UNDER-ACTUATED SPACE ROBOTS: PHYSICAL and ALGORITHMIC CONSTRAINTS in SMOOTH MOTION PLANNING

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ABSTRACT
Space manipulator systems having spacecraft actuators turned off, operate in free-floating mode, and exhibit nonholonomic behaviour due to angular momentum conservation. The system is underactuated and a challenging problem is to control both the location of the end effector and the attitude of the base, using manipulator actuators only. A developed path planning method that perform this task is briefly presented and the constraints of motion, both physical and algorithmic, are examined. The physical limitations are imposed by system's dynamic parameters. Lower and upper bounds for base rotation, due to manipulator motions, are estimated. Smooth functions such as polynomials are employed in the planning scheme, leading to smooth configuration changes in finite and prescribed time, and avoiding many small cyclical motions. Algorithmic limitations of the method in reaching arbitrary final configurations are discussed and examples are presented.

KEY WORDS
Space free-floating robots, underactuated, nonholonomic planning.

1. Introduction

Space missions and on-orbit tasks are major areas where space robots have an increasing role. Such robotic systems are desired because these tasks are too risky or very costly (due to safety support systems) and many times just physically impossible to be executed by humans. Space robots consist of an on-orbit spacecraft fitted with one or more robotic manipulators. In free-flying mode, thruster jets can compensate for manipulator induced disturbances but their extensive use limits a system's useful life span. Many times it is desired for the thrusters to be turned off, as for example during capture operations in order to avoid interaction with the target. In this case the system operates in a free–floating mode, dynamic coupling between the manipulator and the spacecraft exists, and manipulator motions induce disturbances to the spacecraft. This mode of operation is feasible when no external forces or torques act on the system and the total momentum of the system is zero.

A free-floating space robot is an under-actuated system and exhibits a nonholonomic behavior due to the nonintegrability of the angular momentum, [1]. This property complicates the planning and control of such systems, which have been studied by a number of researchers. Vafa and Dubowsky have developed a technique called the Virtual Manipulator, [2]. Inspired by astronaut motions, they proposed a planning technique, which employs small cyclical manipulator joint motions to modify spacecraft attitude. Papadopoulos and Dubowsky studied the Dynamic Singularities of free-floating space manipulator systems, which are not found in terrestrial systems and depend on the dynamic properties of the system, [1, 3]. They also showed that any terrestrial control algorithm could be used to control end-point trajectories, despite spacecraft motions [3]. Nakamura and Mukherjee explored Lyapunov techniques to achieve simultaneous control of spacecraft's attitude and its manipulator joints, [4]. To limit the effects of a certain null space, the authors proposed a bidirectional approach, in which two desired paths were planned, one starting from the initial configuration and going forward and the other starting from the final configuration and going backwards. The method is not immune to null space problems and yields non-smooth trajectories that require that the joints come to a stop at the switching point.

In another attempt to plan a space robot's motion, Papadopoulos proposed a method that allowed Cartesian movement of manipulator's end point, avoiding dynamic singularities, [5]. The method involved small end-effector Cartesian cyclical motions designed to change the attitude of the spacecraft to one that was known of avoiding dynamic singularities, [5], [6]. Recently, Franch et al. have employed flatness theory to plan trajectories for free-floating systems. Their method requires selection of robot parameters so that the system is made controllable and linearizable by prolongations, [7].

In this paper, a developed path planning methodology in joint space for planar free-floating space manipulator systems is briefly presented [11]. Physical limitations, due to system's dynamic parameters, are examined and lower and upper bounds for base attitude, caused by manipulator motions, are estimated. The planning scheme avoids multiple cyclical motions and allows simultaneous manipulator end-point and spacecraft's attitude control, using internal motions (manipulator actuators only). The method is based on mapping of the angular momentum to a space that can be satisfied trivially. Smooth functions, such as polynomials are used, and the system is driven to the desired configuration in finite and prescribed time.

