On Dynamic Analysis and Control of a Novel Orbital Debris Disposer

Georgios Rekleitis and Evangelos Papadopoulos[1]

National Technical University of Athens, Department of Mechanical Engineering
15780 Athens Greece
e-mail: {georek | egpapado@central.ntua.gr}

Abstract. The removal of orbital debris is a special on-orbit service that gains interest. In this paper, we present the novel idea of an orbital debris disposer, based on a number of cooperating robotic free-flyers equipped with a flexible surface, such as a net. A uni-dimensional model of the system is studied as a proof of concept, to help in the understanding of system dynamics and physical constraints. This understanding is used in the design of a control strategy aiming at successful debris captures. The response of a simulated system shows that in principle, the constrained control method can achieve the task, and is sufficiently robust in the presence of inaccurate debris parameters.

1 Introduction

During space activities, astronauts have been required to work on extra-vehicular activities (EVA). Most of these operations fall under the general area of on-orbit servicing (OOS) and include tasks like refueling, upgrading, assembling, and repairing of orbital systems. In recent years, OOS is studied as a potential field of operation for orbital robotic systems, (Landzettel et al., 2004), (Dupuis et al., 2004). Systems must be able to perform the EVA needed, relieving astronauts from hostile environment work, or must be able to perform tasks impossible by EVA, such as salvaging uncontrolled satellites. Examples of such systems include NASA's Robonaut, (Bluethmann et al., 2004), or JAXA's ETS-VII, (Yoshida, Hashizume and Abiko, 2001).

Decades of space program activities have burdened the orbital environment with thousands of orbital debris, sized from one square millimeter to one square meter or even larger. These are the products of several collisions as well as of various intentional or unintentional breakups (NASA, 2004). Adding to these the meteorites and other space objects that end up orbiting around the earth, it is easy to understand why the orbital environment includes tens of thousands of orbiting debris (NASA, 2005). These objects pose serious dangers to operating orbital systems and their presence has initiated several studies to estimate the impact results (Lacome, et al. 2002). The actions taken up to now include tracking large debris and imposing avoidance maneuvers on threatened systems, while letting small ones hit a system's armor. Recently, the danger of collision with debris has become a serious concern. Methods for reducing man-made orbital debris, or for debris mitigation have been studied (Smith, D.A. et al, 2003). A special OOS activity, the removal of debris from the orbital environment, has emerged, and methods such as

[1] Support of this work by the EPAN Cooperation Program 4.3.6.1.b (Greece-USA 035) of the Hellenic General Secretariat for Research and Technology is acknowledged.

re-orbiting debris at much higher altitudes, (Todd and Bowling, 2004), or pushing the debris into catastrophic re-entry, (Patera and Ailor, 1998) are being proposed.

In this paper, the novel idea of an orbital debris disposer is proposed. A number of cooperating robotic free-flyers equipped with a flexible surface, such as a net, are controlled aiming at debris capture. A uni-dimensional model of the system is studied, to help in the understanding of system dynamics and of the physical constraints involved, and in designing control algorithms. A control method is developed, that after contact, forces the robotic system and the object to move with the same speed, keeping thus the debris on the net. Simulation results validate the control method, which is robust in the presence of debris parameter uncertainty.

2 Concept Overview

A main problem in capturing orbital debris is that there is no predetermined way of grabbing them, since they include no attachment points. Moreover, their motion can be complex, as they may be tumbling in orbit, making it even more difficult to track and capture, even if their motion itself is predictable. This work studies a method of capturing orbital debris, employing a coordinated swarm of free-flying robots. Using their manipulators, these free-flyers move a rigid ring with a net, and fly before the debris with a relative velocity such that the debris is slowly approaching the net, see Fig. 1. As the debris is approaching, for safety and fuel economy reasons, the robots turn to free-floating mode, i.e. their satellite base is uncontrolled (Papadopoulos and Dubowsky, 1991), and wait for the collision. The collision must be such that the robot system and the debris will continue moving together, with the debris in the net, thus making it much easier to remove it from the orbit. To do this, the robot system initial velocities, as well as the gains of the manipulator joint control algorithms must be chosen appropriately. In the present work, the study of a uni-dimensional model of the interaction is presented as a proof of concept, but also aiming at a better understanding of the behavior of such systems.

