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Robotics and Autonomous Systems xx (xxxx) xxx–xxx

Robotics and Autonomous Systems
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On point-to-point motion planning for underactuated space manipulator systems

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Received 3 February 2006; received in revised form 26 June 2006; accepted 6 July 2006

Abstract

In free-floating mode, space manipulator systems have their actuators turned off, and exhibit nonholonomic behavior due to angular momentum conservation. The system is underactuated and a challenging problem is to control both the location of the end effector and the attitude of the base, using manipulator actuators only. Here a path planning methodology satisfying this requirement is developed. The method uses high order polynomials, as arguments in cosine functions, to specify the desired path directly in joint-space. In this way, the accessibility of final configurations is extended drastically, and the free parameters are determined by optimization techniques. It was found that this approach leads always to a path, provided that the desired change in configuration lies between physically permissible limits. Physical limitations, imposed by system's dynamic parameters, are examined. Lower and upper bounds for base rotation, due to manipulator motions, are estimated and shown in the implementation section. The presented method avoids the need for many small cyclical motions, and uses smooth functions in the planning scheme, leading to smooth configuration changes in finite and prescribed time.
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Keywords: Space free-floating robots; Underactuated; Nonholonomic planning; Pfaffian

1. Introduction

Space missions and on-orbit tasks will rely increasingly on space robots, since these tasks are either too risky or very costly, due to safety support systems, or just physically impossible to be executed by humans. On-orbit space robotic systems consist of a spacecraft fitted with one or more robotic manipulators. In free-flying mode, thruster jets can compensate for manipulator induced disturbances, but their extensive use reduces useful system life. In many instances, as for example during capture operations, it is desired that the thrusters are turned off to avoid interaction with the target. In this case, the system operates in a free-floating mode, resulting in dynamic coupling between the manipulator and its spacecraft, which is now subject to manipulator induced disturbances. This mode of operation is feasible when no external forces or torques act on the system and the total system momentum is zero.

A free-floating space robot is an underactuated system and exhibits a nonholonomic behavior due to the nonintegrability of the angular momentum, [1,2]. This property complicates the planning and control of such systems, which have been studied by a number of researchers. Vafa and Dubowsky have developed a technique called the Virtual Manipulator, [3]. Inspired by astronaut motions, they proposed a planning technique, which employs small cyclical manipulator joint motions to modify spacecraft attitude. Papadopoulos and Dubowsky studied the Dynamic Singularities of free-floating space manipulator systems, which are not found in terrestrial systems and depend on the dynamic properties of the system, [1,4]. They also showed that any terrestrial control algorithm could be used to control end-point trajectories, despite spacecraft motions, [4]. Nakamura and Mukherjee explored Lyapunov techniques to achieve simultaneous control of a spacecraft's attitude and its manipulator joints, [5]. To limit the effects of a certain null space, the authors proposed a bidirectional approach, in which two desired paths were planned, one starting from the initial configuration and going forward and the other starting from the final configuration and

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doi:10.1016/j.robot.2006.07.003

Please cite this article as: Ioannis Tortopidis, Evangelos Papadopoulos, On point-to-point motion planning for underactuated space manipulator systems, Robotics and Autonomous Systems (2006), doi:10.1016/j.robot.2006.07.003.

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going backwards. The method is not immune to singularities and yields non-smooth trajectories with the joints coming to a stop at the switching point.

Another method that allows for Cartesian motion of a manipulator's end point and avoiding dynamic singularities, has been proposed by Papadopoulos in [6]. The method involved small end-effector Cartesian cyclical motions designed to change the attitude of the spacecraft to one that was known to avoid dynamic singularities, [6].

To avoid representational singularities, Caccavale and Siciliano used quaternions, and developed kinematic control schemes for a redundant manipulator mounted on a free-floating spacecraft, [7]. Franch et al. used flatness theory, to plan trajectories for free-floating systems, [8]. Their method requires selection of robot parameters so that the system is made controllable and linearizable by prolongations. Yoshida tried zero reaction maneuvers on the Japanese experimental space robot ETS-VII [9] and presented flight data.

The path planning problem for a free-floating prismatic-jointed manipulator, is addressed by Pandey and Agrawal [13]. The end-effector is moved to a desired position and orientation, using a prismatic-jointed manipulator. However, the requirement for a desired final base attitude, is not taken into account. Lampariello, et al. presented a motion planning method for free-flying space robots, which gives optimal solutions for spacecraft actuation and movement duration, and avoids unnecessary spacecraft actuation, [14]. However, the final attained spacecraft attitude is unknown beforehand, and is obtained only after the optimal solution is implemented.

An analytical approach to the path planning problem was developed and presented by the authors, [17]. The method allowed for endpoint Cartesian point-to-point control with simultaneous control of the spacecraft's attitude, using manipulator actuators only. It was based on a transformation of the angular momentum to a space where it could be satisfied trivially. Smooth functions, such as polynomials, were used to plan motions in that space, and the system was driven to a desired configuration in finite and prescribed time, without the need for many small cyclical motions. Further work showed that final configuration accessibility improved drastically, when high order polynomials for the joint angles were used as *arguments* in cosine functions. The planning problem was reduced to solving a non-linear equation, representing the integral of motion.

Based on this idea, in this paper a numerical path planning approach is developed for the general case of an N degree-of-freedom (dof) manipulator. First the dynamics of a free-floating space manipulator system is briefly given. Then, the general 3D case path planning problem is formulated as an optimization problem. The applicability of the method is illustrated by various examples, where the manipulator is mounted on an arbitrary point of the base. Also, for the planar system, physical limitations imposed by system dynamic parameters are examined, and lower and upper bounds for base rotation, due to manipulator motions, are estimated analytically. We remind here, that we are talking about paths that lead to the desired final configuration in finite and prescribed time, without the need of many small cyclical motions.

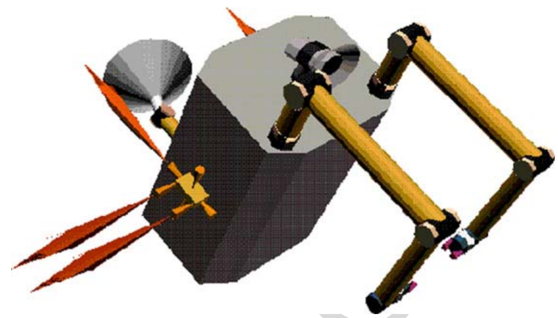

Fig. 1. A free-floating space manipulator system.