2. Free-floating Space Manipulators

Dynamics: A free-floating space manipulator system consists of a spacecraft (base) and a manipulator mounted on it, as shown in Fig. 2. When the system is operating in free-floating mode, the spacecraft's attitude control system is turned off. In this mode, no external forces and torques act on the system, and hence the spacecraft translates and rotates in response to manipulator movements.

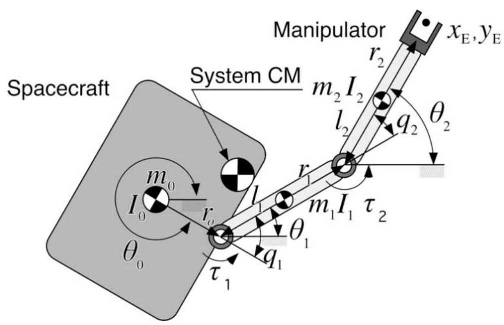

Fig. 2. A Free-Floating Space Manipulator System.

For simplicity, the manipulator is assumed to have revolute joints and an open chain kinematic configuration, so that, in a system with N-degree-of-freedom (dof) manipulator, there will be $N+6$ dof (under-actuated system). Since no external forces act on the system, and the initial momentum is zero, the system Center of Mass (CM) remains fixed in space, and the coordinates origin, O, can be chosen to be the system's CM. The equations of motion for a free-floating system have the form, [3],

$$\mathbf{H}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{C}(\mathbf{q},\dot{\mathbf{q}}) = \boldsymbol{\tau} \tag{1}$$

where $\mathbf{H}(\mathbf{q})$ is a positive definite symmetric matrix, called the reduced system inertia matrix, and $\mathbf{C}(\mathbf{q},\dot{\mathbf{q}})$ contains nonlinear velocity terms. The $N \times 1$ column vectors $\mathbf{q}, \dot{\mathbf{q}}$ and $\boldsymbol{\tau}$ represent manipulator joint angles, velocities, and torques. The base attitude is computed using the conservation of angular momentum, [1],

$${}^{\mathbf{0}}\boldsymbol{\omega}_{\mathbf{0}} = -{}^{\mathbf{0}}\mathbf{D}^{-1}\,{}^{\mathbf{0}}\mathbf{D}_{\mathbf{q}}\dot{\mathbf{q}} \tag{2}$$

where ${}^{\mathbf{0}}\boldsymbol{\omega}_{\mathbf{0}}$ is the base angular velocity in the spacecraft 0^{th} frame, and ${}^{\mathbf{0}}\mathbf{D}$, ${}^{\mathbf{0}}\mathbf{D}_{\mathbf{q}}$ are inertia-type matrices.

For simplicity, we focus on a free-floating robotic system consisting of a two-dof manipulator mounted on a three-dof spacecraft. The spacecraft is constrained to move in the plane perpendicular to the axis of the manipulator rotation. For the planar free-floating space manipulator, (2) is written as,

$$D_0\dot{\theta}_0 + D_1\dot{\theta}_1 + D_2\dot{\theta}_2 = 0 \tag{3}$$

where θ_0, θ_1, θ_2 are spacecraft attitude and manipulator absolute joint angles, and q_1, q_2, are manipulator joint angles. The D_0, D_1 and D_2 are functions of system inertial parameters and of q_1, and q_2, see Fig. 2. The angular momentum, given by (3), cannot be integrated to analytically yield the spacecraft orientation θ_0 as a function of the system's configuration. However, if the joint angle trajectories are known as a function of time, then (3) can be integrated numerically to yield the trajectory for the base orientation. This nonintegrability property introduces nonholonomic characteristics to free-floating systems, and results from the dynamic structure of the system; it is not due to kinematics, as is the case of nonholonomic constraints in mobile manipulators.

