ON SPATIAL TRAJECTORY PLANNING & OPEN-LOOP CONTROL FOR UNDERACTUATED AUVs

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Abstract: This paper considers the problem of spatial planning and open-loop control for underactuated AUVs. Given a smooth inertial 3D trajectory to be followed by some vehicle point, the developed algorithm uses the vehicle dynamic model, and computes the 3D corresponding body-fixed linear and angular velocities as well as vehicle orientations, yielding a feasible 6 DOF trajectory. Tracking controllers are then facilitated by utilizing this trajectory which is consistent with AUV dynamics. The full 6 DOF computed trajectory is further used to compute the efforts for the three available control inputs and to result in an open-loop trajectory controller.

Keywords: underactuated AUV, 3D trajectory planning, tracking.

1. INTRODUCTION[1]

In a great number of typical missions, Autonomous Underwater Vehicles (AUVs), see Fig. 1, are employed in tracking an inertial reference trajectory in 3D environments. Such missions include oceanographic observations, bathymetric surveys, ocean floor analysis, military applications, etc., (Yuh, 2000). Besides their numerous practical applications, these vehicles present a challenging control problem since most of them are underactuated, i.e., they have fewer inputs than degrees of freedom (DOF). Such a control configurations impose nonintegrable acceleration constraints. In addition, AUVs' kinematic and dynamic models are highly nonlinear and coupled, making control design a hard task. Underactuation rules out the use of trivial control schemes e.g. full state-feedback linearization, and the complex hydrodynamics excludes designs based on kinematic models only.

Trajectory tracking requires the design of control laws that guide the vehicle to track an inertial

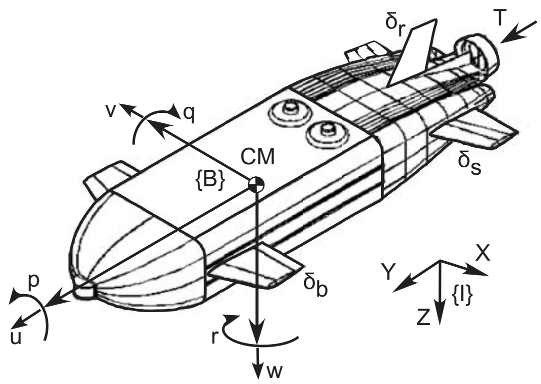

Fig. 1. The AUV with the controls and motion variables.

trajectory, i.e., a 3D path on which a time law is specified. The performance of trajectory tracking controllers is greatly improved when trajectory planning has been designed previously.

The goal of trajectory planning is to generate the reference inputs to the motion control system which ensures that the vehicle executes the planned trajectory. While several researchers have considered

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the tracking control problem of an AUV (or generally a marine vehicle) in 2D, see (Pettersen and Nijmeijer, 2001; Behal et. al., 2002) and few the tracking control problem in 3D, for example (Encarnacao and Pascoal, 2000; Aguiar and Hespanha), to the best of the authors' knowledge, there is no known work that studied the trajectory planning for an underactuated AUV in 3D. A first result on this subject was presented in our previous work (Repoulias and Papadopoulos, 2005) where we studied the combined problem of trajectory planning and tracking control for an underactuated AUV moving on a horizontal plane. In that work, a trajectory tracking controller is designed in two steps: first, the trajectory planning algorithm computes the reference – consistent with vehicle dynamics – variables, and second, a closed-loop backstepping controller, utilizing these reference variables and the open-loop actuation efforts achieves asymptotic tracking.

In this paper, we present a trajectory planning algorithm and an open-loop feed-forward controller for underactuated AUVs moving in space. Given a smooth 3D trajectory in inertial coordinates, the developed algorithm uses the vehicle dynamic model, and computes 3D body-fixed linear and angular velocities as well as vehicle orientation, i.e., yielding a feasible 6 DOF trajectory. Tracking controllers are then facilitated by utilizing this trajectory which is consistent with AUV dynamics, resulting in improved performance. Exploiting the benefits mentioned, we use the three available control inputs to design an open-loop tracking controller. Finally, an application of the algorithm is presented along with simulation results.

