On the Kinematics of Multiple Manipulator Space Free-Flyers and Their Computation

S. Ali A. Moosavian
Department of Mechanical Engineering
K.N. Toosi University of Technology
Tehran, Iran

Evangelos Papadopoulos*
Department of Mechanical Engineering
National Technical University
15773 Zografou, Athens, Greece
e-mail: egpapado@central.ntua.gr

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In this article, two basic approaches for kinematics modelling of multiple manipulator space free-flying robots (SFFRs) are developed. In the barycentric vector approach, the center of mass of the whole system is taken as a representative point for the translational motion of the system, and a set of body-fixed vectors which reflect both geometric configuration and mass distribution of the system are used. On the other hand, the direct path method relies on taking a point on the base body (preferably its center of mass) as the representative point for the translational motion of the system. The consequences of using each of the two approaches in deriving dynamics equations and in control design of SFFRs are discussed. It is revealed that the direct path method is a more appropriate approach for modelling multiple arm systems, in the presence of external forces/torques (i.e., free-flying mode). A 14 degree-of-freedom space free-flying system is considered as a benchmark system and a quantitative comparison between the two approaches is presented. The results show that the direct path method requires significantly less computations for position and velocity analyses.

*To whom all correspondence should be addressed.

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今回の研究では、複数マニピュレータSpace Free-Flying Robots (SFFRs ;空間自由飛行ロボット)の運動モデルを作成するための2つの基本的な手法を開発した。バリセントリック・ベクタ法において、全システムの質量の中心は、システムの並進動作を表す点として使われ、またボディー固定ベクタの集合は、システムの幾何学的配置と質量分布を反映するのに使われる。一方、直接経路法では、基礎ボディー上の点(質量の中心が望ましい)を、システムの並進動作を表す点として使う。そして、各々の手法を使った力学方程式の導出とSFFRsの制御設計について論じる。これによって、外部からの力とトルクが存在する場合(例えば自由飛行モード)、直接経路法の方が、複数アーム・システムのモデリングに適していることがわかる。なお、14 d. o. f.の空間自由飛行システムをベンチマーク試験システムとして、これら2つの手法の定量的な比較について説明する。その結果は、直接経路法では、位置と速度の解析に必要な計算が極めて少なくて済むことを示している。

I. INTRODUCTION

Unlike fixed-based manipulators, the spacecraft (base) of a space free-flying robot (SFFR) can respond to dynamic reaction forces due to manipulator motions. Hence, in order to control such a system, it is essential to consider the dynamic coupling between the manipulators and the base. To this end, one should first derive a proper kinematics model for the system. Early studies focused on space robotic systems with a single manipulator, while the more complex case of multiple manipulators has received some attention recently.

The kinematics of a single manipulator SFFR was described using the *virtual manipulator approach*.[1] Under the assumption of no external forces acting on the system, the system center of mass is fixed in inertial space, enabling the description of the kinematics of a free-floating system by a virtual kinematic chain with a fixed base. This approach has been employed in path planning of space manipulators aiming at minimization of spacecraft attitude disturbances.[2] The *generalized Jacobian matrix* for a free-floating system was presented to reflect both momentum conservation laws and kinematic relations.[3,4] Assuming that no external forces are applied on a rigid robotic system with revolute joints, a generalized Jacobian matrix was derived and employed for control purposes. The proposed generalized Jacobian matrix converges to the conventional Jacobian, when the base body is relatively massive. This Jacobian has been derived for multiple arm systems, and employed in minimizing attitude disturbances due to manipulator motions.[5] The efficient computation of the generalized Jacobian matrix for control purposes, in the case of multiple arm space robots, has been presented.[6] The *barycentric vector approach* was employed to study kinematics and dynamics of a single arm SFFR in free-floating mode.[7,8] Taking the center of mass of the whole system as a representative point for the translational motion, and using barycentric vectors which reflect both geometric configuration and system mass distribution, results in decoupling the total linear and angular momentum equations from the rest of the dynamic equations, and therefore in a system equation reduction. This approach was also applied to obtain the dynamics and to control a multiple arm SFFR in free-flying mode.[9,10]