The main advantages of this method, are briefly mentioned here: (a) it is always possible to find a path, provided that the desired change in configuration lies between physically permissible limits, (b) it is possible to obtain paths leading to almost maximum permissible change in base attitude, in one "simple" point-to-point motion, i.e. a path that avoids many small cyclical motions, (c) determination of free parameters is automated, and (d) additional requirements such as joint limits or obstacle avoidance can be achieved by adding more freedom in the path via the use of higher order polynomials. As in [17], smooth functions are employed and the system is driven to the desired configuration, in finite and prescribed time, without requiring many small cyclical motions.

2. Dynamics of free-floating space manipulators

Free-floating space manipulator systems consist of a spacecraft (base) and one or more manipulators mounted on it, see Fig. 1. In free-floating mode, no external forces and torques act on the system, the attitude control system of the spacecraft is turned off, and the spacecraft translates and rotates in response to manipulator motions.

The manipulator has revolute joints and an open chain kinematic configuration, so that, in a system with N-degree-of-freedom (dof) manipulator, there are $N + 6$ dof in total. The system is underactuated, and controlling both the end-effector position and the attitude of the base, using manipulator actuators only, is a non trivial task which can be achieved by exploiting the nonholonomic nature of the system, [3,5].

Since no external forces act on the system, and the initial momentum is zero, the system Center of Mass (CM) remains fixed in space, and the coordinates origin, O, can be chosen to be the system's CM. Essentially this removes three of the six underactuated dof of the system, leaving three more to deal with, and is a result of the integrability of the translational equations of motion written for the system CM. The equations of motion, have the form, [4],

$$\mathbf{H}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{C}(\mathbf{q}, \dot{\mathbf{q}}) = \boldsymbol{\tau} \tag{1}$$

where $\mathbf{H}(\mathbf{q})$ is a positive definite symmetric matrix, called the reduced system inertia matrix, and $\mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})$ contains nonlinear velocity terms. The $N \times 1$ column vectors $\mathbf{q}, \dot{\mathbf{q}}$ and $\boldsymbol{\tau}$ represent manipulator joint angles, velocities, and torques. The

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base attitude is computed using the conservation of angular momentum, [1],

$$ {}^{\mathbf{0}}\omega_{\mathbf{0}} = \underbrace{-\,{}^{\mathbf{0}}\mathbf{D}^{-\mathbf{1}}(\mathbf{q})\;{}^{\mathbf{0}}\mathbf{D}_{\mathbf{q}}(\mathbf{q})}_{\mathbf{F}_1(\mathbf{q})}\,\dot{\mathbf{q}} \tag{2}$$

where ${}^{\mathbf{0}}\omega_{\mathbf{0}}$ is the base angular velocity in the spacecraft 0th frame, and ${}^{\mathbf{0}}\mathbf{D}$ (3×3), ${}^{\mathbf{0}}\mathbf{D}_{\mathbf{q}}$ $(3\times N)$ are inertia-type matrices. Eq. (1) describes the reduced system dynamics in joint space (N-dofs). However, integration of these equations does not yield the attitude of the spacecraft, as these equations are independent of the spacecraft attitude. Therefore, to compute the resulting spacecraft attitude, one needs to append to Eq. (1) the angular momentum equation, given by (2).

If we use ZYX Euler angles (Yaw–Pitch–Roll), to represent the attitude of the base $\psi_0 = [\ \theta_1 \quad \theta_2 \quad \theta_3\ ]^{\mathrm{T}}$, [15,16], then the base angular velocity can be expressed as a function of the Euler rates,

$$\begin{aligned} {}^{\mathbf{0}}\omega_{\mathbf{0}} \;&=\; \mathbf{E}(\psi_0)\dot{\psi}_0 = \begin{bmatrix} -s_2 & 0 & 1 \\ c_2 s_3 & c_3 & 0 \\ c_2 c_3 & -s_3 & 0 \end{bmatrix} \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{\theta}_3 \end{bmatrix} \\ &\Leftrightarrow\; \dot{\psi}_0 = \mathbf{E}^{-1}(\psi_0)\,{}^{\mathbf{0}}\omega_{\mathbf{0}} = \frac{1}{c_2}\begin{bmatrix} 0 & s_3 & c_3 \\ 0 & c_2 c_3 & -c_2 s_3 \\ c_2 & s_2 s_3 & s_2 c_3 \end{bmatrix} {}^{\mathbf{0}}\omega_{\mathbf{0}}. \end{aligned} \tag{3}$$

In the above equation, $\mathbf{E}^{-1}$ is non-singular except at discrete configurations, i.e. for pitch-angle, around the y-axis: $\theta_2 = \pm\pi/2 \Leftrightarrow c_2 = 0$. If such a configuration occurs, this can be dealt with by the use of an alternative set of Euler angles. Alternatively, Euler parameters can be used to describe the base attitude. However, the analysis that follows is independent of the method chosen to represent the base attitude.

Using (2) and (3), the conservation of angular momentum is rewritten as

$$\dot{\psi}_0 = \mathbf{E}^{-1}(\psi_0)\mathbf{F}_1(\mathbf{q})\,\dot{\mathbf{q}} \tag{4}$$

where $\mathbf{F}_1(\mathbf{q})$ is a function of the configuration defined in (2). It is well known that (4) cannot be integrated to analytically yield the spacecraft orientation ψ_0 as a function of the system's configuration, [1,6]. However, if the joint angle trajectories are known as a function of time, then (4) can be integrated numerically to yield the trajectory for the base orientation. This nonintegrability property introduces nonholonomic characteristics to free-floating systems, and results from the dynamic structure of the system; it is not due to kinematics, as is the case with nonholonomic constraints in mobile robots with wheels, [11].

3. Nonholonomic path planning

The main problem we address here is to find a path which connects a given initial configuration $(\psi_0^{in}, \mathbf{q}^{in})$ to a final one $(\psi_0^{\mathrm{fin}}, \mathbf{q}^{\mathrm{fin}})$, by actuating manipulator joints only. The problem must be solved in finite time and with simple motions, i.e. a large number of small cyclical motions is undesirable. It is well known that this problem is not trivial, since one must satisfy (4) which introduces nonholonomic behavior, and at the same time, achieve a change in a $(N+3)$ dimensional configuration space with only N controls (underactuated system).