Nonholonomic Path Planning: In this section we briefly describe a method leading to a path which connects initial configuration $(\theta_0^{in}, \theta_1^{in}, \theta_2^{in})$ to a final $(\theta_0^{fin}, \theta_1^{fin}, \theta_2^{fin})$ one, by actuating manipulator joints only (for a free-floating space manipulator system shown in Fig. 2.). Note that if this is possible, then one can also design a path to change both the attitude and the manipulator endpoint to desired values, since the endpoint location x_E and y_E is given by, [1],

$$\begin{aligned} x_E &= a\cos\theta_0 + b\cos\theta_1 + c\cos\theta_2 \\ y_E &= a\sin\theta_0 + b\sin\theta_1 + c\sin\theta_2 \end{aligned} \tag{4}$$

where, a, b, c are constant terms, functions of the mass properties of the system, [1]. It is well known that this problem is not trivial, since one must satisfy the nonholonomic constraint and achieve a change in a three-dimensional configuration space with only two controls.

Using (3), the scleronomic constraint can be written in the Pfaffian form,

$$D_0(\theta_0,\theta_1,\theta_2)\,d\theta_0 + D_1(\theta_0,\theta_1,\theta_2)\,d\theta_1 + D_2(\theta_0,\theta_1,\theta_2)\,d\theta_2 = 0 \tag{5}$$

Note that (5) contains three differentials. Planning is facilitated if this form is transformed to one containing two differentials. This is in principle possible and requires finding appropriate functions *u, v, w* of $\theta_0, \theta_1, \theta_2$, for which (3) can be transformed to [8-10],

$$du + v\,dw = 0 \tag{6}$$

Then planning is done as follows. If we choose functions $w = f(t)$, $u = g(w)$ and set,

$$v = -g'(w) = -(du / dw) \tag{7}$$

then (6) is satisfied identically. Therefore, the planning problem reduces to choosing functions f and g such that they satisfy the values of *u, v,* and *w* at the initial and final time. Then the trajectories for $\theta_0, \theta_1, \theta_2$ (or θ_0, x_E, y_E) are found using the inverse transformation from *u, v, w* to $\theta_0, \theta_1, \theta_2$. Functions f and g can be polynomials, splines, or any other smooth functions. One possibility is to choose *f* as a fifth order polynomial, so that the system initial and final configuration, velocity and acceleration can be specified, and g as a third order polynomial, so that initial and final system configurations can be specified . The method of finding the proper functions *u, v, w* is analytically presented in [9]. Note that although more than one transformations can be found, not all of them are equivalent in terms of complexity.

Next, this methodology is applied to a free-floating space manipulator system in which the manipulator is mounted on the spacecraft's CM. A forward transformation for the case where the manipulator is not mounted at the spacecraft CM is available but is very complex. Mounting the manipulator at the spacecraft CM simplifies the resulting transformation and eliminates Dynamic Singularities from the workspace, [1, 3]. It is easy to show that the angular momentum in this case remains non-integrable and therefore the system still exhibits nonholonomic characteristics. The coefficients of the nonholonomic constraint (5) and the transformation used, are given in Appendix A.

3. Constraints of Motion

Physical Constraints: It is expected that for a given change in configuration variables, a limited change in base attitude would occur, due to dynamic system's properties. In other words not all configurations are reachable from an initial one. For example, it is not rational to expect large base rotations with a small point-to-point manipulator motion. Similarly, it may not be possible to move to any Cartesian point and have the final spacecraft attitude unchanged.

Here we will try to specify lower and upper bounds for base rotation $\Delta\theta_0$, caused by $(\Delta\theta_1, \Delta q_2)$. Using the Pfaffian form (5), we can write

$$d\theta_0 = -(D_1 / D_0)d\theta_1 - (D_2 / D_0)d\theta_2 \qquad (8)$$

Since $\theta_2 = \theta_1 + q_2$, it is $d\theta_2 = d\theta_1 + dq_2$, and

$$\cos(\theta_1 - \theta_2) = \cos(q_2) \triangleq c_2 \qquad (9)$$

so (8) can be written as

$$d\theta_0 = -\frac{(D_1 + D_2)}{D_0}d\theta_1 - \frac{D_2}{D_0}dq_2 \triangleq f_1(c_2)d\theta_1 + f_2(c_2)dq_2 \qquad (10)$$

see (A.1).