2. AUV KINEMATICS AND DYNAMICS

In this section, the kinematic and dynamic equations of motion for an AUV moving in a 3D space are presented.

To describe the kinematics, two reference frames are employed, the inertial reference frame $\{I\}$ and a body-fixed frame $\{B\}$, see Fig. 1. As shown, the origin of $\{B\}$ frame coincides with the AUV center of mass (CM) while the center of buoyancy (CB) is on the negative z body axis for static stability. Using the standard notation of ocean engineering, the general motion of an AUV in 6 DOF can be described by the following vectors:

$$\begin{aligned} &\boldsymbol{\eta}=[\boldsymbol{\eta}_1^T,\boldsymbol{\eta}_2^T]^T; \quad \boldsymbol{\eta}_1=[x,y,z]^T; \quad \boldsymbol{\eta}_2=[\phi,\theta,\psi]^T; \\ &\mathbf{v}=[\mathbf{v}_1^T,\mathbf{v}_2^T]^T; \quad \mathbf{v}_1=[u,\upsilon,w]^T; \quad \mathbf{v}_2=[p,q,r]^T; \end{aligned} \qquad (1)$$

In (1), $\boldsymbol{\eta}_1$ denotes the inertial position of the CM and $\boldsymbol{\eta}_2$ the orientation of $\{B\}$ – in terms of Euler angles – with respect to the $\{I\}$ frame. Vector $\mathbf{v}_1$ denotes the linear velocity of the CM and $\mathbf{v}_2$ the angular velocity of $\{B\}$ with respect to $\{I\}$ frame, both expressed in the body-fixed $\{B\}$ frame.

In guidance and control applications, for the representation of rotations, it is customary to use the xyz (roll-pitch-yaw) convention defined in terms of Euler angles adopted in the present work or quaternions. Hence, the velocity transformation between $\{B\}$ and $\{I\}$ frames is expressed as

$$\dot{\boldsymbol{\eta}}_1=\mathbf{J}_1(\boldsymbol{\eta}_2)\mathbf{v}_1 \qquad (2)$$

where

$$\mathbf{J}_1(\boldsymbol{\eta}_2)=\begin{bmatrix} c\psi c\theta & -s\psi c\phi+c\psi s\theta s\phi & s\psi s\phi+c\psi c\phi s\theta \\ s\psi c\theta & c\psi c\phi+s\phi s\theta s\psi & -c\psi s\phi+s\theta s\psi c\phi \\ -s\theta & c\theta s\phi & c\theta c\phi \end{bmatrix} \qquad (3)$$

The body-fixed angular velocities and the time rate of the Euler angles are related through

$$\dot{\boldsymbol{\eta}}_2=\mathbf{J}_2(\boldsymbol{\eta}_2)\mathbf{v}_2 \qquad (4)$$

where

$$\mathbf{J}_2(\boldsymbol{\eta}_2)=\begin{bmatrix} 1 & s\phi t\theta & c\phi t\theta \\ 0 & c\phi & -s\phi \\ 0 & s\phi/c\theta & c\phi/c\theta \end{bmatrix} \qquad (5)$$

where $s\cdot=\sin(\cdot)$, $c\cdot=\cos(\cdot)$, $t\cdot=\tan(\cdot)$.