Next, a planning methodology is described that allows for a systematic approach in the planning of systems subject to nonholonomic constraints of the form of (4). We use high order polynomials for $\mathbf{q}(t)$, in order to specify a trajectory directly in joint-space. Let

$$q_i = q_i(t, \mathbf{b}_i), \qquad \mathbf{b}_i((k_i+1)\times 1), \quad i = 1, \ldots, N \tag{5}$$

be polynomials of time t, of order k_i, and $\mathbf{b}_i$ the corresponding coefficients to be specified for joint-i, i.e. there exist $n_f = (k_1+k_2+\cdots+k_N+N)$ free parameters. The minimum number of constraints (n_c) to be satisfied include 6 boundary conditions per joint, i.e. the desired initial and final positions, and zero initial and final velocities and accelerations, plus at least three for the integrals of motion, representing the angular momentum conservation equation in three axes, i.e. $n_c \geq 6N+3$. Since it should be $n_f \geq n_c$, we have

$$\begin{aligned} &(k_1+k_2+\cdots+k_N) \geq 5N+3 \\ &k_i \geq 5. \end{aligned} \tag{6}$$

For each joint, the 6 boundary conditions mentioned above are imposed. If $k_i = 5$, then the corresponding trajectory of joint-i is determined. In a different case, where additional freedom is introduced to joint-i, i.e. $k_i \geq 6$, $\mathbf{b}_i$ contains (k_i-5) additional free parameters to be determined, which contribute to the satisfaction of the integrals of motion. All the other parameters in $\mathbf{b}_i$ are expressed as linear functions of these (k_i-5) free parameters.

Let $\mathbf{b} \in R^k$, the vector containing the remaining free parameters of all $\mathbf{b}_i$ $(i = 1, \ldots, N)$, after boundary conditions for all joints are satisfied. Then (5), can be written in vector form as

$$\mathbf{q} = \mathbf{q}(t, \mathbf{b}). \tag{7}$$

Using (4) and (7), we can write

$$\dot{\psi}_0 = \mathbf{F}(\psi_0, \mathbf{b}, t). \tag{8}$$

These free parameters $\mathbf{b} \in R^k$, as said earlier, should be at least three $(k \geq 3)$ and satisfy the integrals of motion

$$\begin{aligned} &\psi_0^{\mathrm{fin}}(\mathbf{b}) = \psi_0^{\mathrm{fin}}(\mathrm{des}), \quad \text{or} \\ &\mathbf{h}(\mathbf{b}) \triangleq \psi_0^{\mathrm{fin}}(\mathbf{b}) - \psi_0^{\mathrm{fin}}(\mathrm{des}) = \mathbf{0}. \end{aligned} \tag{9}$$

Eq. (8) represents a system of very complex, highly nonlinear and dynamically coupled differential equations which, as said earlier, cannot be integrated analytically. However, these equations can also be seen as a dynamic system with a constant input $\mathbf{b}$, state vector ψ_0, and t as a parameter. Then, for given joint trajectories, i.e. given $\mathbf{b}$, and initial base attitude ψ_0^{in}, (8) can be numerically integrated, on $[t_{in}, t_{\mathrm{fin}}]$, yielding the final base attitude $\psi_0(\mathbf{b}, t_{\mathrm{fin}}) \triangleq \psi_0^{\mathrm{fin}}(\mathbf{b})$. In this way the problem reduces to determining the unknown vector $\mathbf{b}$, numerically, so that (9) is satisfied.

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We should note here the existence of a physical limitation which is that for a given change in manipulator joints, only a limited change in base attitude is expected, due to the dynamic system's properties. In other words, not all configurations are reachable from an initial one, in prescribed finite time and with simple motions (without the use of many small cyclical motions). For example, it is not rational to expect large base reorientations with small manipulator displacements. Similarly, it may be impossible to move to any Cartesian point and have the final spacecraft attitude unchanged. The path planning method, presented here, is expected to give at least one solution, provided that the desired change in base attitude, due to a given change in manipulator's configuration, is feasible.

If the number of free parameters to be determined is three, i.e. if $\mathbf{b} \in R^3$, we can solve Eq. (9) numerically. The Jacobian matrix $\mathbf{J} = \partial \mathbf{h}/\partial \mathbf{b}$, required in the Newton–Raphson method, is unavailable and the finite-difference determination of this Jacobian is time and computer resource consuming. For this reason, we employ here a quasi-Newton method, often called the *secant* method, which uses a computationally inexpensive approximation of the needed Jacobian.

If the desired change in base attitude is feasible, but (9) does not yield to a solution for $\mathbf{b} \in R^k (k = 3)$, additional freedom to one or more joints can be introduced. Then, $\mathbf{b} \in R^k$ with $(k > 3)$, is determined using optimization techniques, as follows:

$$\|\mathbf{b}\| \to \min : \quad \mathbf{h}(\mathbf{b}) = \mathbf{0} \tag{10}$$

i.e., a search is initiated for a minimum norm $\mathbf{b} \in R^k$, which satisfies the boundary values for the integrals of motion (9). It was found that this approach leads to a path always, provided that the desired change in the configuration lies within the physically permissible limits. To implement a solution to the problem defined by (10), i.e. an optimization with equality constraints, we used MATLAB's fmincon() function, which uses a Sequential Quadratic Programming (SQP) method. These methods are based on an approximation of the Hessian of the Lagrangian function, using a quasi-Newton updating method.

We should clarify here, that although it is known that an optimization problem may have multiple local minima, all these minima still satisfy (9), i.e. they still solve the problem of finding a parameter vector $\mathbf{b}$ that will lead to the desired system configuration. In other words, although the problem is generally formulated as an optimization problem with equality constraints, we are mostly interested in finding solution(s) satisfying the equality constraints (9) which represent the integrals of motion, rather than finding the global minimum for $\mathbf{b}$.

Some observations of the method outlined above are presented here: First, additional requirements such as attitude change maximization, and joint limit or obstacle avoidance can be achieved by adding more freedom in the end-point path via the use of higher order polynomials for one or more joints. Also, even if there are only three parameters to be determined, (9) may result in multiple solutions, due to the periodicity of the trigonometric functions involved, elements of $\mathbf{F}_1(\mathbf{q})$, see (2),

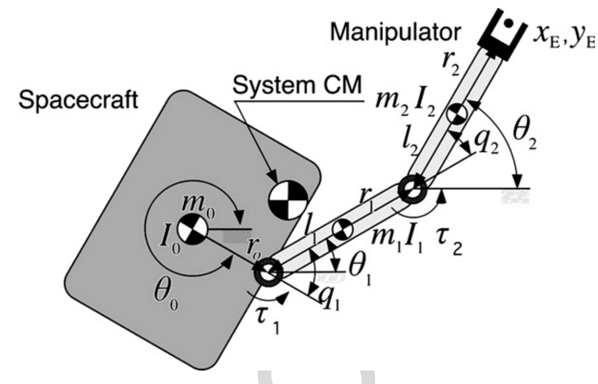

Fig. 2. A planar free-floating space manipulator system.