Integrating (10) we have

$$\Delta\theta_0 = \int_{\theta_1^{in}}^{\theta_1^{fin}} f_1(c_2)d\theta_1 + \int_{q_2^{in}}^{q_2^{fin}} f_2(c_2)dq_2 \triangleq \Delta\theta_{01} + \Delta\theta_{02} \qquad (11)$$

where $\Delta\theta_{01}, \Delta\theta_{02}$, represent the contribution of $\Delta\theta_1$ and Δq_2 in $\Delta\theta_0$ respectively.

The function $f_1(c_2)$, using (10) and (A.1), is given by

$$f_1(c_2) = -(\alpha_1 + \alpha_2 + 2\alpha_3 c_2) / \alpha_0 < 0, \quad \forall c_2 \qquad (12)$$

and is always negative, since $\alpha_i > 0$ and $\alpha_1 + \alpha_2 \pm 2\alpha_3 > 0$ (for system's parameters used and shown in examples section). Bounds for f_1, are given by

$$f_{1,\min} \le f_1(c_2) \le f_{1,\max} < 0 \qquad (13)$$

where

$$\begin{aligned} f_{1,\min} &= -(\alpha_1 + \alpha_2 + 2\alpha_3) / \alpha_0, \quad (q_2 = 0^o) \\ f_{1,\max} &= -(\alpha_1 + \alpha_2 - 2\alpha_3) / \alpha_0, \quad (q_2 = 180^o) \end{aligned} \qquad (14)$$

The minimum value for f_1 (absolute maximum), occurs when the arm is fully extended ($q_2 = 0^o$ or $c_2 = 1$). Similarly $f_{1,\max}$ (absolute minimum) occurs when (or $c_2 = -1$). The corresponding bounds for $\Delta\theta_{01}$ are estimated by (11), (13) and (14) as

$$\begin{cases} f_{1,\min}\Delta\theta_1 \le \Delta\theta_{01} \le f_{1,\max}\Delta\theta_1 < 0, & (\Delta\theta_1 > 0) \\ f_{1,\max}\Delta\theta_1 \le \Delta\theta_{01} \le f_{1,\min}\Delta\theta_1, & (\Delta\theta_1 < 0) \end{cases} \qquad (15)$$

Function $f_2(c_2)$ is given by

$$f_2(c_2) = -(\alpha_2 + \alpha_3 c_2) / \alpha_0 \qquad (16)$$

Since $(\alpha_2 - \alpha_3) < 0$, two bounds for f_2 are

$$f_{2,\min} \le f_2(c_2) \le f_{2,\max} \qquad (17)$$

where

$$\begin{aligned} f_{2,\min} &= -(\alpha_2 + \alpha_3) / \alpha_0 < 0, \quad (q_2 = 0^o) \\ f_{2,\max} &= -(\alpha_2 - \alpha_3) / \alpha_0 > 0, \quad (q_2 = 180^o) \end{aligned} \qquad (18)$$

The corresponding bounds for $\Delta\theta_{02}$ are estimated using (11), (17) and (18), and are given by

$$\begin{cases} f_{2,\min}\Delta q_2 \le \Delta\theta_{02} \le f_{2,\max}\Delta q_2, & (\Delta q_2 > 0) \\ f_{2,\max}\Delta q_2 \le \Delta\theta_{02} \le f_{2,\min}\Delta q_2, & (\Delta q_2 < 0) \end{cases} \qquad (19)$$