The dynamic model of the AUV used for the illustration of the method is taken from (Encarnacao and Pascoal, 2000; Fossen, 1994). It is a simplified model developed for control design tasks, which captures the main dynamical characteristics of a flat-fish shaped AUV moving in 3D space, see Fig. 1. The vehicle is underactuated, i.e., it has less control inputs than the number of DOF. Specifically, in the following equations of motion, the three controls are T for surge propulsion, rudder angle δ_r for yaw rotation and stern and bow plane angles $\delta_s=-\delta_b$ for pitch rotation.

$$\begin{aligned}(m-r_3X_{\dot u})\dot u &= (r_3X_{wq}-m)wq+(r_3X_{vr}+m)\upsilon r \\ &\quad +r_2X_{uu}u^2+r_2X_{vv}\upsilon^2+T\end{aligned} \qquad (6a)$$

$$(m-r_3Y_{\dot v})\dot\upsilon=(r_3Y_r-m)ur+(r_3Y_{wp}+m)wp+r_2Y_vu\upsilon \qquad (6b)$$

$$(m-r_3Z_{\dot w})\dot w=(r_3Z_q+m)uq+(r_3Z_{vp}-m)\upsilon p+r_2Z_wuw \qquad (6c)$$

$$(I_x-r_5K_{\dot p})\dot p=r_5K_{qr}qr+r_4K_pup+z_{CB}c\theta s\phi B \qquad (6d)$$

$$\begin{aligned}(I_y-r_5M_{\dot q})\dot q &= (r_5M_{pr}+I_z-I_x)pr+r_4M_{uq}uq \\ &\quad +r_3M_{uw}uw+r_3u^2(M_{ds}\delta_s+2M_{db}\delta_b) \\ &\quad +z_{CB}s\theta B\end{aligned} \qquad (6e)$$

$$\begin{aligned}(I_z-r_5N_{\dot r})\dot r &= (r_5N_{qp}+I_x-I_y)pq+r_3N_vu\upsilon \\ &\quad +r_4N_rur+r_3u^2N_{dr}\delta_r\end{aligned} \qquad (6f)$$

A brief explanation of the various terms in (6) follows: m is the vehicle's mass and I_x, I_y, I_z are the moments of inertia about the body x, y, and z, axes respectively. B is the buoyancy force acting on the CB. z_{CB} is the $z-$coordinate of the CB. $r_i = (\rho/2)L^i$, $i = 1,...,5$, where ρ is the water density and L the AUV's length. $X_{\dot{u}}$, $Y_{\dot{v}}$, $Z_{\dot{w}}$ are added mass and $K_{\dot{p}}$, $M_{\dot{q}}$, $N_{\dot{r}}$ are added moments of inertia terms. X_{wq}, X_{vr}, Y_r, Y_{wp}, Z_q, Z_{vp}, K_{qr}, M_{pr}, and N_{qp} are added mass cross terms. X_{uu}, X_{vv}, Y_v, Z_w, K_p, M_{uq}, M_{uw}, M_{ds}, M_{db}, N_v, N_r, and N_{dr} are drag and body lift, force and moment terms. Detailed description of the model parameters can be found in (Fossen, 1994). The lack of control actuation in sway υ, heave w, and roll p motions renders the system underactuated.

3. TRAJECTORY PLANNING

In this section, we describe the trajectory planning methodology, i.e., the algorithm that maps an inertial trajectory of the 3D space to body-fixed velocities and orientation. The only restriction on the inertial trajectory is that it must be sufficiently "smooth", i.e., three times differentiable with respect to time.

3.1 Inertial trajectory geometry

We choose the CM of the vehicle as the point of interest. Let us assume that the trajectory which must be tracked by this point is given as a time function of the inertial variables x_R, y_R, z_R and their time derivatives up to the third order. The subscript "R" indicates a reference variable. The position and the magnitude of the velocity vector of a point P on the trajectory are given by

$$\mathbf{s}_P = [x_R, y_R, z_R]^T \tag{7}$$

$$v_p = \|\mathbf{v}_p\| = \|\dot{\mathbf{s}}_P\| = \sqrt{\dot{x}_R^2 + \dot{y}_R^2 + \dot{z}_R^2} \tag{8}$$