Most studies discuss the kinematics of space free-flyers in brief, and as a prelude of the dynamics and control. This article focuses on the kinematics of systems of multiple manipulators mounted on a spacecraft, and studies the consequences of using different kinematics approaches on deriving dynamics equations and on system control. To this end, two fundamental approaches for the kinematics modelling of a rigid multi-arm space robotic system are developed, and the results obtained are compared. In section II, free-flyer kinematics are developed using a minimum set of body-fixed *barycentric vectors*. Position analysis based on the definition of these vectors, and velocity analysis leads to a derivation of a Jacobian matrix of the system. In section III, free-flyer kinematics are developed based on the *direct path method*,[11] using a set of body-fixed vectors. Comparisons of the developed approaches allow the conclusion that the direct path method results in equations with simpler terms, and requires significantly less computations for position and velocity analyses. Therefore, it emerges as a more appropriate approach for modelling multiple arm systems, especially in the presence of external forces.

II. THE BARYCENTRIC VECTOR APPROACH (BVA)

In this section, the kinematics of a rigid multiple arm free-flying space robotic system is developed, using a minimum set of body-fixed *barycentric vectors*. The motion of the system center of mass (CM) is used to describe system translation with respect to an inertial frame of reference, **XYZ**. The body 0 in Figure 1, represents the spacecraft of the free-flyer, which is connected to n manipulators or appendages, each with N_m links. Manipulator joints are revolute with a single degree-of-freedom (DOF).

The joint angles and rates are represented by $K \times 1$ column vectors $\boldsymbol{\theta} = (\boldsymbol{\theta}^{(1)^T}, \boldsymbol{\theta}^{(2)^T}, \ldots, \boldsymbol{\theta}^{(n)^T})^T$, and $\dot{\boldsymbol{\theta}} = (\dot{\boldsymbol{\theta}}^{(1)^T}, \dot{\boldsymbol{\theta}}^{(2)^T}, \ldots, \dot{\boldsymbol{\theta}}^{(n)^T})^T$, where $\boldsymbol{\theta}^{(m)}$ is an $N_m \times 1$ column vector which contains the joint angles of mth manipulator, and $K = \sum_{m=1}^{n} N_m$. The total number of DOF of the system, including the spacecraft, is $N = K + 6$.

The inertial position of an arbitrary point P, $\mathbf{R}_P$, can be written as

$$\mathbf{R}_P = \mathbf{R}_{\mathrm{CM}} + \boldsymbol{\rho}_P \tag{1}$$

and

$$\boldsymbol{\rho}_P = \boldsymbol{\rho}_{C_i} + \mathbf{r}_{p/C_i} \tag{2}$$

where $\boldsymbol{\rho}_P$ is the position vector of P with respect to the system CM, $\mathbf{R}_{\mathrm{CM}}$ is the inertial position of the system CM, C_i is the CM of the ith body, $\boldsymbol{\rho}_{C_i}$ is its position vector with respect to the system CM, and $\mathbf{r}_{p/C_i}$ is the position vector of P with respect to C_i. Next, $\boldsymbol{\rho}_{C_i}$ can be computed and expressed in terms of barycentric vectors. Note that for simplicity, additional subscripts and superscripts are not added in the above equations. When a more precise specification is required, subscript "0" is used for the base, a right superscript (e.g., "m") to indicate a specific manipulator, and a right subscript (e.g., "i") is used to indicate a specific body of that manipulator.

Definition of Barycentric Vectors: Vectors $\boldsymbol{\rho}_{C_i}$ in Eq. (2), are position vectors of the CM of the ith