(4) and (11) and (A.1). This physically means that we can have alternative or multiple rotations for some joints, leading to the same final configuration. These solutions result in end-effector paths of different length. In other words, the method does not exclude multiple rotations as solutions. More generally, the joints are not restricted by the method to vary monotonically from initial to final values, and therefore greater flexibility is achieved. However, long paths are in general undesired, because for given motion duration, they result in high joint velocities. Determination of $\mathbf{b}$ according to (10) is expected to yield shorter paths.

Once joint-space trajectories are specified, the base attitude $\psi_0(t)$ is calculated by integrating (8). Following the estimation of $\mathbf{b}$, initial and final values for the base attitude ψ_0 are satisfied. Also, the initial and final velocities and accelerations of ψ_0 are necessarily zero because the joint variables have zero initial and final velocities and accelerations, and in addition, $\dot{\psi}_0(t)$ satisfies (4). Finally, since the path is defined directly in the joint space, it is always feasible and will never be subject to Dynamic Singularity problems in the system's workspace, [1].

4. Implementation

In this section, we implement the methodology outlined earlier on a free-floating robotic system consisting of a two-dof manipulator mounted on an arbitrary point of a three-dof spacecraft. This choice reduces the complexity of the problem so that it can be presented in more detail. However, the proposed method is general, and can be applied to systems with N-dof manipulators without restrictions.

4.1. System description

The spacecraft is constrained to move in the plane perpendicular to the axis of manipulator joint rotations, see Fig. 2.

For the planar free-floating space manipulator, the conservation of angular momentum equation, Eq. (2), can be written as,

$$D_0(\mathbf{q})\dot{\theta}_0 + D_1(\mathbf{q})\,\dot{\theta}_1 + D_2(\mathbf{q})\dot{\theta}_2 = 0 \tag{11}$$

where $\theta_0, \theta_1, \theta_2$ are spacecraft attitude and manipulator absolute joint angles. The D_0, D_1 and D_2 are functions of

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system inertial parameters and of the manipulator joint angles q_1 and q_2, see Appendix A. The problem to be addressed is to find a path that connects a given initial system configuration $(\theta_0^{in}, \theta_1^{in}, \theta_2^{in})$ to a final one $(\theta_0^{\text{fin}}, \theta_1^{\text{fin}}, \theta_2^{\text{fin}})$, by actuating manipulator joints only. The endpoint location x_E and y_E is given by [1],

$$\begin{aligned} x_E &= a\cos\theta_0 + b\cos\theta_1 + c\cos\theta_2 \\ y_E &= a\sin\theta_0 + b\sin\theta_1 + c\sin\theta_2 \end{aligned} \tag{12}$$

where, a, b, c, are constant terms, functions of the mass properties of the system, given analytically by (26).

4.2. Physical constraints of motion

As mentioned earlier, for a given change in manipulator joints, a limited change in base attitude is expected, depending on system dynamic properties. In other words, not all configurations are reachable from an initial one, in prescribed finite time and with simple motions (without the use of many small cyclical motions). In this section, we specify bounds for base attitude changes induced by manipulator motions, for the planar system, shown in Fig. 2. In the general 3D case, the change in the base attitude due to manipulator motions, although more difficult to be derived analytically, is bounded too, and the basic notion presented here remains valid.

Using (11), the scleronomic constraint can be written in the Pfaffian form, as

$$D_0 \mathrm{d}\theta_0 + D_1 \mathrm{d}\theta_1 + D_2 \mathrm{d}\theta_2 = 0 \tag{13}$$

$$\underbrace{(D_0 + D_1 + D_2)}_{D} \mathrm{d}\theta_0 + (D_1 + D_2)\mathrm{d}q_1 + D_2 \mathrm{d}q_2 = 0 \tag{14}$$

since, from Fig. 2., it is

$$\theta_1 = \theta_0 + q_1, \quad \text{and} \quad \theta_2 = \theta_1 + q_2 = \theta_0 + q_1 + q_2. \tag{15}$$

Next, lower and upper bounds for base rotation $\Delta\theta_0$, caused by $(\Delta q_1, \Delta q_2)$ are computed. Using (14), we write

$$\begin{aligned} \mathrm{d}\theta_0 &= -\frac{(D_1 + D_2)}{D}\mathrm{d}q_1 - \frac{D_2}{D}\mathrm{d}q_2 \\ &\triangleq g_1(\mathbf{q})\mathrm{d}q_1 + g_2(\mathbf{q})\mathrm{d}q_2 \end{aligned} \tag{16}$$

where $g_1(\mathbf{q}), g_2(\mathbf{q})$ are defined by (16) and in the Appendix by (A.1). Integrating (16) yields,

$$\Delta\theta_0 = \int g_1(\mathbf{q})\mathrm{d}q_1 + \int g_2(\mathbf{q})\mathrm{d}q_2 \triangleq \Delta\theta_{01} + \Delta\theta_{02} \tag{17}$$

where $\Delta\theta_{01}, \Delta\theta_{02}$, represent the contribution of changes Δq_1 and Δq_2 in $\Delta\theta_0$ respectively. The functions $g_1(\mathbf{q}), g_2(\mathbf{q})$ are bounded, and therefore,

$$g_{1,\min} \le g_1(\mathbf{q}) \le g_{1,\max} \tag{18}$$

$$g_{2,\min} \le g_2(\mathbf{q}) \le g_{2,\max} \tag{19}$$

where the bounds in (18) and (19) are given by

$$\begin{aligned} g_{1,\min} &= \min_{\mathbf{q}} g_1(\mathbf{q}), \quad & g_{1,\max} &= \max_{\mathbf{q}} g_1(\mathbf{q}) \\ g_{2,\min} &= \min_{\mathbf{q}} g_2(\mathbf{q}), \quad & g_{2,\max} &= \max_{\mathbf{q}} g_2(\mathbf{q}). \end{aligned} \tag{20}$$

Table 1
System parameters

Body	l_i [m]	r_i [m]	m_i [kg]	I_i [kg m^2]
0	1.0	0.5	400.0	66.67
1	0.5	0.5	40.0	3.33
2	0.5	0.5	30.0	2.50