Of great importance in the derivation is the concept of an orientation frame associated with the curve corresponding to the desired trajectory. To every point of the above curve, we can associate an orthonormal triad of vectors, i.e., a set of unit vectors that are mutually orthogonal, namely, the tangent $\mathbf{e}_t$, the normal $\mathbf{e}_n$, and the binormal $\mathbf{e}_b$, see Fig. 2. Properly arranging these vectors in a 3×3 matrix, we obtain a description of the curve orientation, (Angeles, 1997). The corresponding reference frame is the Frenet-Serret one. The unit vectors are then defined as

$$\mathbf{e}_t = \dot{\mathbf{s}}_P / v_P \tag{9a}$$

$$\mathbf{e}_b = (\dot{\mathbf{s}}_P \times \ddot{\mathbf{s}}_P) / \|\dot{\mathbf{s}}_P \times \ddot{\mathbf{s}}_P\| \tag{9b}$$

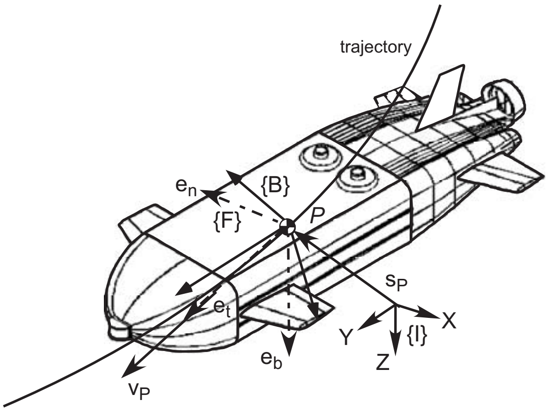

Fig. 2. The inertial, the Frenet-Serret and the body frame during tracking.

$$\mathbf{e}_n = \mathbf{e}_b \times \mathbf{e}_t = (\dot{\mathbf{s}}_P \times \ddot{\mathbf{s}}_P) \times \dot{\mathbf{s}}_P / \|\dot{\mathbf{s}}_P \times \ddot{\mathbf{s}}_P\| \|\dot{\mathbf{s}}_P\| \tag{9c}$$

According to the notation of rotational transformations used in robotics literature (Sciavicco and Siciliano, 1996), we can express the coordinates of a vector given in the Frenet-Serret frame $\{F\}$ to the $\{I\}$ frame with the matrix

$$\mathbf{R}_F^I = [\mathbf{e}_t \quad \mathbf{e}_n \quad \mathbf{e}_b] \tag{10}$$

Also, we shall use the fact that

$$\mathbf{R}_I^F = (\mathbf{R}_F^I)^T \tag{11}$$

In (aero)nautical applications it is more convenient to express the various velocities in the current – body – frame, (Baruh, 1999). Thus, the angular velocity of the $\{F\}$ frame with respect to the $\{I\}$ frame expressed in $\{F\}$ frame is given by

$$\mathbf{\Omega}^F = \mathbf{R}_I^F \dot{\mathbf{R}}_F^I \tag{12a}$$

where $\mathbf{\Omega}^F$ is a skew-symmetric matrix containing the components of the angular velocity vector, and defined as follows:

$$\mathbf{\Omega}^F = \begin{bmatrix} 0 & -\omega_3^F & \omega_2^F \\ \omega_3^F & 0 & -\omega_1^F \\ -\omega_2^F & \omega_1^F & 0 \end{bmatrix} \tag{12b}$$

From (12b), we collect the components in a vector:

$$\boldsymbol{\omega}_I^F = [\omega_1^F, \omega_2^F, \omega_3^F]^T = \mathbf{f}_{\omega^F}(\text{traj., variables}) \tag{13}$$

where "$\mathbf{f}_{\omega^F}(\text{traj., variables})$" means a vector function of the first, second, and third derivatives of the trajectory variables x_R, y_R, z_R.