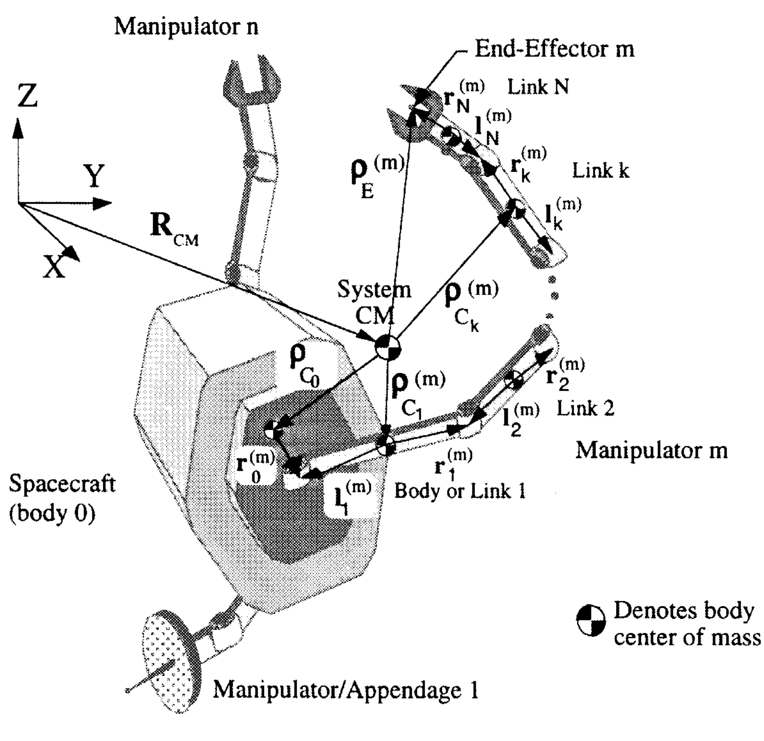

Figure 1. Developing barycentric vector approach for a free-flying space robotic system with n manipulators.

body with respect to the system CM. These vectors can be computed based on the definition of the CM location,

$$m_0 \boldsymbol{\rho}_{C_0} + \sum_{m=1}^{n} \sum_{i=1}^{N_m} m_i^{(m)} \boldsymbol{\rho}_{C_i}^{(m)} = \mathbf{0} \tag{3}$$

and using the following geometrical relationships,

$$\begin{aligned}
&\boldsymbol{\rho}_{C_i}^{(1)} - \boldsymbol{\rho}_{C_{i-1}}^{(1)} = \mathbf{r}_{i-1}^{(1)} - \mathbf{l}_i^{(1)} \quad i = 1, \ldots, N_1 \\
&\vdots \\
&\boldsymbol{\rho}_{C_i}^{(m)} - \boldsymbol{\rho}_{C_{i-1}}^{(m)} = \mathbf{r}_{i-1}^{(m)} - \mathbf{l}_i^{(m)} \quad i = 1, \ldots, N_m \\
&\vdots \\
&\boldsymbol{\rho}_{C_i}^{(n)} - \boldsymbol{\rho}_{C_{i-1}}^{(n)} = \mathbf{r}_{i-1}^{(n)} - \mathbf{l}_i^{(n)} \quad i = 1, \ldots, N_n
\end{aligned} \tag{4}$$

Equations (3) and (4) represent a system of $K+1$ vector equations with $K+1$ unknowns ($\boldsymbol{\rho}_{C_i}$) which can be solved to yield

$$\boldsymbol{\rho}_{C_0} = \tilde{\mathbf{e}}_0 + \sum_{m=1}^{n} \sum_{k=1}^{N_m} \tilde{\mathbf{l}}_k^{(m)} \tag{5a}$$

$$\boldsymbol{\rho}_{C_i}^{(m)} = \tilde{\mathbf{r}}_0^{(m)} + \sum_{\substack{j=1 \\ j \neq m}}^{n} \sum_{k=1}^{N_j} \tilde{\mathbf{l}}_k^{(j)} + \sum_{k=1}^{N_m} \tilde{\mathbf{v}}_{ki}^{(m)} \quad \begin{cases} m = 1, \ldots, n \\ i = 1, \ldots, N_m \end{cases} \tag{5b}$$

where $(\tilde{\bullet})$ denotes body-fixed *barycentric vectors* defined as

$$\tilde{\mathbf{v}}_{ki}^{(m)} = \begin{cases} \tilde{\mathbf{r}}_k^{(m)} = \mathbf{r}_k^{(m)} - \mathbf{e}_k^{(m)} & k < i \\ \tilde{\mathbf{e}}_k^{(m)} = -\mathbf{e}_k^{(m)} & k = i \\ \tilde{\mathbf{l}}_k^{(m)} = \mathbf{l}_k^{(m)} - \mathbf{e}_k^{(m)} & k > i \end{cases} \quad \begin{cases} m = 1, \ldots, n \\ i = 1, \ldots, N_m \end{cases} \tag{6}$$