We note here, that slightly tighter limits for $\Delta\theta_{02}$, could be evaluated using initial and final values of q_2 rather than $0^o / 180^o$. So if $c_{2i} = \cos(q_2^{in})$ and $c_{2f} = \cos(q_2^{fin})$, we could use the following values for $f_{2,\min}$, $f_{2,\max}$ to bound $\Delta\theta_{02}$:

$$f_{2,\min} = f_2(c_{2,\max}), c_{2,\max} = \begin{cases} 1, \ \textit{if } q_2 \textit{ crosses } 0^o \ (\textit{during motion}) \\ \max\{c_{2i}, c_{2f}\}, \ \textit{in any other case} \end{cases} \quad (20)$$

$$f_{2,\max} = f_2(c_{2,\min}), c_{2,\min} = \begin{cases} -1, \ \textit{if } q_2 \textit{ crosses } 180^o \ (\textit{during motion}) \\ \min\{c_{2i}, c_{2f}\}, \ \textit{in any other case} \end{cases} \quad (21)$$

Finally, bounds for $\Delta\theta_0$ are estimated by adding the corresponding terms of (15) and (19):

$$\Delta\theta_{0,\min} < \Delta\theta_0 < \Delta\theta_{0,\max} \quad (22)$$

In any case, the bounds presented earlier are valid in the sense that there is no path connecting a given initial with a desired (final) configuration, leading to $\Delta\theta_0$ out of the limits given above.

Algorithmic restrictions: The inverse transformation given by (A.4) is defined if and only if

$$-1 \leq \frac{v - \alpha_1}{2\alpha_3} \leq 1 \quad (23)$$

To satisfy (23), we add more freedom by increasing the order of polynomial $u(w)$ by one and by assuming that the final spacecraft orientation θ_0^{fin} is free [11]:

$$u(w) = b_4 w^4 + b_3 w^3 + b_2 w^2 + b_1 w + b_0 \quad (24)$$

Because of (7), $v(w)$ is

$$v(w) = -4 b_4 w^3 - 3 b_3 w^2 - 2 b_2 w - b_1 \quad (25)$$

Using the initial and final system conditions, b_i, $i = 0, \ldots, 3$ are found as functions of b_4 and θ_0^{fin}. The problem reduces to finding a range of values for b_4 and θ_0^{fin} which lead to paths that satisfy (23) for all $w \in [w_{in}, w_{fin}]$. Obviously, some limitations in the reachable configurations result because of this reason. For example, it is expected that it will not be always possible to achieve extreme values for $\Delta\theta_0$ by a single point to point motion, mainly due to the order of the polynomial u(w). In any case, the method can still be used, leading to maximum changes in base rotation, if the motion is implemented in three steps: (i) the arm is fully extended / retracted $(q_2 : q_2^{in} \to 0^o \,/\, 180^o)$ having $\theta_1 \equiv \theta_1^{in}$, (ii) next θ_1 changes to θ_1^{fin} with $q_2 \equiv 0^o \,/\, 180^o$ and finally (iii) the arm is retracted / extended to q_2^{fin} having $\theta_1 \equiv \theta_1^{fin}$. Applications of the planning methodology are illustrated in the following section.

4. Application Examples

To illustrate the methodology described above, the free-floating space manipulator shown in Fig. 2 is employed. The system parameters used are shown in Table 1.

Table 1. System Parameters

Body	l_i $[m]$	r_i $[m]$	m_i $[kg]$	I_i $[kgm^2]$
0	1.0	0.0	400.0	66.67
1	0.5	0.5	40.0	3.33
2	0.5	0.5	30.0	2.50

For this system, a, b, and c, in (4), are given by:

$$a = r_0 m_0 (m_0 + m_1 + m_2)^{-1} = 0$$

$$b = (r_1(m_0 + m_1) + l_1 m_0)(m_0 + m_1 + m_2)^{-1} = 0.89m$$

$$c = (l_2(m_0 + m_1))(m_0 + m_1 + m_2)^{-1} + r_2 = 0.97m$$

For the examples presented here, the duration of motion is chosen equal to 10 *s*. Increasing or decreasing this time has no effect on the path taken, but increases or decreases the torque requirements and the magnitude of velocities or accelerations. For this system, the reachable workspace is computed to be a hollow disk with an external radius equal to R_{max} = 1.86 *m* and an internal one R_{min} = 0.08 *m*. All workspace points on this hollow disk are, in principle, accessible by the end-effector.