3.2 Vehicle's dynamics during tracking

Consider next the dynamics of the AUV when its CM tracks accurately the motion of the point P, and let u_R, υ_R, w_R, p_R, q_R, r_R denote the reference body-fixed velocities. The magnitude of the total linear velocity vector of the CM is given by

$$v_R = \| \mathbf{v}_R \| = \| [u_R, \upsilon_R, w_R]^T \| = \sqrt{u_R^2 + \upsilon_R^2 + w_R^2} \quad (14)$$

As far as the reference orientation $[\phi_R, \theta_R, \psi_R]^T$ of the body-fixed $\{B\}$ frame with respect to the inertial $\{I\}$ frame is concerned, we have the following: Due to the strong hydrodynamic effects, the vehicle $\{B\}$ frame does not coincide with the Frenet-Serret frame $\{F\}$, but undergoes a further rotation with respect to the latter, see Fig. 2, to eventually coincide with the reference – desired – $\{R\}$ frame that provides the orientation consistent with the AUV dynamics. Therefore, when the AUV CM tracks the curve, the body $\{B\}$ coincides with the reference frame $\{R\}$.

The rotation of $\{B\}$ frame from $\{F\}$ frame to $\{R\}$ frame can be expressed using customary aeronautical notation by considering the sideslip angle β and angle of attack α, (McCormic, 1994):

$$\beta = \sin^{-1}(\upsilon_R / v_P) \quad (15)$$

$$\alpha = \sin^{-1}(w_R / \sqrt{u_R^2 + w_R^2}) \quad (16)$$

The overall rotation is composed by a rotation about body-z axis through the angle β, followed by a rotation about body-y axis through the angle α and is expressed by the matrix

$$\mathbf{R}_F^R = \mathbf{R}_y(\alpha)\mathbf{R}_z(\beta) \quad (17)$$

where the matrix $\mathbf{R}_F^R$ represents the rotation between the $\{F\}$ and the reference or desired frame $\{R\}$. The angular velocity of the $\{R\}$ frame with respect to the $\{F\}$ frame, expressed in $\{R\}$, is computed by

$$\mathbf{\Omega}^R = \mathbf{R}_F^R \dot{\mathbf{R}}_R^F \quad (18)$$

The associated angular velocity vector is as before

$$\boldsymbol{\omega}_F^R = [\omega_1^R, \omega_2^R, \omega_3^R]^T = \mathbf{f}_{\omega^R}(\text{traj., body variables}) \quad (19)$$

where "$\mathbf{f}_{\omega^R}(\text{traj., body variables})$" means a function of the derivatives of x_R, y_R, and z_R up to the second order as well as of velocities u_R, υ_R, w_R and their first order derivatives.

Finally, the reference orientation between the inertial $\{I\}$ frame and the reference $\{R\}$ frame is given by

$$\mathbf{R}_I^R = \mathbf{R}_F^R \mathbf{R}_I^F \quad (20)$$

From (20), we can extract the reference angles using the following , (Sciavicco and Siciliano, 1996)

$$\phi_R = \text{atan2}(r_{23}, r_{33}) \quad (21a)$$

$$\theta_R = \text{atan2}(-r_{13}, \sqrt{r_{23}^2 + r_{33}^2}) \quad (21b)$$

$$\psi_R = \text{atan2}(r_{12}, r_{11}) \quad (21c)$$

where r_{ij} denotes the ij element of $\mathbf{R}_I^R$.

During tracking, the magnitude of the tangent vector to the trajectory $\mathbf{v}_P$ equals the magnitude of the vehicle's velocity vector $\mathbf{v}_R$. From (8) and (14) it is:

$$v_P = v_R \Rightarrow \sqrt{\dot{x}_R^2 + \dot{y}_R^2 + \dot{z}_R^2} = \sqrt{u_R^2 + \upsilon_R^2 + w_R^2} \quad (22)$$

From (22) and (8), the reference sway velocity is

$$\upsilon_R = \pm\sqrt{\dot{x}_R^2 + \dot{y}_R^2 + \dot{z}_R^2 - u_R^2 - w_R^2} = \pm\sqrt{v_P^2 - u_R^2 - w_R^2} \quad (23)$$

where "$\pm$" indicates that υ_R may be positive or negative depending on the trajectory curvature. Differentiating (23) with respect to time yields,

$$\dot{\upsilon}_R = \pm(v_P \dot{v}_P - u_R \dot{u}_R - w_R \dot{w}_R) / \sqrt{v_P^2 - u_R^2 - w_R^2} \quad (24)$$

where $\dot{v}_P$ is given by a simple differentiation of (8). In this way, we have also expressed υ_R and $\dot{\upsilon}_R$ by the trajectory variables and by u_R, $\dot{u}_R$, w_R, $\dot{w}_R$.