Using (17) and (18) the resulting bounds for $\Delta\theta_{01}$ are estimated as,

$$\begin{cases} g_{1,\min}\Delta q_1 \le \Delta\theta_{01} \le g_{1,\max}\Delta q_1, & (\Delta q_1 > 0) \\ g_{1,\max}\Delta q_1 \le \Delta\theta_{01} \le g_{1,\min}\Delta q_1, & (\Delta q_1 < 0) \end{cases} \tag{21}$$

whereas bounds for $\Delta\theta_{02}$ are estimated, using (17) and (19), as

$$\begin{cases} g_{2,\min}\Delta q_2 \le \Delta\theta_{02} \le g_{2,\max}\Delta q_2, & (\Delta q_2 > 0) \\ g_{2,\max}\Delta q_2 \le \Delta\theta_{02} \le g_{2,\min}\Delta q_2, & (\Delta q_2 < 0). \end{cases} \tag{22}$$

Finally, bounds for $\Delta\theta_0$ are estimated by adding the corresponding terms of (21) and (22):

$$\Delta\theta_{0,\min} < \Delta\theta_0 < \Delta\theta_{0,\max}. \tag{23}$$

We note here that maximum absolute base rotation is achieved, as expected, when the arm is fully extended ($q_2 = 0°$), while absolute minimum base rotation occurs when the arm is fully retracted ($q_2 = 180°$). This is because the effect of the manipulator is maximized when its inertia is maximum, and minimized when its inertia is minimum. The bounds presented in this section are valid in the sense that if the desired attitude change is not within these bounds, then no path exists that can connect the initial to the desired (final) configuration.

4.3. Path planning and integral of motion

The path planning method presented in Section 3 is applied. Using (7), (16) and (9), it is reduced to a single integral of motion, which should be satisfied

$$h(\mathbf{b}) \triangleq -\Delta\theta_0^{\text{des}} + \underbrace{\int_{t_{in}}^{t_{\text{fin}}} (g_1\dot{q}_1 + g_2\dot{q}_2)\,\mathrm{d}t}_{I(\mathbf{b})} = 0. \tag{24}$$

Here, the unknown vector $\mathbf{b} \in R^k$, contains at least one parameter, i.e. $k \ge 1$, to satisfy the integral of motion (24). After $\mathbf{b} \in R^k$ is determined, by the methods presented in Section 3, the joint-space trajectories are known, and $\theta_0(t)$ is given by

$$\theta_0(t) = \theta_0^{in} + \int_{t_{in}}^{t} (g_1\dot{q}_1 + g_2\dot{q}_2)\,\mathrm{d}t. \tag{25}$$

All other configuration variables can be calculated using (7) and (25).

5. Examples

In the following examples, the free-floating space manipulator shown in Fig. 2 is employed. The system parameters used are shown in Table 1.

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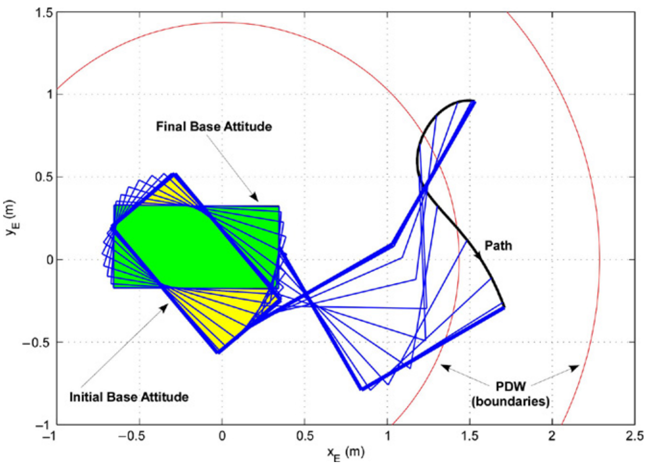

Fig. 3. Snapshots of a free-floater moving to a desired θ_0, x_E, y_E.

For this system, a, b, and c, in (12), are given by the following equations

$$
\begin{aligned}
a &= r_0 m_0 (m_0 + m_1 + m_2)^{-1} = 0.43 \text{ m} \\
b &= (r_1(m_0 + m_1) + l_1 m_0)(m_0 + m_1 + m_2)^{-1} = 0.89 \text{ m} \quad (26) \\
c &= (l_2(m_0 + m_1))(m_0 + m_1 + m_2)^{-1} + r_2 = 0.97 \text{ m}.
\end{aligned}
$$

The reachable workspace is computed to be a disk with an external radius equal to $R_{\max,2} = a + b + c = 2.29$ m. The outer ring of the Path Dependent Workspace (PDW), i.e. the subworkspace in which Dynamic Singularities may occur, is defined by $R_{\min,2} = b + c - a = 1.44$ m and $R_{\max,2}$, [1].

For the examples presented here, the duration of motion is chosen equal to 10 s. Since the constraints are scleronomic, increasing or decreasing this time has no effect on the path taken, but instead increases or decreases the torque requirements and the magnitude of velocities or accelerations.

5.1. New system configuration motion

The free-floater has to move its manipulator endpoint to a new (desired) location and at the same time change its spacecraft attitude to a new (desired) one. The initial system configuration is $(\theta_0, x_E, y_E)^{in} = (-50°, 1.53 \text{ m}, 0.96 \text{ m})$ and $(\theta_0, x_E, y_E)^{\text{fin}} = (0°, 1.71 \text{ m}, -0.29 \text{ m})$, or $(\theta_0, q_1, q_2)^{in} = (-50, 80, 30)°$ and $(\theta_0, q_1, q_2)^{\text{fin}} = (0, -60, 90)°$.

The equivalent change in q_1, q_2 is $(\Delta q_1, \Delta q_2) = (-140, 60)°$ and bounds for base rotation, calculated using (20)–(23), result in bounds for the desired change, given by $\Delta\theta_0 \in (1.4, 72.2)°$. Here, $\Delta\theta_0^{\text{des}} = 50°$, which lies between the permissible bounds, so it is expected that at least one path exists that can connect the given (initial) to the desired (final) configurations. The path planning method presented in Sections 3 and 4.3 is employed here to specify the desired path.

A fifth and a sixth order polynomial of t, is specified for q_1, q_2 respectively, i.e. we assume initially that $k_1 = 5$ and $k_2 = 6$, see (5). Here only one free parameter exists, (b_{26}), to satisfy the integral of motion (24). All the other parameters of $\mathbf{b}_2$ were calculated as functions of b_{26}, using initial and final positions of joints, with zero initial and final velocities and accelerations. Solving the nonlinear Eq. (24) numerically, results to infinite solutions for b_{26}.