A. New Configuration: The free-floater has to move its manipulator endpoint to a new location and at the same time change its spacecraft attitude to a desired one. Only manipulator actuators are to be used. The initial system configuration is $(\theta_0, x_E, y_E)^{in} = (-50^o, 1.26m, 1.28m)$ and the final $(\theta_0, x_E, y_E)^{fin} = (0^o, 1.28m, -0.29m)$. In this case we have $(\Delta\theta_1, \Delta q_2) = (-90.0, 60.0)^o$ and the corresponding bounds for base rotation (calculated using (14), (15), (19), (20) and (21)) are $\Delta\theta_0 \in (8.9, 91.8)^o$. In this example $\Delta\theta_0^{des} = 50^o$ which lies within the permissible range. The path planning method is applied, which leads to a desired path for $b_4 = -15$. Fig. 3 depicts snapshots of the free-floater motion as it changes its configuration. The manipulator roughly rotates counter-clockwise, adjusting its inertia appropriately, while the spacecraft rotates in the opposite direction, reaching the desired final attitude.

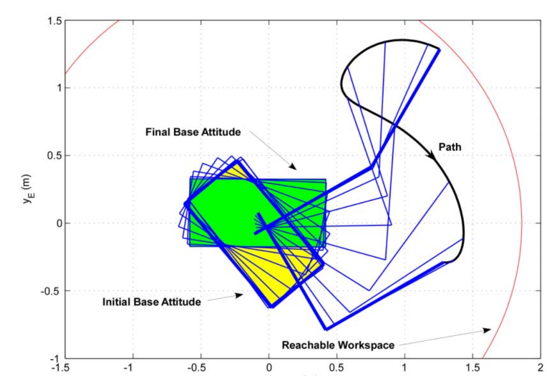

Fig. 3. Snapshots of a free-floater moving to a desired θ_0, θ_1, θ_2.

In Fig. 4, is shown that the desired configuration is reached in the specified time.

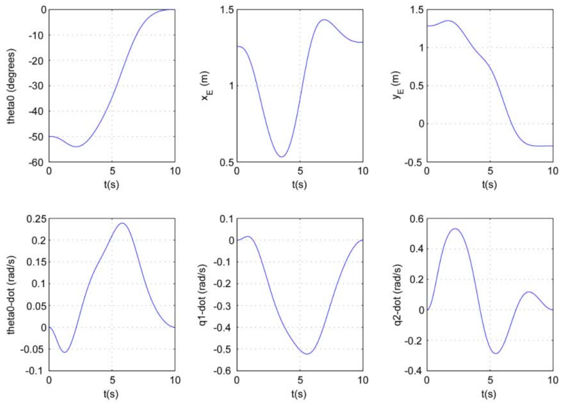

Fig. 4. Configuration variables and spacecraft orientation and joint angles rate trajectories that correspond to snapshots in Fig. 3.

Also, all trajectories are smooth throughout the motion, and the system starts and stops smoothly at zero velocities, as predicted. This is an important characteristic of the method employed and is due to the use of smooth functions, such as polynomials.

The corresponding joint torques are given in Fig. 5. These torques are computed using (1) and the elements of the reduced inertia matrix, given in [1]. As shown in Fig. 5, the required torques are small and smooth while they can be made arbitrarily small, if the duration of the maneuver is increased. The implication of this fact is that joint motors can apply such torques with ease and therefore the resulting configuration maneuver is feasible.

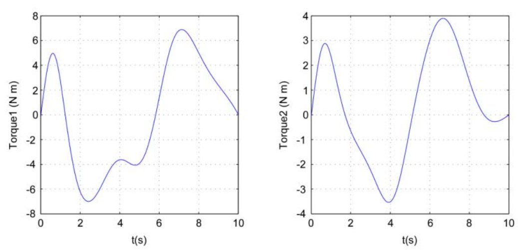

Fig. 5. Manipulator torques required for the motion shown in Fig. 3.