Referring to Figure 1, vectors $\mathbf{l}_i^{(m)}$ and $\mathbf{r}_i^{(m)}$ are invariant body-fixed vectors which describe position of joints i and $i+1$ with respect to C_i, respectively, and $\mathbf{e}_0$ and $\mathbf{e}_i^{(m)}$ are computed as

$$\mathbf{e}_0 = \sum_{m=1}^{n} \mathbf{r}_0^{(m)} \mu_1^{(m)} \tag{7a}$$

$$\mathbf{e}_i^{(m)} = \mathbf{l}_i^{(m)} (1 - \mu_i^{(m)}) + \mathbf{r}_i^{(m)} \mu_{i+1}^{(m)} \tag{7b}$$

The quantity $\mu_i^{(m)}$ describes the ratio of the outboard mass after the ith joint of the mth manipulator with respect to the total mass, and is given by

$$\mu_i^{(m)} = \sum_{k=i}^{N_m} \frac{m_k^{(m)}}{M} \qquad i = 1, \ldots, N_m \quad \text{and} \quad \mu_{N_m+1}^{(m)} = 0 \tag{7c}$$

where M is the total mass of the system, and $m_k^{(m)}$ is the mass of the kth body of the mth manipulator. Considering Eqs. (6) and (7), it can be seen that barycentric vectors are physically meaningful. For the ith link of mth manipulator, if an augmented body is formed by concentrating the inboard and outboard masses at the corresponding joint of both ends, then $\mathbf{e}_i^{(m)}$ describes the CM position of this augmented body with respect to the real CM of the link. Taking the CM of the augmented body as a reference point, vectors $\tilde{\mathbf{e}}_i^{(m)}$, $\tilde{\mathbf{l}}_i^{(m)}$, and $\tilde{\mathbf{r}}_i^{(m)}$ describe the CM position of the link, and the position of joints i and $i+1$ with respect to that point, respectively.

Substitution of Eqs. (5a) and (5b) for $\boldsymbol{\rho}_{C_i}$ into Eq. (2), and the result into Eq. (1) completes the position analysis and yields

$$P \in \text{Base: } \mathbf{R}_p^{(0)} = \mathbf{R}_{\text{CM}} + \tilde{\mathbf{e}}_0 + \sum_{m=1}^{n} \sum_{k=1}^{N_m} \tilde{\mathbf{l}}_k^{(m)} + \mathbf{r}_{\rho/C_0} \tag{8a}$$

$$P \in \text{Link}_i^{(m)}\text{: } \mathbf{R}_{\rho_i}^{(m)} = \mathbf{R}_{\text{CM}} + \tilde{\mathbf{r}}_0^{(m)} + \sum_{\substack{j=1 \\ j \neq m}}^{n} \sum_{k=1}^{N_j} \tilde{\mathbf{l}}_k^{(j)} + \sum_{k=1}^{N_m} \tilde{\mathbf{v}}_{ki}^{(m)} + \mathbf{r}_{p/C_i^{(m)}} \tag{8b}$$

Note that the position vectors in the above equations are written in terms of invariant body-fixed vectors. To obtain scalar equations, appropriate transformation matrices for each term must be employed. Also, note that based on the spacecraft attitude and corresponding joint angles, the orientation of any link of the system can also be obtained.

Velocity Analysis: To obtain the inertial velocity of point P, $\dot{\mathbf{R}}_p$, Eqs. (1) and (2) are differentiated with respect to time, which results in

$$\dot{\mathbf{R}}_p = \dot{\mathbf{R}}_{\text{CM}} + \dot{\boldsymbol{\rho}}_{C_i} + \boldsymbol{\omega}_i \times \mathbf{r}_{p/C_i} \tag{9}$$

where $\dot{\mathbf{R}}_{\text{CM}}$ is velocity of the system center of mass, and $\dot{\boldsymbol{\rho}}_{C_i}$ is obtained by differentiation of Eqs. (5a) and (5b) which describe $\boldsymbol{\rho}_{C_i}$ in terms of barycentric vectors. Note that barycentric vectors, according to the definition, are body-fixed vectors with constant length (as long as system mass distribution does not change). Therefore, differentiation of Eqs. (5a) and (5b) yields

$$\dot{\boldsymbol{\rho}}_{C_0} = \boldsymbol{\omega}_0 \times \tilde{\mathbf{e}}_0 + \sum_{m=1}^{n} \sum_{k=1}^{N_m} \boldsymbol{\omega}_k^{(m)} \times \tilde{\mathbf{l}}_k^{(m)} \tag{10a}$$