Using the optimization formulation defined in (10), the minimum $b_{26} = -0.552 \times 10^{-4}$ is found yielding the shortest path, as shown in Fig. 3. The coefficients of joint trajectories are then computed as,

$$
\begin{aligned}
\mathbf{b}_1 &= [-7\pi/150\,000, 7\pi/6000, -7\pi/900, 0, 0, 4\pi/9] \\
\mathbf{b}_2 &= [b_{26}, (-1\,500\,000 * b_{26} + \pi)/50\,000, \\
&\qquad (600\,000 * b_{26} - \pi)/2000, \\
&\qquad (-300\,000 * b_{26} + \pi)/300, 0, \pi/6].
\end{aligned}
\tag{27}
$$

We can see that the manipulator roughly rotates clockwise, adjusting its inertia appropriately, while the spacecraft rotates in the opposite direction, reaching the desired final attitude.

In Fig. 4, we can see that the desired configuration is reached in the specified time. Also, all trajectories are smooth throughout the motion, and the system starts and stops smoothly at zero velocities, as expected. This is an important characteristic of the method employed and is due to the use of smooth functions, such as polynomials.

The corresponding joint torques are given in Fig. 5. These torques are computed using (1) and the elements of the reduced inertia matrix, given in [1].

As shown in Fig. 5, the required torques are small and smooth while they can be made arbitrarily small, if the duration of the maneuver is increased. The implication of this fact is that joint motors can apply such torques with ease and therefore the resulting configuration maneuver is feasible.

5.2. Change in base attitude and same initial–final manipulator configuration motion

In this example, we want to find a path that changes the base orientation while at the end of the maneuver, the manipulator's configuration is identical to the initial one. Specifically, the initial system configuration is $(\theta_0, \theta_1, \theta_2)^{in} = (0, 0, -20)°$ and the desired (final) one is $(\theta_0, \theta_1, \theta_2)^{\text{fin}} = (90, 90, 70)°$, i.e. $\mathbf{q}^{in} = \mathbf{q}^{\text{fin}} = (0, -20)°$, while $\Delta\theta_0^{\text{des}} = 90°$.

Here, $(\Delta\theta_1, \Delta q_2) = (90, 0)°$. Using the analysis in Section 4.2, the bounds for the attitude are found to be $\Delta\theta_0 \in (-84.0, -7.9)°$. Since $\Delta\theta_0^{\text{des}}$ does not belong in the previous interval, there is no path connecting initial to final configurations, with $\Delta\theta_1 = 90°$. This is reasonable, for a relatively "simple" point-to-point motion (without many small cyclical motions) performed in finite prescribed time because, due to angular momentum conservation, if the manipulator moves counter-clockwise, the base is expected to rotate clockwise, resulting in $\Delta\theta_0 < 0$.

However, a solution still exists, if we let the manipulator rotate clockwise, i.e. setting $(\theta_0, \theta_1, \theta_2)^{\text{fin}} = (90, -270, 70)°$, which represents the same final manipulator configuration as above. Now it is $\mathbf{q}^{\text{fin}} = (-360, 340)°$. Additional freedom is given to the first joint, i.e. a sixth and a fifth order polynomial of t are assigned to q_1, q_2 respectively $(k_1 = 6, k_2 = 5)$, see (5). Consequently, there is only one free parameter (b_{16}),

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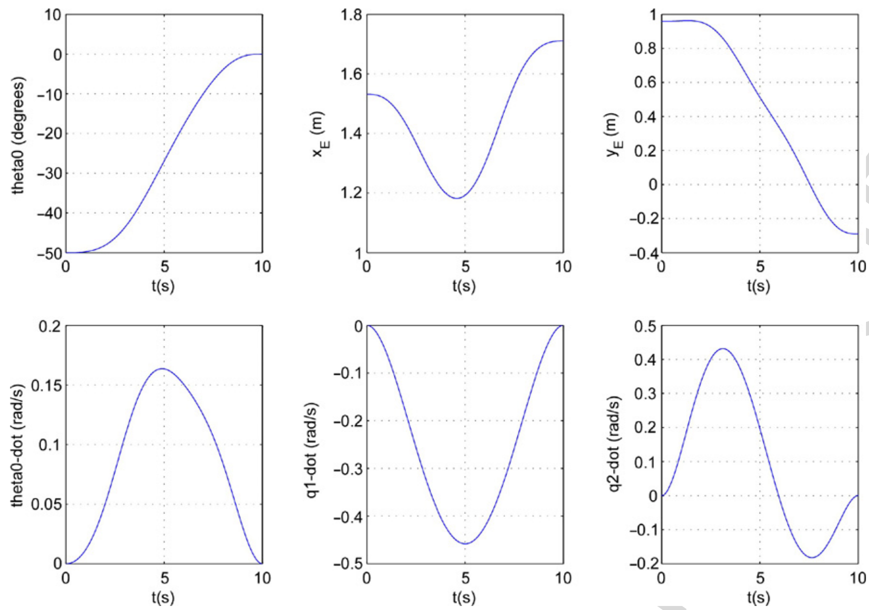

Fig. 4. End-effector coordinates, spacecraft orientation, and orientation and joint angle rate trajectories that correspond to the motion in Fig. 3.

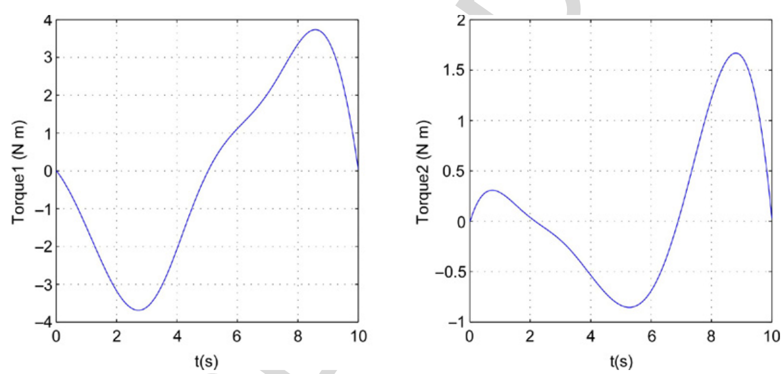

Fig. 5. Manipulator torques required for the motion shown in Fig. 3.

to satisfy the integral of motion (24). All other elements of $\mathbf{b}_1$ are calculated as functions of b_{16}, using initial and final positions of joints, with zero initial and final velocities and accelerations. Solving the nonlinear Eq. (24) numerically, two discrete values for b_{16} are obtained. Using the smallest one, $b_{16} = 0.3362 \times 10^{-4}$, which is identical to that we obtained using optimization (10), the shortest path results, see Fig. 6. The coefficients of the joint trajectories are then computed as,

$$\begin{aligned}\mathbf{b}_1 \;&=\; [b_{16}, -3 * (250\,000 * b_{16} + \pi)/25\,000,\\ &\qquad 3 * (100\,000 * b_{16} + \pi)/1000,\\ &\qquad -(50\,000 * b_{16} + \pi)/50, 0, 0, 0]\\ \mathbf{b}_2 &= [3\pi/25\,000, -3\pi/1000, \pi/50, 0, 0, -\pi/9].\end{aligned} \tag{28}$$

Note that in this case, the free-floating robot configuration is the same at the beginning and end of the motion, while the spacecraft attitude has changed as requested, following a smooth path and at the given time.

