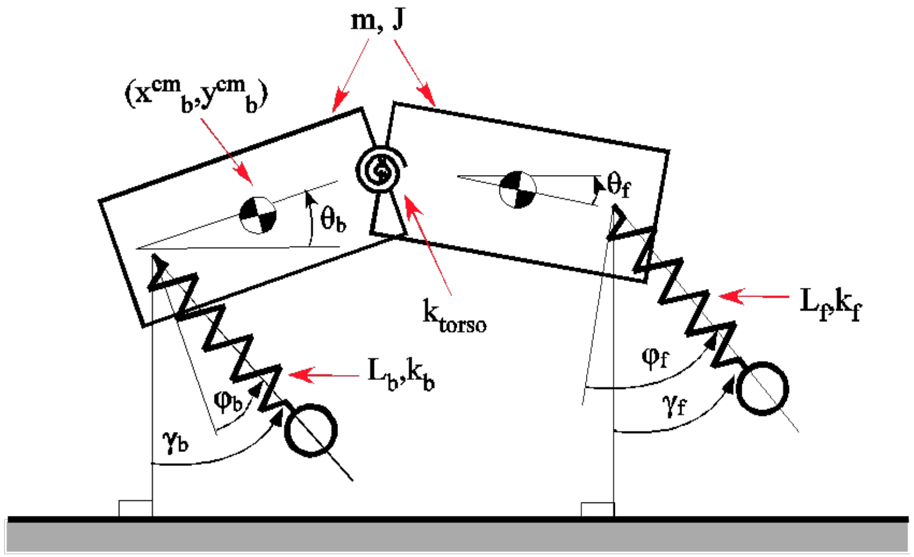
<u>Assumptions & simplifications</u>

- Motion at sagittal plane (2D) only
- Simplified galloping = bounding
- Two – segment body
- Pinned spine joint with torsional spring
- Massless springy legs
- No sliding during foot-ground contact
- Passive model





Modeling and Dynamics: Double Stance



Equations derived with Lagrange's methodology

Generalized coordinates

x^{cm}_{b}, y^{cm}_{b}, θ_{b}, θ_f

Control inputs

φ^{td}_{b}, φ^{td}_f

$$2m\ddot{x}_b^{cm}+m(-d\dot{\theta}_b^2\cos\theta_b-d\dot{\theta}_f^2\cos\theta_f-d\ddot{\theta}_b^2\sin\theta_b-d\ddot{\theta}_f^2\sin\theta_f)+k_b(L_b-l_b)\sin\gamma_b+k_f(L_f-l_f)\sin\gamma_f=0$$

$$2m\ddot{y}_b^{cm}+m(-d\dot{\theta}_b^2\sin\theta_b-d\dot{\theta}_2^2\sin\theta_f+d\ddot{\theta}_b^2\cos\theta_b-d\ddot{\theta}_f^2\cos\theta_f)-k_b(L_b-l_b)\cos\gamma_b-k_f(L_f-l_f)\cos\gamma_f+2mg=0$$

$$\ddot{\theta}_b(\mathrm{J}+md^2)+\ddot{\theta}_f md^2-\ddot{x}md\sin\theta_b+\ddot{y}md\cos\theta_b-dk_b(L_b-l_b)\cos(\gamma_f-\theta_b)+-dk_f(L_f-l_f)\cos(\gamma_b-\theta_b)+$$
$$+k_t(\theta_b-\theta_f)+mgd\cos\theta_b=0$$

$$\ddot{\theta}_f(\mathrm{J}+md^2)+\ddot{\theta}_b md^2-\ddot{x}md\sin\theta_f+\ddot{y}md\cos\theta_f-2dk_f(L_f-l_f)\cos(\gamma_f-\theta_f)-k_t(\theta_b-\theta_f)+mgd\cos\theta_f=0$$