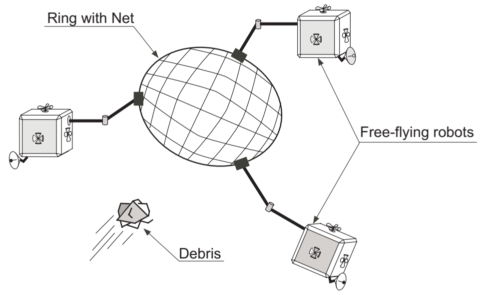

Figure 1. Graphic representation of the Orbital Debris Disposer.

3 Simplified Model Analysis

The system in Fig. 1 is modeled as a uni-dimensional three-body system, see Fig. 2. Body 1 represents the debris, body 2, the spring k, and the damper, b, represent the net, and the body 3, the free-flying satellites. The manipulators apply forces u. The natural spring length is x_0.

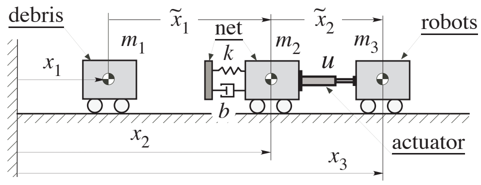

Figure 2. A 1D model of the orbital debris disposer.

The system equations of motion, obtained after the debris / net contact, are:

$$\begin{aligned} m_1\ddot{x}_1 + b(\dot{x}_1 - \dot{x}_2) + k\left(x_0 - (x_2 - x_1)\right) &= 0 \\ m_2\ddot{x}_2 - b(\dot{x}_1 - \dot{x}_2) - k\left(x_0 - (x_2 - x_1)\right) &= -u \\ m_3\ddot{x}_3 \qquad\qquad\qquad\qquad\qquad\qquad &= u \end{aligned} \tag{1}$$

The above system has no external forces acting on it, therefore its linear momentum is conserved. This can be seen by adding all three of them and integrating the result once to obtain,

$$m_1\dot{x}_1 + m_2\dot{x}_2 + m_3\dot{x}_3 = L \tag{2}$$

where L is the system linear momentum. Note that if two of the velocities are driven to their desired values, the third will be given by Eq. (2). This allows us to study the first two equations of motion, in which x_3 is missing. Without loss of generality, we assume that $x_0 = 0$, and obtain,

$$\begin{aligned} m_1\ddot{x}_1 + b(\dot{x}_1 - \dot{x}_2) + k(x_1 - x_2) &= 0 \\ m_2\ddot{x}_2 - b(\dot{x}_1 - \dot{x}_2) - k(x_1 - x_2) &= -u \end{aligned} \tag{3}$$

4 Control Law and Constraints

The problem addressed next is how to actuate the manipulators so that after the collision transient, all bodies move with the *same* velocity. This is important, because otherwise, in the absence of mechanical handles or special latching mechanisms, after the impact, the debris will separate from the net and be lost before it is captured.

From an analytical point of view, this means that two relative speeds must be set with a single control force (underactuation). To achieve this, the control force must be such that the net (spring-damper) returns to its equilibrium point without overshooting, while at the same time the net and the debris acquire the desired speed. Should the spring-damper system overshoot the equilibrium, the debris will acquire a velocity resulting in its loss from the net.

The analysis starts by observing that if all body steady state velocities $\dot{x}_{i,ss}$ are equal, then they must be equal to the system center of mass (CM) velocity given by:

$$\dot{x}_{1,ss} = \dot{x}_{2,ss} = \dot{x}_{3,ss} = L / M \tag{4}$$

where M is the total system mass. This observation directs us to choose the controller force as:

$$u(\dot{x}_1, \dot{x}_2, L) = K_1\left(\dot{x}_2(t) - \dot{x}_1(t)\right) + K_2\left(\dot{x}_2(t) - L / M\right) \quad (5)$$

where K_1 and K_2 are unknown control gains. In Eq. (5), the first term attempts to make the relative velocity of the debris and the net zero, while the second one to make the absolute velocity of the net equal to the CM velocity. If two of the three bodies' velocities attain the velocity given by Eq. (4), then, Eq (2) guarantees that the third will also be the same. Therefore, in principle this controller can achieve the goal of equal velocities.