Again all system trajectories are smooth, and shown in Fig. 7. The required torques are shown in Fig. 8, and as previously, they can be applied easily.

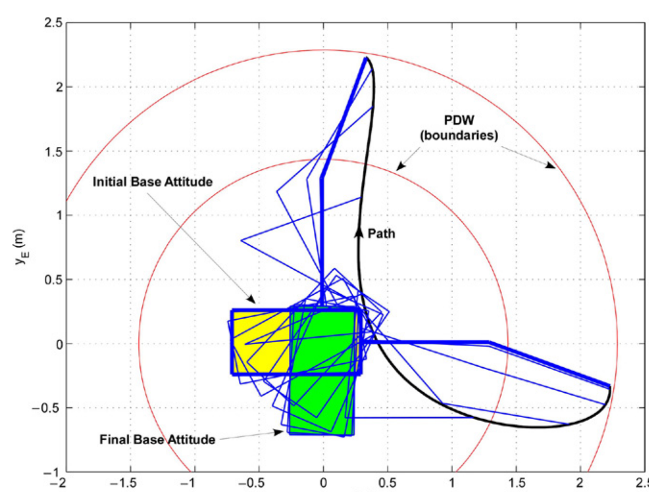

Fig. 6. Snapshots of a free-floater, while changing its base orientation.

5.3. New end-point position motion

In this example, the manipulator end-point is desired to move to a new location, while at the end of the motion the

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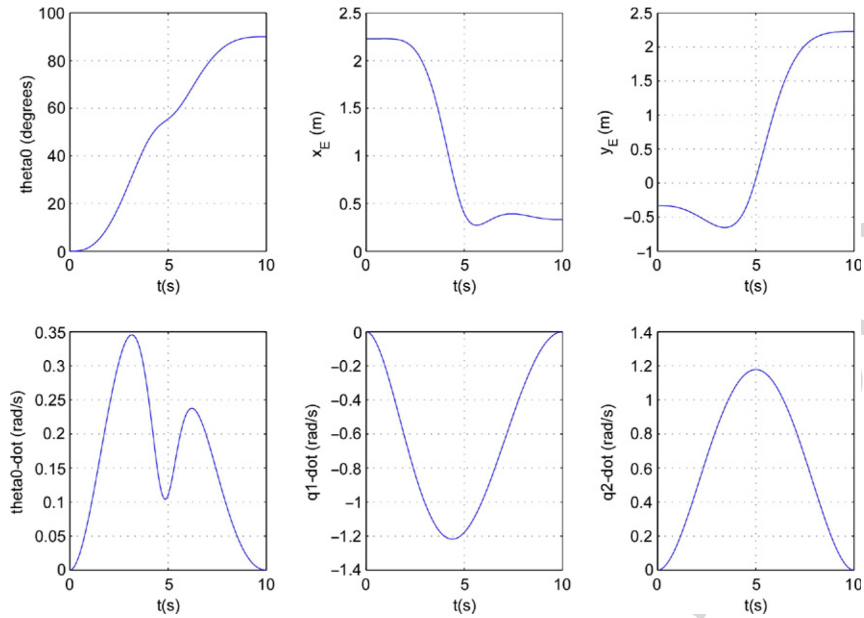

Fig. 7. End-effector coordinates, spacecraft orientation, and orientation and joint angle rate trajectories that correspond to the motion in Fig. 6.

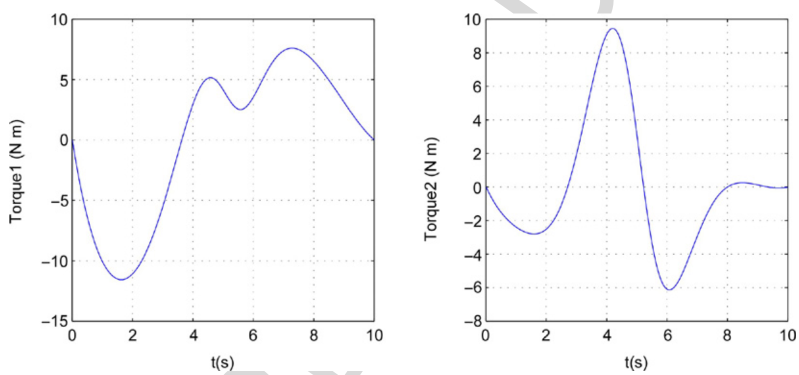

Fig. 8. Manipulator torques required for the motion shown in Fig. 6.

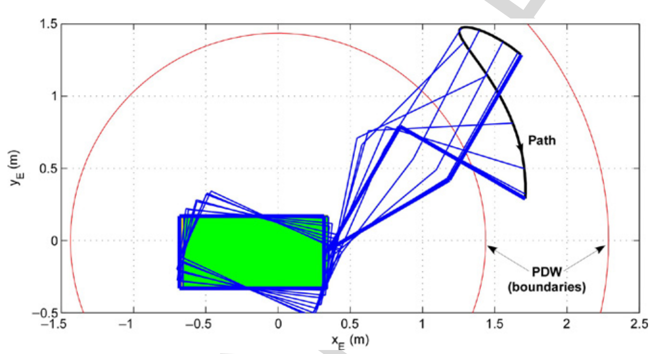

Fig. 9. Snapshots of a free-floater moving to a desired x_E, y_E, with identical initial and final spacecraft attitude.

base must be at its initial attitude, i.e. $\theta_0^{\text{fin}} = \theta_0^{in}$. Fig. 9 shows snapshots of the free-floater motion when it moves from $(\theta_0, \theta_1, \theta_2)^{in} = (0, 30, 60)°$ to $(\theta_0, \theta_1, \theta_2)^{\text{fin}} = (0, 60, -30)°$, or equivalently from $(\theta_0, x_E, y_E)^{in} = (0°, 1.68 \text{ m}, 1.29 \text{ m})$ to $(\theta_0, x_E, y_E)^{\text{fin}} = (0°, 1.71 \text{ m}, 0.29 \text{ m})$. The equivalent change in q_1, q_2 is $(\Delta q_1, \Delta q_2) = (30, -120)°$ and bounds for base rotation, calculated using (20)–(23), are given by $\Delta\theta_0 \in (-23.8, 17.3)°$. Here, $\Delta\theta_0^{\text{des}} = 0°$, which lies between the permissible limits. Therefore, it is expected that there is at least one path connecting the given (initial) with the desired (final) configurations.