Modeling and Dynamics: Poincaré Map

Poincaré section

Taken at the extended flight phase, at the apex of the spinal joint

Fixed points

Proper initial conditions + “control” inputs = continuous motion

$$\mathbf{z}_f[k+1] = P(\mathbf{z}_f[k], \alpha_f[k])$$

$$\mathbf{z}_f = [y_b^{cm}, \theta_b, \theta_f, \dot{x}_b^{cm}, \dot{\theta}_b, \dot{\theta}_f]^T$$

$$\alpha_f = [\varphi_b^{td}, \varphi_b^{td}]^T$$

Simulation: Parameters & Initial Conditions

Initial conditions

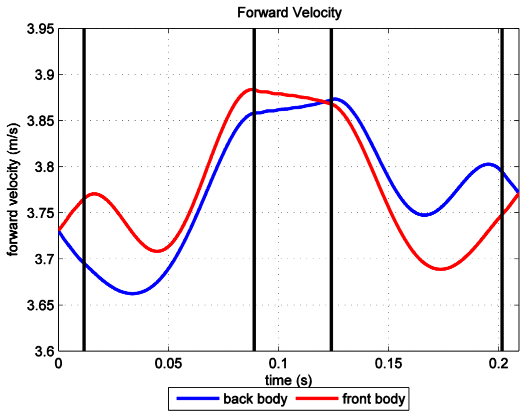
Parameter	Value
$\dot{x}^{cm}_{b,0}$	3.73 m/s
$\dot{y}^{cm}_{b,0}$	0.41 m/s
$y^{cm}_{b,0}$	0.29 m
$\theta_{b,0}$	-10.73 deg
$\theta_{f,0}$	10.73 deg
$\dot{\theta}_{b,0}$	-172 deg/s
$\dot{\theta}_{f,0}$	-172 deg/s

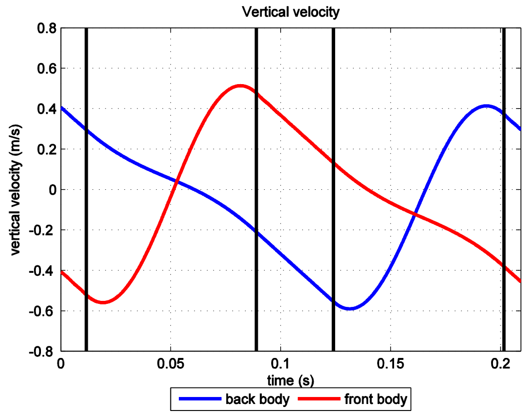
Robot parameters

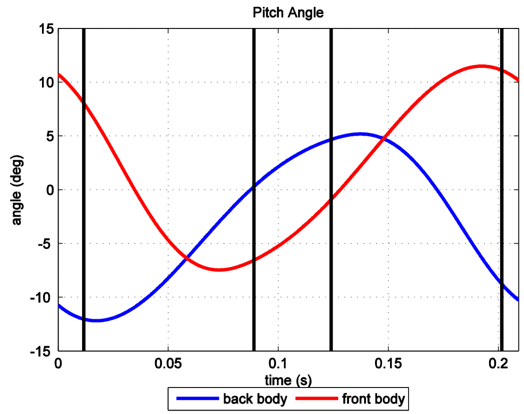
Parameter	Value
m	10.432 kg
J	0.339 kg m^2
$L_f = L_b$	0.323 m
d	0.138 m
$k_f = k_b$	7046 N/m
K_{torso}	165 N/m

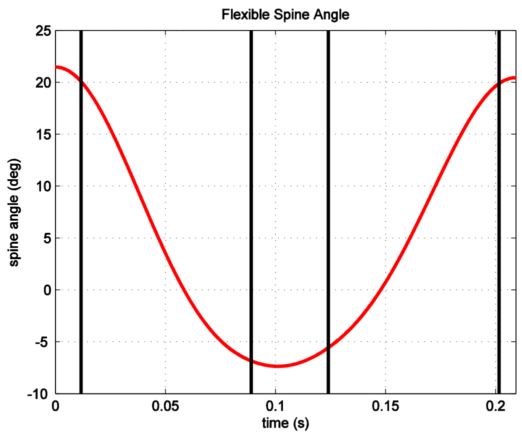
Control input	Value
φ^{td}_b	28.14 deg
φ^{td}_f	12.12 deg