The general form for the response for Eqs. (3) and (5) is given by:

$$x_1(t) = c_{11}t + c_{12} + c_{13}e^{\lambda_1 t} + c_{14}e^{\lambda_2 t} + c_{15}e^{\lambda_3 t} \quad (6a)$$

$$x_2(t) = c_{21}t + c_{22} + c_{23}e^{\lambda_1 t} + c_{24}e^{\lambda_2 t} + c_{25}e^{\lambda_3 t} \quad (6b)$$

where the c_{ij} (i =1, 2, j = 1,...,5) coefficients depend on system parameters and initial conditions, while the λ_i eigenvalues depend on system parameters, only.

Next, the constraints under which the system will yield the desired response are studied. These constraints are classified as (a) "stability" constraints (SC), so that the velocities of the debris, the net and the robots will converge to certain real values, (b) "contact" constraints (CC), so that the debris will not separate from the net after the impact and, (c) "assumption" constraints (AC), so that the development of the previous types of constraints hold true. For example, in Eqs. (6) real λ_i 's are assumed and this is an AC type of constraint. Different ACs result in different groups of constraints to be satisfied. SC, CC, and AC combinations are grouped together and used to compute feasible solutions. In more detail, using a set of initial conditions, namely the observed relative velocities, and the system parameters, such as the body masses and the spring-damper coefficients, the various groups of constraints result in some feasible control pairs K_1 and K_2. If at least one such pair exists, it will ensure the desired response of the system; otherwise, the approach velocity of the free-flyer / net system must be modified till a feasible pair is found. For brevity, here we present the analysis of one such group of constraints, only.

With the AC of real λ_i 's, it can be shown that the following condition must be hold,

$$c_{11} = c_{21} = L / M \quad (7)$$

By differentiating Eqs. (6), while incorporating Eq. (7), it can be shown that all steady state velocities can be made equal to L / M if,

$$\lambda_1, \lambda_2, \lambda_3 < 0 \quad (8)$$

Eq. (8) yields the only set of SC constraints, which must be satisfied to attain the desired goal.

Next, the required CCs are derived. These result from the fact that, after the first time at which the net starts deforming, the debris must be in contact with the net. Using Eq. (7), Eqs. (6) yield the initial form of the single CC, (where for simplicity we take $x_0 = 0$):

$$x_2(t) - x_1(t) = (c_{22} - c_{12}) + \sum_{i=3}^{5}\left((c_{2i} - c_{1i})e^{\lambda_{i-2} t}\right) \le x_0 = 0 \quad (9)$$

Next, Eq. (9) is analyzed aiming at obtaining conditions that are independent of time. To this aim, additional conditions are derived from Eq. (9). Different assumptions will lead to a different development of the contact constraint and, thus, to a different group of constraints.

The steady state of Eqs. (6) is reached at infinite time. Then, the following must hold,

$$\lim_{t\to\infty}\left(x_2(t)-x_1(t)\right)=x_0=0\Rightarrow c_{22}-c_{12}=0 \tag{10}$$

Thus, the CC given by Eq. (9) becomes:

$$(c_{23}-c_{13})e^{\lambda_1 t}+(c_{24}-c_{14})e^{\lambda_2 t}+(c_{25}-c_{15})e^{\lambda_3 t}\le 0 \tag{11}$$

At $t = 0$ (first contact) we have $x_2(0) - x_1(0) = x_0 = 0$, thus, from Eqs. (6), (7) and (10), we have:

$$(c_{23}-c_{13})+(c_{24}-c_{14})+(c_{25}-c_{15})=0 \tag{12}$$

It is clear that the three terms in Eq. (12) cannot have the same sign. Two possibilities exist here, i.e. we may have (a) two negative terms and one positive or (b) two positive terms and one negative. For brevity, we will present only case (b). Without loss of generality, we assume:

$$c_{23}-c_{13}<0,\quad c_{24}-c_{14},\ c_{25}-c_{15}\ge 0 \tag{13}$$

Inequalities in Eq. (13) yield another set of AC, which must hold in order for the following development of the CC given by Eq. (11) to hold. Dividing Eq. (11) by $c_{23}-c_{13}$, the CC yields:

$$\frac{c_{24}-c_{14}}{c_{23}-c_{13}}e^{(\lambda_2-\lambda_1)t}+\frac{c_{25}-c_{15}}{c_{23}-c_{13}}e^{(\lambda_3-\lambda_1)t}\ge -1 \tag{14}$$