$$\dot{\boldsymbol{\rho}}_{C_i}^{(m)} = \boldsymbol{\omega}_0 \times \tilde{\mathbf{r}}_0^{(m)} + \sum_{\substack{j=1 \\ j \neq m}}^{n} \sum_{k=1}^{N_j} \boldsymbol{\omega}_k^{(j)} \times \tilde{\mathbf{l}}_k^{(j)} + \sum_{k=1}^{N_m} \boldsymbol{\omega}_k^{(m)} \times \tilde{\mathbf{v}}_{ki}^{(m)} \qquad \begin{cases} m = 1, \ldots, n \\ i = 1, \ldots, N_m \end{cases} \tag{10b}$$

where $\boldsymbol{\omega}$s are angular velocities and individual bodies.

Substitution of Eqs. (10a) and (10b), for $\dot{\boldsymbol{\rho}}_{C_i}$, into Eq. (9) completes the velocity analysis

$$P \in \text{Base: } \dot{\mathbf{R}}_p^{(0)} = \dot{\mathbf{R}}_{\text{CM}} + \boldsymbol{\omega}_0 \times \tilde{\mathbf{e}}_0 + \sum_{m=1}^{n} \sum_{k=1}^{N_m} \boldsymbol{\omega}_k^{(m)} \times \tilde{\mathbf{l}}_k^{(m)} + \boldsymbol{\omega}_0 \times \mathbf{r}_{p/C_0} \tag{11a}$$

$$P \in \text{Link}_i^{(m)}\text{: } \dot{\mathbf{R}}_{p_i}^{(m)} = \dot{\mathbf{R}}_{\text{CM}} + \boldsymbol{\omega}_0 \times \tilde{\mathbf{r}}_0^{(m)} + \sum_{\substack{j=1 \\ j \neq m}}^{n} \sum_{k=1}^{N_j} \boldsymbol{\omega}_k^{(j)} \times \tilde{\mathbf{l}}_k^{(j)} + \sum_{k=1}^{N_m} \boldsymbol{\omega}_k^{(m)} \times \tilde{\mathbf{v}}_{ki}^{(m)} + \boldsymbol{\omega}_i^{(m)} \times \mathbf{r}_{p/C_i^{(m)}} \tag{11b}$$

Again, in order to obtain scalar equations, appropriate transformation matrices for each term must be used.

For single DOF joints, the angular velocity of an individual body can be obtained as

$$\boldsymbol{\omega}_k^{(m)} = \boldsymbol{\omega}_0 + \sum_{i=1}^{k} \dot{\theta}_i^{(m)} \mathbf{z}_i^{(m)} \qquad \begin{cases} m = 1, \ldots, n \\ k = 1, \ldots, N_m \end{cases} \tag{12}$$

where $\mathbf{z}_i^{(m)}$ is a unit vector along the axis of rotation of the ith joint of the mth manipulator, and $\dot{\theta}_i^{(m)}$ is the corresponding joint angle rate.

Note that choosing a set of coordinates as system *generalized coordinates*, the linear velocity of an arbitrary point P, and the angular velocity of the corresponding body can be related to the time derivative of generalized coordinates (i.e., generalized speeds) through a Jacobian matrix. For instance, if point P belongs to the ith body of the mth manipulator, then

$$\begin{Bmatrix} \dot{\mathbf{R}}_p \\ \boldsymbol{\omega}_i^{(m)} \end{Bmatrix} = \mathbf{J}_{i,p}^{(m)} \boldsymbol{\nu} \tag{13}$$

where $\mathbf{J}_{i,p}^{(m)}$ represents a Jacobian matrix, relating the vector of generalized speeds $\boldsymbol{\nu}$ to the linear velocity of point P and the angular velocity of body m. The vector of generalized speeds is selected as

$$\boldsymbol{\nu} = \left(\dot{\mathbf{R}}_{\text{CM}}^T, \boldsymbol{\omega}_0^T, \dot{\boldsymbol{\theta}}^T \right)^T \tag{14}$$