B. New End-Point Position: In this example, the manipulator end-point is desired to move to a new location, while at the end of the motion the base must be at its initial attitude, i.e. $\theta_0^{fin} = \theta_0^{in}$. Fig. 6 shows snapshots of the free-floater motion when it moves from $(\theta_0, x_E, y_E)^{in} = (0^o, 1.32m, 1.31m)$ to $(\theta_0, x_E, y_E)^{fin} = (0^o, 0.16m, -0.09m)$. The equivalent change in θ_1, q_2 is $(\Delta\theta_1, \Delta q_2) = (30.0, -160.0)^o$ and calculated bounds for base rotation are given by $\Delta\theta_0 \in (-42.3, 45.2)^o$. Here, $\Delta\theta_0^{des} = 0^o$, and by choosing $b_4 = 10$, the resulting path leads to the desired final configuration. Smooth trajectories for output variables are shown in Fig. 7.

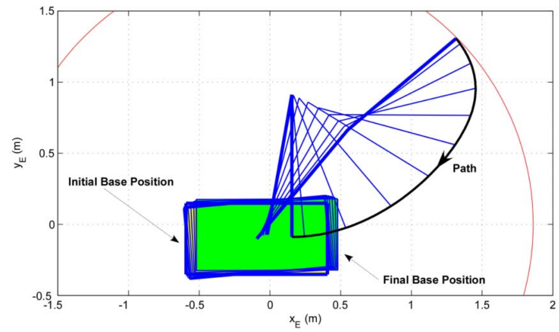

Fig. 6. Snapshots of a free-floater moving to a desired x_E, y_E.

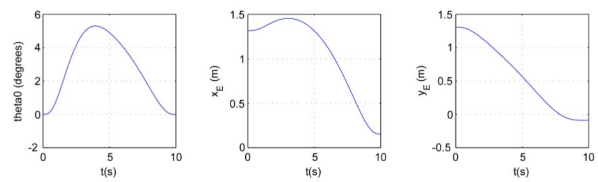

Fig. 7. Configuration variable trajectories that correspond to Fig. 6.

C. Arm fully extended, during the whole motion: This extreme case, leads to the maximum possible $|\Delta\theta_0|$. The method can be applied for this special case: $q_2 \equiv 0^o \Leftrightarrow v(w) \equiv \alpha_1 + 2\alpha_3 = v_{\max}$. The system is to move from $(\theta_0, x_E, y_E)^{in} = (0^o, 1.86m, 0m)$ to $(\theta_0, x_E, y_E)^{fin} = (-50.18^o, 1.32m, 1.32m)$. Here $(\Delta\theta_1, \Delta q_2) = (45.0, 0.0)^o$ which leads to $\Delta\theta_0 \in (-50.2, -14.0)^o$. Using $b_4 = 0$, the method gives the unique path that leads to the desired (maximum) change in base attitude. Fig. 8 depicts motion and Fig. 9 the configuration variables.

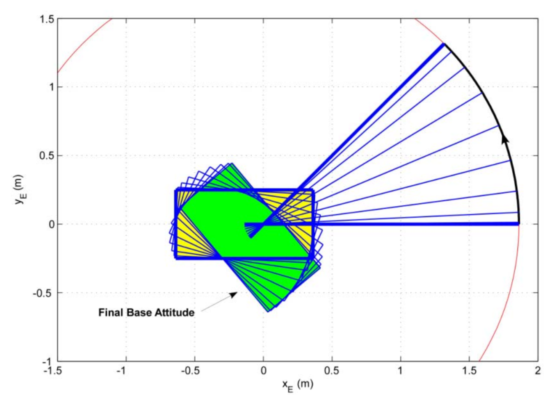

Fig. 8. Snapshots of a free-floater moving to a desired x_E, y_E.