From Eq. (12), we see that constraint (14) holds as equality for $t = 0$. Thus, Eq. (14) holds true iff:

$$\lambda_2-\lambda_1<0 \text{ and } \lambda_3-\lambda_1<0 \tag{15}$$

Inequalities (15) are the only needed CC's in this case. SC (8), AC (13) and CC (15) along with the AC that λ_i are real, make the first group of constraints that, if satisfied, the system will have the desired response. For different assumptions, such as Ineqs. (13), different groups of constraints have been derived. As mentioned above, to each such group, a range of control gains may correspond and yield the desired response. To this end, for a set of system parameters and initial conditions, plots of a group of constraints such as of Eqs. (8), (13) and (15) are drawn in the gain space, for identifying the feasible subspaces in which the acceptable sets of control gains lie. If such a subspace does not exist, a different set of constraints is tried, until a feasible set of gains results. Otherwise, since the debris velocity cannot be changed, the approach velocity of the capturing system may have to be adjusted by firing its thrusters.

5 Simulation Results

The system described by Eq. (1) is simulated in MATLAB. The system parameters are m_1 = 20 kg, m_2 = 20 kg, m_3 = 140 kg, k = 500 kg/s^2, b = 40 kg/s, and x_0 = 0. The initial conditions are $x_1(0)$ = 200 m/s, $x_2(0)$ = 199 m/s and $x_3(0)$ = 196.5 m/s. The result of the impact of the three bodies, for a given set of system parameters and initial conditions, with random control gains, are shown in Fig. 3. As seen in Fig. 3 by the relative distance $x_2 - x_1$, the debris bounces off the net at the moment this distance exceeds x_0=0, and then they move with different speeds, parting rapidly.

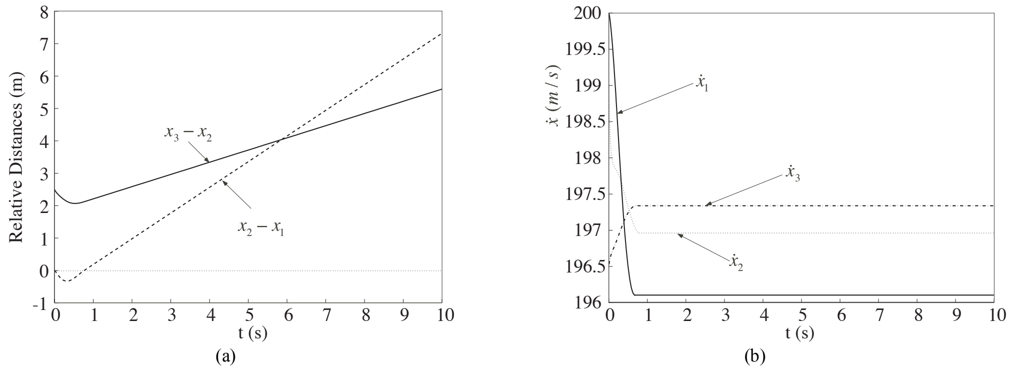

Figure 3. (a) Relative positions response and (b) velocities for control gains K_1 = 80 kg/s and K_2 = 400 kg/s, that do not satisfy the constraints for successful capture.

Next, the control gains are chosen according to the presented method, while the initial conditions and system parameters remain the same as before. Note that the presented gains are obtained by the same group of constraints as the one presented at the previous section. The velocity response of the three objects for this case is shown in Fig. 4, where we can see that they end up moving with the same speed $\dot{x}_1 = \dot{x}_2 = L / M$.