Then, based on Eqs. (11b), and (12), $\mathbf{J}_{i,p}^{(m)}$ is obtained as

$$\mathbf{J}_{i,p}^{(m)} = \begin{bmatrix} \mathbf{1}_{3\times 3} & \mathbf{J}_1^{(m)} & \mathbf{J}_2^{(m)} \\ \mathbf{0}_{3\times 3} & \mathbf{1}_{3\times 3} & \mathbf{J}_3^{(m)} \end{bmatrix}_{6\times N} \tag{15a}$$

where

$$\mathbf{J}_1^{(m)} = -\left[\mathbf{T}_0 \, {}^0\tilde{\mathbf{r}}_0^{(m)} + \sum_{\substack{j=1 \\ j \neq m}}^{n} \sum_{k=1}^{N_j} \mathbf{T}_k^{(j)} \, {}^k\tilde{\mathbf{l}}_k^{(j)} + \sum_{k=1}^{N_m} \mathbf{T}_k^{(m)} \, {}^k\tilde{\mathbf{v}}_{ki,p}^{(m)} \right]^{\times} \tag{15b}$$

$$\mathbf{J}_2^{(m)} = -\sum_{\substack{j=1 \\ j \neq m}}^{n} \sum_{k=1}^{N_j} \left[\mathbf{T}_k^{(j)} \, {}^k\tilde{\mathbf{l}}_k^{(j)} \right]^{\times} \mathbf{E}_k^{(j)} - \sum_{k=1}^{N_m} \left[\mathbf{T}_k^{(m)} \, {}^k\tilde{\mathbf{v}}_{ki,p}^{(m)} \right]^{\times} \mathbf{E}_k^{(m)} \tag{15c}$$

$$\mathbf{J}_3^{(m)} = \mathbf{E}_i^{(m)} \tag{15d}$$

The $\mathbf{T}_0$ and $\mathbf{T}_j^{(k)}$ are rotation matrices between body-fixed frames and the inertial frame, $[\bullet]^{\times}$ is the *cross product operator* defined as

$$[\mathbf{r}]^{\times} = \begin{bmatrix} 0 & -r_z & r_y \\ r_z & 0 & -r_x \\ -r_y & r_x & 0 \end{bmatrix} \tag{16}$$

and

$$\tilde{\mathbf{v}}_{ji,p}^{(k)} = \tilde{\mathbf{v}}_{ji}^{(k)} + \delta_{ji} \mathbf{r}_{p/C_i^{(m)}} \tag{17a}$$

$$\mathbf{E}_j^{(k)} = \begin{bmatrix} \mathbf{0}_{3\times b} & \mathbf{T}_1^{(k)} \, {}^1\mathbf{z}_1^{(k)} & \cdots & \mathbf{T}_j^{(k)} \, {}^j\mathbf{z}_j^{(k)} & \mathbf{0} \end{bmatrix}_{3\times K} \tag{17b}$$

where δ_{ji} is *Kronecker delta*, $b = \sum_{l=1}^{k-1} N_l$, and ${}^{j}\mathbf{z}_j^{(k)} \equiv (0, 0, 1)^T$ is a unit vector along axis of rotation of the jth joint of the kth manipulator expressed in its own body-fixed frame. Note that a left superscript refers to the frame in which the corresponding vector is expressed, and it disappears for the inertial frame.

Similarly, based on Eqs. (11a), and (12), $\mathbf{J}_p^{(0)}$ can be obtained for a point P on the spacecraft as

$$\mathbf{J}_p^{(0)} = \begin{bmatrix} \mathbf{1}_{3\times 3} & \mathbf{J}_1^{(0)} & \mathbf{J}_2^{(0)} \\ \mathbf{0}_{3\times 3} & \mathbf{1}_{3\times 3} & \mathbf{0} \end{bmatrix}_{6\times N} \tag{18a}$$

where

$$\mathbf{J}_1^{(0)} = -\left[\mathbf{T}_0\left({}^{0}\tilde{\mathbf{e}}_0 + \mathbf{r}_{p/c_0}\right) + \sum_{m=1}^{n}\sum_{k=1}^{N_j} \mathbf{T}_k^{(m)}\,{}^{k}\tilde{\mathbf{l}}_k^{(m)}\right]^{\times} \tag{18b}$$

$$\mathbf{J}_2^{(0)} = -\sum_{m=1}^{n}\sum_{k=1}^{N_j}\left[\mathbf{T}_k^{(m)}\,{}^{k}\tilde{\mathbf{l}}_k^{(m)}\right]^{\times}\mathbf{E}_k^{(m)} \tag{18c}$$

The above equations provide a complete set of kinematic relations useful in developing equations of motion and in control of multiple SFFRs. In the next section, kinematic equations are developed using the direct path method.