Now, the reference angular velocities are obtained by the succession of the angular velocity of the $\{F\}$ frame with respect to the $\{I\}$ frame and the angular velocity of the reference orientation frame $\{R\}$ with respect to $\{F\}$, all expressed in $\{R\}$:

$$\boldsymbol{\omega}_I^R = \boldsymbol{\omega}_F^R + \mathbf{R}_F^R \boldsymbol{\omega}_I^F = [p_R, q_R, r_R]^T \quad (25)$$

Substituting in (25), the quantities from (13), (17) and (19) and taking into account (23) and (24), we obtain to express the body-frame reference angular velocities as functions of the trajectory variables as well as of the body-fixed variables u_R, $\dot{u}_R$, w_R, $\dot{w}_R$.

Considering now the two unactuated dynamical equations (6b) and (6c) during tracking, and substituting in them the expressions (23), (24), and (25) that give υ_R, $\dot{\upsilon}_R$, p_R, q_R, and r_R, a system of two coupled nonlinear time-varying differential equations results:

$$\begin{aligned} \dot{u}_R &= f_u(x_R, \dot{x}_R, \ddot{x}_R, \dddot{x}_R, y_R, \dot{y}_R, \ddot{y}_R, \dddot{y}_R, \\ &\quad z_R, \dot{z}_R, \ddot{z}_R, \dddot{z}_R, u_R, w_R), \\ u_{R,o} &= u_R(t=0) \end{aligned} \quad (26)$$

$$\begin{aligned} \dot{w}_R &= f_w(x_R, \dot{x}_R, \ddot{x}_R, \dddot{x}_R, y_R, \dot{y}_R, \ddot{y}_R, \dddot{y}_R, \\ &\quad z_R, \dot{z}_R, \ddot{z}_R, \dddot{z}_R, u_R, w_R), \\ w_{R,o} &= w_R(t=0) \end{aligned} \quad (27)$$

Since the time-varying inputs x_R, y_R, z_R, and their derivatives are known, numerical integration of (26) and (27) yields the values of u_R and w_R as functions of time.

Having now these functions, we can go back and compute the rest of the reference variables: υ_R from (23), p_R, q_R and r_R from (25), and ϕ_R, θ_R, and ψ_R from (21). Therefore, at this step, all feasible trajectory variables are known.

4. OPEN-LOOP CONTROL AND SIMULATIONS

In this section, we design an open-loop controller in order for the AUV to track a reference trajectory. Then, simulation results are presented to illustrate the way the above planning methodology applies.

4.1 Open-loop tracking control

Once the body-fixed variables, i.e., linear and angular velocities and Euler angles, required for the AUV to track the reference trajectory have been computed, it is straightforward to construct the corresponding open-loop controls using the dynamic model. Indeed, from (6a), (6e), and (6f) it is respectively:

$$\begin{aligned} T_R &= (m - r_3 X_{\dot{u}})\dot{u}_R - (r_3 X_{wq} - m) w_R q_R \\ &\quad -(r_3 X_{vr} + m)\upsilon_R r_R - r_2 X_{uu} u_R^2 - r_2 X_{vv} \upsilon_R^2 \end{aligned} \tag{28a}$$

$$\begin{aligned} \delta_{sR} &= (1/r_3 u_R^2 (M_{ds} - 2M_{db}))[(I_y - r_5 M_{\dot{q}})\dot{q}_R \\ &\quad -(r_5 M_{pr} + I_z - I_x) p_R r_R - r_4 M_{uq} u_R q_R \\ &\quad -r_3 M_{uw} u_R w_R - z_{CB} s\theta_R B] \end{aligned} \tag{28b}$$