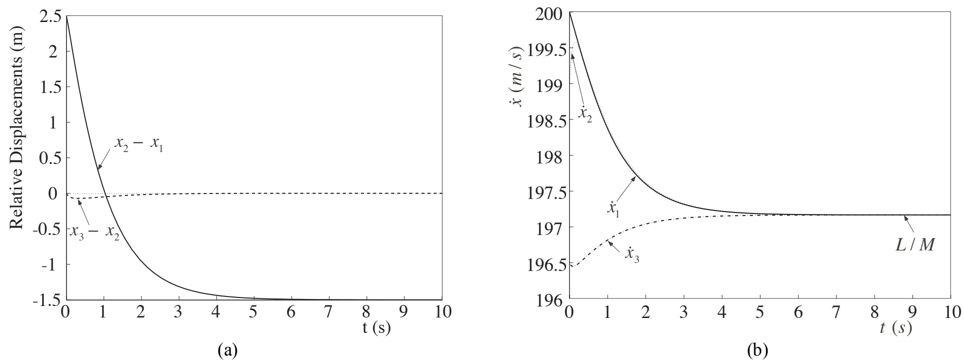

Figure 4. (a) Relative positions response and (b) velocities for control gains K_1 = 250 kg/s and K_2 = 30 kg/s, that satisfy the constraints for successful capture.

From Fig. 4, we can see that the distance between debris and net, does not exceed $x_0 = 0$ after the first impact, as desired, while the net is displaced for about -1.5*m* with respect to the robot base. Fig. 5 shows three snapshots of the system motion, namely the instant of the first contact, that of the net's maximum deformation and finally the instant in which the steady state is reached.

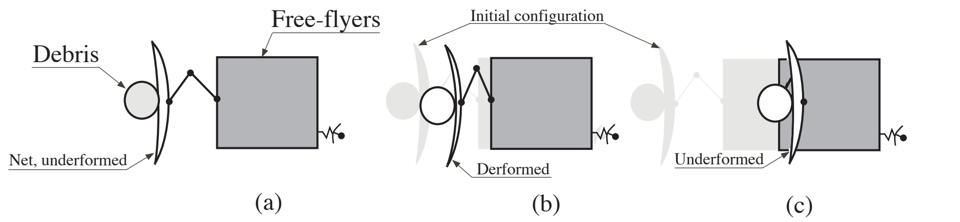

Figure 5. Motion snapshots of the orbital debris disposer during capture. (a) t = 0.0 s, first contact, (b) t = 0.32 s, net's maximum deformation, (c) t = 7.8 s, all bodies move together.

Next, the robustness of the controller strategy is investigated. Typically, the debris mass, its initial velocity, or both, may be inaccurately estimated. Thus, another simulation was run with estimated debris mass 10% less than the actual. As shown in Fig. 6, the velocities of the debris and the net converge on the estimated CM velocity L_{est}/M_{est} and not on the correct L/M. Due to momentum conservation, this results in the robot bases moving at a lower steady state velocity. In this case, at the instance the debris and the net attain their common velocity, a convergence force is applied between the net and the robot bases, so as to decelerate the net and accelerate the robot bases. Then, the system reaches a common velocity, avoiding the possibility of a extending the manipulator to its limits. Also, note that since this force decelerates the net, it will not result in debris loss, but will keep it captured. For a 3 *m* reach manipulator, we assume that the common velocity must be attained before the net carrying end-effector reaches 2.75 *m*. In Fig. 6, the transient of the debris-net system is over when the end-effector is at 2.60 *m*, i.e. less than 2.75 *m*. At this point, the convergence force starts acting to make all three velocities equal to L/M.

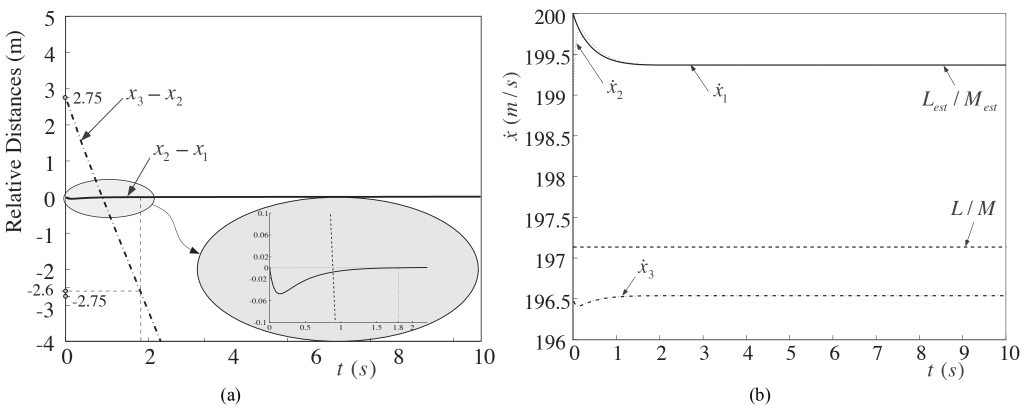

Figure 6. (a) Relative positions response and (b) velocities for control gains K_1 = 250 kg/s and K_2 = 30 kg/s that satisfy the constraints for successful capture, but with inaccurate estimation of the debris mass.