III. THE DIRECT PATH METHOD (DPM)

In this section, the kinematics of a rigid multiple arm free-flying space robotic system is developed using a set of body-fixed geometric vectors. The motion of the *spacecraft CM* is used to describe the system translation with respect to an inertial frame of reference, **XYZ**. The rest of the definitions described in section II, are applicable here too.

Considering Figure 2, the inertial position of an arbitrary point P, $\mathbf{R}_P$, is written as

$$\mathbf{R}_P = \mathbf{R}_{C_0} + \mathbf{r}_P \tag{19a}$$

and

$$\mathbf{r}_P = \mathbf{r}_{C_i} + \mathbf{r}_{p/C_i} \tag{19b}$$

where $\mathbf{R}_{C_0}$ is the inertial position of the spacecraft CM, $\mathbf{r}_p$ is the position vector of point P with respect to the spacecraft CM, and $\mathbf{r}_{C_i}$ is the CM position vector of the ith body with respect to the spacecraft CM. Referring to Figure 2, $\mathbf{r}_{C_i}$ can be expressed as

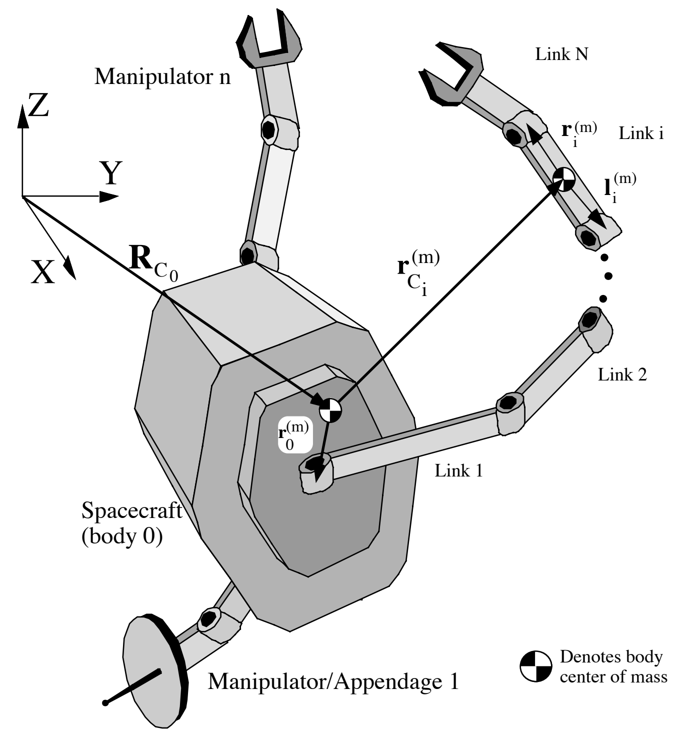

Figure 2. Developing direct path method for a free-flying space robotic system with n manipulators.

follows,

$$\mathbf{r}_{C_0} = \mathbf{0} \tag{20a}$$

$$\mathbf{r}_{C_i}^{(m)} = \mathbf{r}_0^{(m)} + \sum_{k=1}^{i-1} (\mathbf{r}_k^{(m)} - \mathbf{l}_k^{(m)}) - \mathbf{l}_i^{(m)} \qquad \begin{cases} m = 1, \ldots, n \\ i = 1, \ldots, N_m \end{cases} \tag{20b}$$

where, as before, vectors $\mathbf{l}_i^{(m)}$ and $\mathbf{r}_i^{(m)}$ are body-fixed vectors which describe the position of joints i and $i+1$ with respect to C_i, see Figure 2.