$$\begin{aligned} \delta_{rR} &= (1/r_3 u_R^2 N_{dr})[(I_z - r_5 N_{\dot{r}})\dot{r}_R \\ &\quad -(r_5 N_{qp} + I_x - I_y) p_R q_R - r_3 N_v u_R \upsilon_R \\ &\quad -r_4 N_r u_R r_R] \end{aligned} \tag{28c}$$

where, for (28b), we have used that $\delta_{bR} = -\delta_{sR}$. In the case of tracking with constant reference velocities $\dot{u}_R$, $\dot{q}_R$, and $\dot{r}_R$ are zero; else, one must take surge acceleration from (26), and differentiate q_R and r_R from (25). Note that in order for the controls δ_{sR} and δ_{rR} to be valid functions, the surge velocity u_R must be different from zero. This is a natural requirement since for such kind of actuation to be functional the vehicle must be in forward motion.

The presented trajectory planning algorithm and the direct resulted open-loop controller may be incorporated in a two-step, closed-loop, trajectory-tracking controller design. Forward feeding the actuators of an AUV with the above computed open-loop controls, consistent with vehicle dynamics, one can design a feedback controller to take care of the small remaining errors. Such a controller will not require high gains and, hence, will have an improved performance. A further advantage of the method is the ease of checking the possibility that the computed controls result in actuator saturation; if this is the case, the designer can replan the trajectory choosing slower functions of time for the representation of the inertial curve. Moreover, we observe that this methodology can be applied to vehicles with similar dynamic models and actuation, such as unmanned aeroplanes and helicopters, moving in 3D space.

4.2 Simulation results

In this section, we present an example of trajectory planning and open-loop controlled motion. The CM inertial trajectory is a helix given by:

$$x_R(t) = 60\cos(0.01t) \tag{29a}$$

$$y_R(t) = 60\sin(0.01t) \tag{29b}$$

$$z_R(t) = 0.1t \tag{29c}$$

Differentiating (29) three times and following the above designed trajectory planning procedure, provides the reference variables which are depicted in the following diagrams in dotted lines; the actual variables are given in solid lines. Since there is no error feedback in the open-loop controls, we must start the simulation using the reference initial values for the AUV dynamics in order for the application to make sense.

In Fig. 3, the planned reference path is depicted. In Fig. 4, the angles β and α, computed from (15) and (16) are shown. In Fig. 5, the constant values of the surge force and the angles of rotation of the rudder as well as the bow and stern planes computed from (28) are shown. In Fig. 6, we see the perfect inertial path following of the reference position for the CM of the AUV. As far as the reference orientation concerns, negligible errors are only present in the roll Euler angle. Such errors are attributed to numerical errors and to inaccurate setting the initial values for the simulation. Very good tracking performance we observe in Fig. 7 where the linear and angular velocities, actual and reference, are depicted.

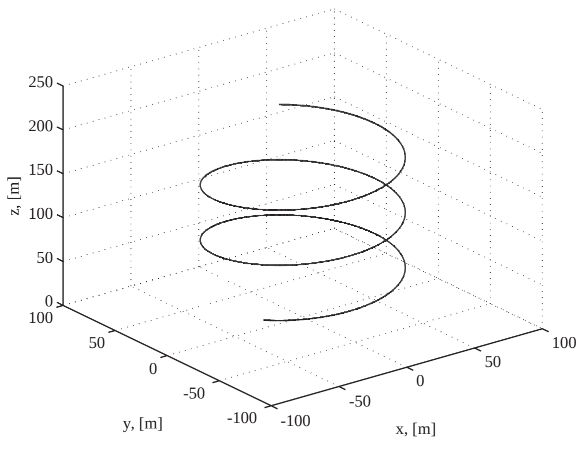

Fig. 3. The actual and the reference 3D space path.