If the debris mass is overestimated, then the free-flying robots move at steady state with greater velocity than the net-debris system. A way to compensate for this, after the net and the debris reach the same velocity, is to fire the jets of the free-flyers, to slowly decelerate the whole system. This motion will keep the debris on the net, while at the same time will stop the distancing of the robots from the net.

This one-dimensional model analysis demonstrates the concept feasibility. During our current work, we extend the dynamics and control scheme, as well as the accompanying constraints to a more realistic system with higher dimension.

6 Conclusions

A novel concept for an orbital debris capturing system has been presented, including a number of orbital free-flying robots, holding a net, by which the orbital debris is to be captured. In order to better understand the behavior of this complex system, a simplified uni-dimensional system has been derived. The dynamics and control of the system and the desired system response led to the derivation of sets of constraints that must be hold between system parameters and initial conditions. Satisfying the constraints results to a set of control gains that are used in a velocity-based controller. The response of the simulated system has confirmed the constrained control method. The robustness of the method was studied in the case of inaccurate estimation of the debris parameters. It was found that the method shows robustness, keeping the debris in the net, although an additional force may be needed.

Bibliography

Bluethmann, W. et al. (2004). Building an Autonomous Humanoid Tool User. *Proc. IEEE-RAS Int. Conf. on Humanoid Robotics.* Los Angeles, CA, Nov. 2004.

Dupuis, E. et al. (2004). Autonomous Operations for Space Robots. *55th International Astronautical Congress.* Paper #IAC-04-IAA.U.5.03. Vancouver, Canada, Oct. 4-8, 2004.

Lacome, J. L., Espinosa Ch. and Gallet, C. (2002). Simulation of Hypervelocity Spacecrafts and Orbital Debris Collisions Using Smoothered Particle Hydrodynamics in LS-DYNA. *5th Dynamics and Control of Systems and Structures in Space Conference*. King's College, Cambridge, UK, July 14th – 18th.

Landzettel, K. et al. (2004). System Prerequisites and Operational Modes for On Orbit Servicing. *Int. Symposium Space Technology and Science (ISTS)*, Miyazaki, Japan, May 30–June 06 2004.

NASA (2005) *The Orbital Debris Quarterly News*. Vol. 9, Issue 1, pp 3. Available on-line in the address: http://www.orbitaldebris.jsc.nasa.gov/newsletter/newsletter.html

NASA (2004) *History Of On-Orbit Satellite Fragmentations*, 13th Edition. Lyndon B. Johnson Space Center, Houston, TX, May 2004.

Papadopoulos, E. and Dubowsky, S., On the Nature of Control Algorithms for Free-floating Space Manipulators. *IEEE Tr. on Robotics and Automation*, Vol. 7, No. 6, December 1991, pp. 750-758.

Patera, R. P. and Ailor, W. H. (1998) The Realities of Reentry Disposal. *Proceedings of the AAS/AIAA Space Flight Mechanics Meeting*, Monterey, California, February 9–11; also in *Advances in Astronautical Science* Vol. 99, pp 1059–1071, September 1998.

Smith, D.A. et al. (2003) An Assessment of Geostationary Intervention Missions. *Proc. 54th Int. Astronautical Congress*, Vol. 109, pp. 167-182. Sept. 29-Oct. 3, 2003, Bremen, Germany.

Todd, L. and Bowling, T. (2004). Debris Mitigation in Geostationary Earth Orbit. *6th Dynamics and Control of Systems and Structures in Space Conference.* Rionaggiore, Italy, 2004.

Yoshida, K., Hashizume, K. and Abiko, S. (2001) Zero Reaction Maneuver: Flight Validation with ETS-VII Space Robot and Extension to Kinematically Redundant Arm. *Proceedings of the IEEE International Conference on Robotics and Automation*, pp. 441-446, Seoul, Korea, 2001.