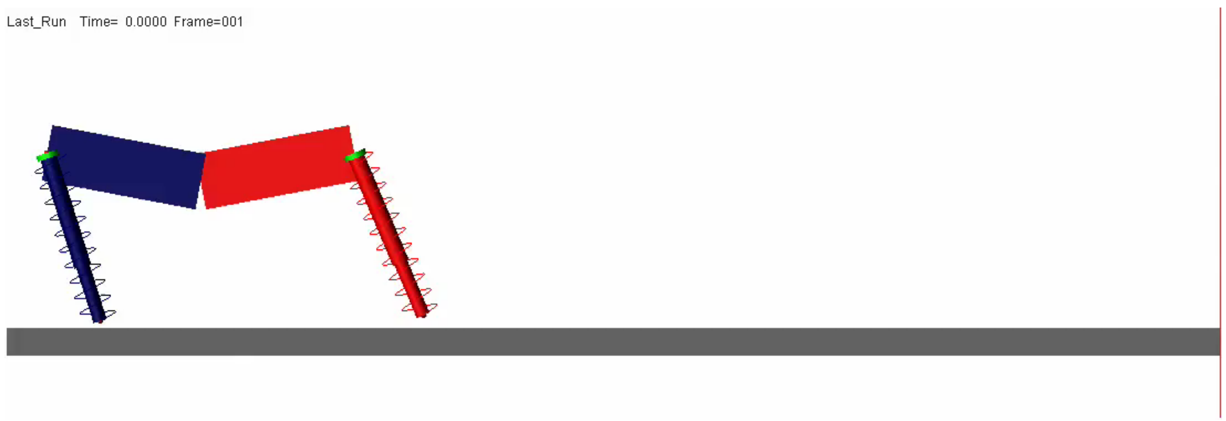
Simulation - Results











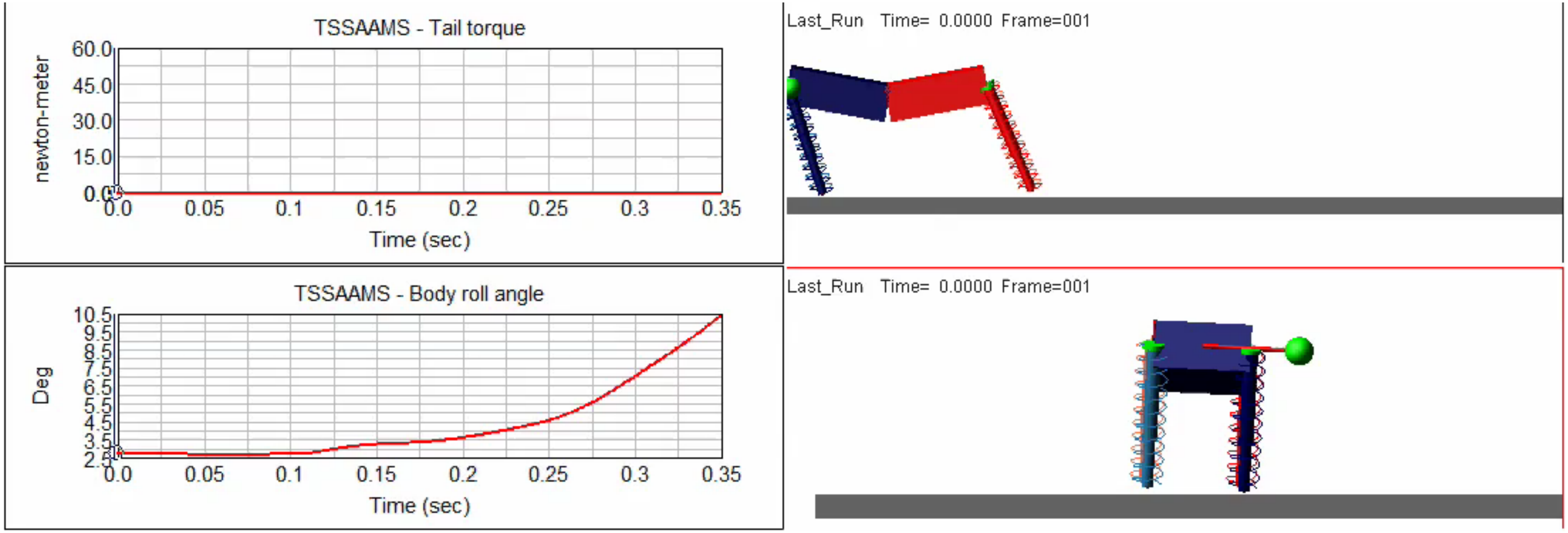
- Reversing time symmetry $\mathbf{x}_b(-t)=\mathbf{x}_f(t)$