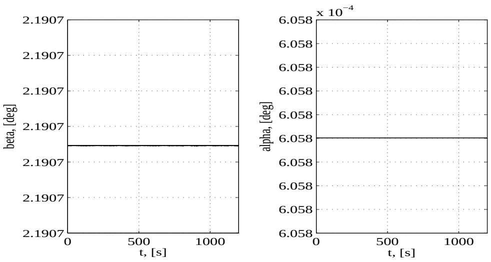

Fig. 4. The angles β and α .

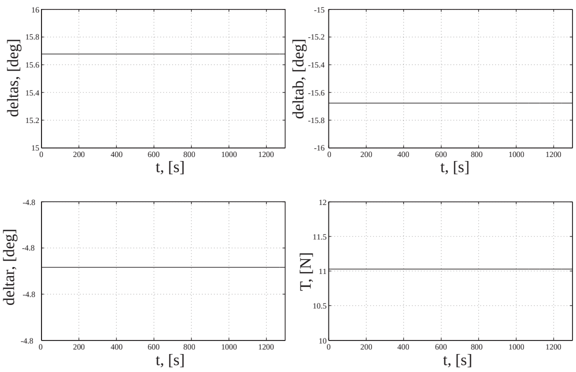

Fig. 5. Open-loop control inputs.

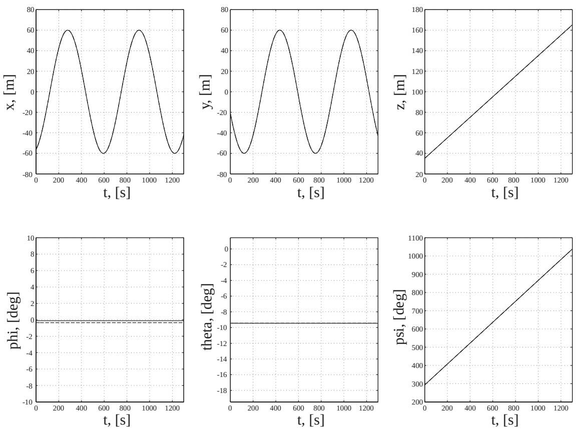

Fig. 6. Inertial position variables of the CM and the Euler angles.

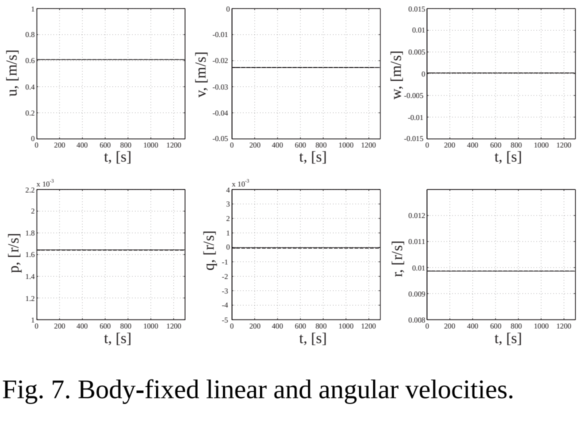

Fig. 7. Body-fixed linear and angular velocities.

3. CONCLUSIONS

In this paper, we considered the problem of spatial planning and open-loop control for underactuated AUVs. Given a smooth inertial 3D trajectory to be followed by the CM of the vehicle, the developed algorithm used the vehicle dynamic model, and computed the 3D corresponding body-fixed linear and angular velocities as well as vehicle orientations, yielded a feasible 6 DOF trajectory. Utilizing the consistent with AUV's dynamics variables, the design of tracking controllers is assisted best. The full 6 DOF computed trajectory is further used to compute the efforts for the three available control inputs and to result in an open-loop trajectory controller.

As a future work, we consider the design of a robust closed-loop controller in order to counteract parametric uncertainties and environmental disturbances. Also, in order to avoid rotational kinematics singularities, representation of kinematics by means of quaternions will be considered.

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