The presented path planning methodology can also be used for the extreme case where $q_2 \equiv 180^o$ during the whole motion.

In any case, the method can be used to tackle many important points that either were not possible before, or required a great number of small cyclical manipulator joint or endpoint motions.

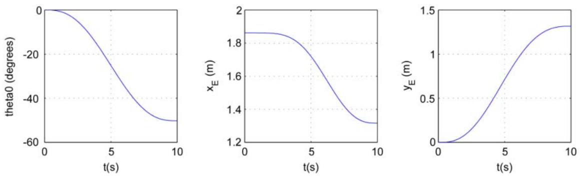

Fig. 9. Configuration variable trajectories that correspond to Fig. 8.

5. Conclusions

The constraints during the motion of a free–floating space manipulator system were examined. Physical constraints are due to the dynamic parameters of the system. Lower and upper bounds for attitude changes caused by manipulator motions, were estimated. A developed path planning method was briefly presented, which allows endpoint Cartesian location control, and simultaneous control of the spacecraft's attitude. The method is based on mapping the angular momentum conservation equation to a space where it can be satisfied trivially. Smooth functions such as polynomials are employed, driving the system to a desired configuration in finite and prescribed time. The methodology avoids previous solutions that required a large number of small joint space or Cartesian space motions, and therefore were not practical. Limitations on reaching arbitrary final configurations were discussed and illustrative examples were presented.

6. Acknowledgements

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Appendix A

The coefficients of (5) are given by

$$
\begin{aligned}
D_0(\theta_0,\theta_1,\theta_2) &= \alpha_0 \\
D_1(\theta_0,\theta_1,\theta_2) &= \alpha_1 + \alpha_3\cos(\theta_1 - \theta_2) = \alpha_1 + \alpha_3 c_2 \\
D_2(\theta_0,\theta_1,\theta_2) &= \alpha_2 + \alpha_3\cos(\theta_1 - \theta_2) = \alpha_2 + \alpha_3 c_2
\end{aligned}
\tag{A.1}
$$

$M = m_0 + m_1 + m_2$, the coefficients α_i are given by

$$
\begin{aligned}
\alpha_0 &= I_0 \\
\alpha_1 &= I_1 + (m_0 m_1 l_1^2 + m_1 m_2 r_1^2 + m_0 m_2 (l_1 + r_1)^2)/M \\
\alpha_2 &= I_2 + m_2 (m_0 + m_1) l_2^2 / M \\
\alpha_3 &= (m_1 m_2 r_1 l_2 + m_0 m_2 l_2 (l_1 + r_1))/M
\end{aligned}
\tag{A.2}
$$

and all variables in (A.2) are defined in Fig. 2.
Following the procedure in [9], the following transformation is chosen among the possible ones:

$$
\begin{aligned}
u(\theta_0,\theta_1,\theta_2) &= \alpha_0 \cdot \theta_0 + \alpha_2 \cdot \theta_2 - \alpha_3 \cdot \sin(\theta_1 - \theta_2) \\
v(\theta_0,\theta_1,\theta_2) &= \alpha_1 + 2 \cdot \alpha_3 \cdot \cos(\theta_1 - \theta_2) \\
w(\theta_0,\theta_1,\theta_2) &= \theta_1
\end{aligned}
\tag{A.3}
$$

The inverse transformation from u, v, w to $\theta_0, \theta_1, \theta_2$ is

$$
\begin{aligned}
\theta_0 &= \frac{1}{\alpha_0}\left[u - \alpha_2 w - \alpha_2 \cos^{-1}\left(\frac{v-\alpha_1}{2\alpha_3}\right) - \alpha_3\sqrt{1-\left(\frac{v-\alpha_1}{2\alpha_3}\right)^2}\right] \\
\theta_1 &= w \\
\theta_2 &= w + \cos^{-1}\left(\frac{v-\alpha_1}{2\alpha_3}\right)
\end{aligned}
\tag{A.4}
$$

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