  where $\mathbf{x}_i=[y_i^{cm}, \theta_i, \dot{\theta}_i, \dot{\mathrm{x}}_i^{cm}, \dot{y}_i^{cm}]^T, \quad \mathrm{i}\in\{\mathrm{f,b}\}$

3D Model with Tail & Flexible Torso

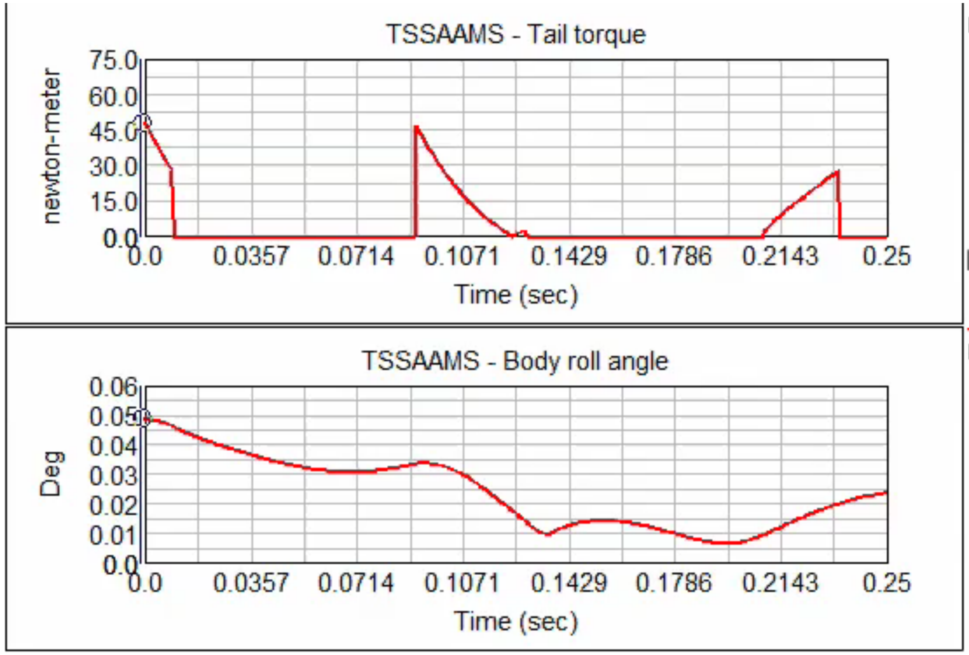
- Model characteristics
  - 3D model in MSC Adams simulation environment
  - Flexible torso and tail in a single model
  - Initial conditions and leg touch down angles derived from sagittal plane fixed point
  - 3 deg initial roll
  - Tail active only during flight – passive during stance
  - Simple tail control law
- Aim
  - To correct the initial roll angle and conserve the motion