Additional freedom is introduced to the first joint, i.e. we assigned a sixth and a fifth order polynomial of t, to q_1, q_2 respectively ($k_1 = 6, k_2 = 5$). Solving the nonlinear Eq. (24) numerically, a unique solution for $b_{16} = -0.622 \times 10^{-4}$ is obtained, and the resulting path is shown in Fig. 9. The coefficients of the joint trajectories are then computed as,

$$\begin{aligned}\mathbf{b}_1 &= [b_{16},\ (-3\,000\,000 * b_{16} + \pi)/100\,000,\\ &\quad (1\,200\,000 * b_{16} - \pi)/4000,\\ &\quad (-600\,000 * b_{16} + \pi)/600, 0, 0, \pi/6]\\ \mathbf{b}_2 &= [-\pi/25\,000,\ \pi/1000,\ -\pi/150, 0, 0, \pi/6].\end{aligned} \tag{29}$$

Smooth resulting trajectories of the system are shown in Fig. 10, with the required torques in Fig. 11.

In general, due to the physics of the problem, it is expected that motion of a $q_i(t)$ closer to the spacecraft base, will result in a greater change in base attitude, since a larger inertia is involved. This has some practical consequences. For example,

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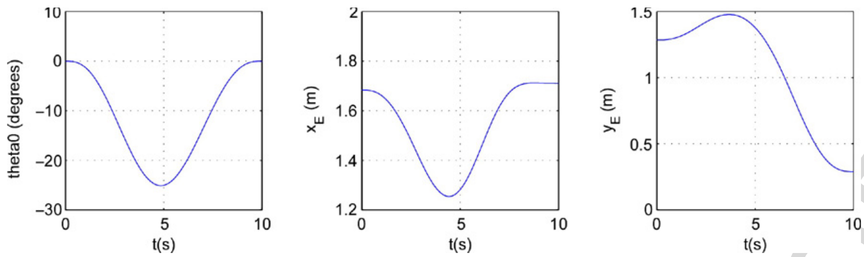

Fig. 10. Trajectories that correspond to snapshots in Fig. 9.

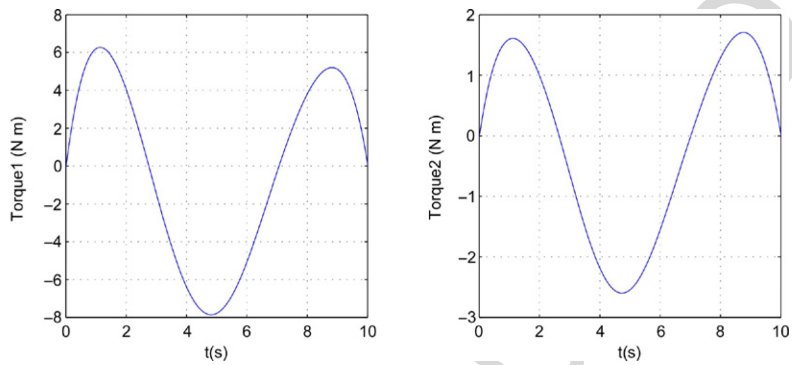

Fig. 11. Manipulator torques required for the motion shown in Fig. 9.

if extra freedom is initially added to $q_2(t)$, i.e. $k_2 \geq 6$, and the minimum path resulting from the optimization is relatively long, perhaps due to joint-2 multiple full rotations, then one can set $k_2 = 5$, and introduce additional freedom to $q_1(t)$, i.e. $k_1 \geq 6$, resulting in a shorter path. Based on this observation, the initial choice of k_i, and the possibility of freedom distribution among the joints, can be easily automated.

6. Conclusions

A point-to-point path planning method, for underactuated space manipulator systems, has been developed and presented in this paper. The method allows for endpoint location and simultaneous control of the spacecraft's attitude, using manipulator actuators only. Building on our previous work, the method uses high order polynomials, to specify the desired path directly in joint-space, and is presented for the general 3D-case, with a N-dof manipulator. The accessibility of final configurations is extended drastically, and free parameters are determined by optimization techniques. The developed method uses smooth functions in the planning scheme, and avoids the need for many small cyclical motions. The system reaches the desired configuration smoothly, in finite and prescribed time. It was found that this approach leads always to a path, provided that the desired change in configuration lies between physically permissible limits. Physical limitations, imposed by system dynamic parameters, are examined. Lower and upper bounds for base rotation, due to manipulator motions, are estimated for the planar case and shown in the implementation section.

Uncited references

[10] and [12].

Acknowledgments

Support by the EPAN Cooperation Program 4.3.6.1.b (Greece-USA 035) of the Hellenic General Secretariat for Research and Technology and the NTUA Senator Committee of Basic Research Programme "Protagoras", R.C. No. 10, is acknowledged. The work of I. Tortopidis has been supported by the Greek State Scholarships Foundation, (IKY).

Appendix A

The coefficients of (11) are given by

$$\begin{aligned} D_0(q_1, q_2) &= d_1 + d_2\cos(q_1) + d_3\cos(q_1 + q_2) \\ D_1(q_1, q_2) &= d_4 + d_5\cos(q_1) + d_6\cos(q_2) \\ D_2(q_1, q_2) &= d_7 + d_8\cos(q_2) + d_9\cos(q_1 + q_2) \end{aligned} \tag{A.1}$$

where $M = m_0 + m_1 + m_2$, the coefficients d_i are given by

$$\begin{aligned} d_1 &= I_0 + m_0(m_1 + m_2)r_0^2/M \\ d_2 &= d_5 = m_0 r_0((m_1 + m_2)l_1 + m_2\, r_1)/M \\ d_3 &= d_9 = m_0 r_0 m_2 l_2/M \\ d_4 &= I_1 + (m_0 m_1 l_1^2 + m_1 m_2 r_1^2 + m_0 m_2(l_1 + r_1)^2)/M \\ d_6 &= d_8 = m_2 l_2(m_0(l_1 + r_1) + m_1 r_1)/M \\ d_7 &= I_2 + m_2(m_0 + m_1)l_2^2/M \end{aligned} \tag{A.2}$$

and all variables in (A.2) are defined in Fig. 2.

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