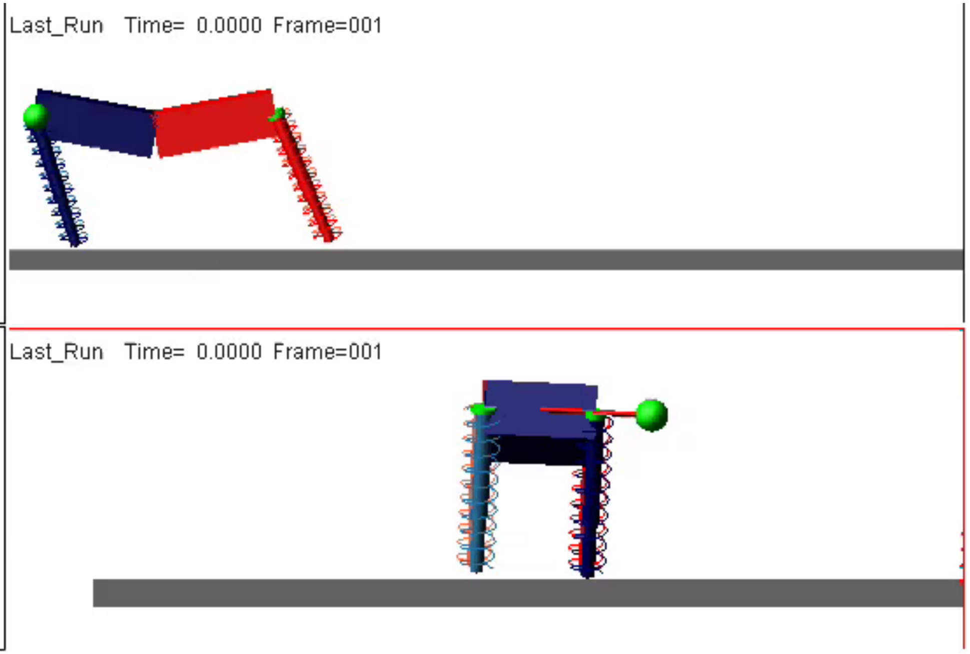
Simulation – Passive Tail (no actuation)



- Roll angle increases since no control is applied
- Not enough foot clearance
- Incapable of completing one stride

Simulation – Success with Active Tail





- Tail actuation during flight phase only
- Roll angle correction within one stride
- Completed the stride *successfully*

Conclusions

- Derived the 2D passive dynamics of a quadruped with a *flexible torso*.
  - A *fixed point* of the motion was found.
- Derived the 2D model of a quadruped with a *tail-like appendage*.
  - A controller was developed to stabilize the main body attitude on the coronal plane using a *tail*.
- A complex 3D model with both *a flexible torso* and a *tail* was simulated with and without tail actuation on the coronal plane.
  - When starting with a roll angle of 3^{o}, the robot *fails* to make a second stride without tail actuation.
  - With the tail correcting the roll angle of the body during flight, the 3D robot follows a *stable* motion corresponding to the fixed point found in the 2D analysis.

Acknowledgement

This research has been financed by the European Union (European Social Fund – ESF) and Greek national funds through the Operational Program “Education and Lifelong Learning” of the National Strategic Reference Framework (NSRF) – Research Funding Program: **ARISTEIA**: Reinforcement of the interdisciplinary and/ or inter-institutional research and innovation.

European Union
European Social Fund

OPERATIONAL PROGRAMME
EDUCATION AND LIFELONG LEARNING
investing in knowledge society
MINISTRY OF EDUCATION & RELIGIOUS AFFAIRS
MANAGING AUTHORITY

NSRF
2007-2013
programme for development
EUROPEAN SOCIAL FUND

Co-financed by Greece and the European Union