Substitution of Eqs. (20a) and (20b) for $\mathbf{r}_{C_i}$, into Eq. (19b), and the resulting equation into Eq. (19a) completes the position analysis and yields

$$P \in \text{Base:}\ \mathbf{R}_p^{(0)} = \mathbf{R}_{C_0} + \mathbf{r}_{p/C_0} \tag{21a}$$

$$P \in \text{Link}_i^{(m)}\text{:}\ \mathbf{R}_{p_i}^{(m)} = \mathbf{R}_{C_0} + \mathbf{r}_0^{(m)} + \sum_{k=1}^{i-1} (\mathbf{r}_k^{(m)} - \mathbf{l}_k^{(m)}) - \mathbf{l}_i^{(m)} + \mathbf{r}_{p/C_i^{(m)}} \tag{21b}$$

Velocity Analysis: To obtain the inertial velocity of point P, Eq. (19) is differentiated to yield

$$\dot{\mathbf{R}}_p = \dot{\mathbf{R}}_{C_0} + \dot{\mathbf{r}}_{C_i} + \boldsymbol{\omega}_i \times \mathbf{r}_{p/C_i} \tag{22}$$

where $\dot{\mathbf{R}}_{C_0}$ is velocity of the spacecraft CM. The velocity $\dot{\mathbf{r}}_{C_i}$ is obtained by differentiation of Eqs. (20) which yields

$$\dot{\mathbf{r}}_{C_0} = \mathbf{0} \tag{23a}$$

$$\dot{\mathbf{r}}_{C_i}^{(m)} = \boldsymbol{\omega}_0 \times \mathbf{r}_0^{(m)} + \sum_{k=1}^{i-1} \boldsymbol{\omega}_k^{(m)} \times (\mathbf{r}_k^{(m)} - \mathbf{l}_k^{(m)}) - \boldsymbol{\omega}_i^{(m)} \times \mathbf{l}_i^{(m)} \qquad \begin{cases} m = 1, \ldots, n \\ i = 1, \ldots, N_m \end{cases} \tag{23b}$$

where $\boldsymbol{\omega}$s are angular velocities of individual bodies.

Substitution of Eqs. (23a) and (23b), for $\dot{\mathbf{r}}_{C_i}$, into Eq. (22) completes the velocity analysis,

$$P \in \text{Base:}\ \dot{\mathbf{R}}_p^{(0)} = \dot{\mathbf{R}}_{C_0} + \boldsymbol{\omega}_0 \times \mathbf{r}_{p/C_0} \tag{24a}$$

$$P \in \text{Link}_i^{(m)}\text{:}\ \dot{\mathbf{R}}_{p_i}^{(m)} = \dot{\mathbf{R}}_{C_0} + \boldsymbol{\omega}_0 \times \mathbf{r}_0^{(m)} + \sum_{k=1}^{i-1} \boldsymbol{\omega}_k^{(m)} \times (\mathbf{r}_k^{(m)} - \mathbf{l}_k^{(m)}) - \boldsymbol{\omega}_i^{(m)} \times \left(\mathbf{l}_i^{(m)} - \mathbf{r}_{p/C_i^{(m)}}\right) \tag{24b}$$

Note that the angular velocity of any individual body, in a treelike structure of bodies with single DOF joints, can be obtained using Eq. (12).

To obtain a typical Jacobian matrix, the linear velocity of an arbitrary point P on the ith body of the mth manipulator, and angular velocity of the corresponding body are expressed as

$$\begin{Bmatrix} \dot{\mathbf{R}}_p \\ \boldsymbol{\omega}_i^{(m)} \end{Bmatrix} = \mathbf{J}_{i,p}^{(m)} \boldsymbol{\nu} \tag{25}$$

where $\mathbf{J}_{i,p}^{(m)}$ represents a Jacobian matrix, and $\boldsymbol{\nu}$ is the vector of generalized speeds, which is defined as

$$\boldsymbol{\nu} = \left(\dot{\mathbf{R}}_{C_0}^T, \boldsymbol{\omega}_0^T, \dot{\boldsymbol{\theta}}^T\right)^T \tag{26}$$

Note that the generalized speeds are here a function of $\dot{\mathbf{R}}_{C_0}$ and not of $\dot{\mathbf{R}}_{\text{CM}}$, see Eq. (13b). Then, based on Eqs. (12) and (24b), $\mathbf{J}_{i,p}^{(m)}$ is computed as

$$\mathbf{J}_{i,p}^{(m)} = \begin{bmatrix} \mathbf{1}_{3\times 3} & \mathbf{J}_1^{(m)} & \mathbf{J}_2^{(m)} \\ \mathbf{0}_{3\times 3} & \mathbf{1}_{3\times 3} & \mathbf{J}_3^{(m)} \end{bmatrix}_{6\times N} \tag{27a}$$

where

$$\mathbf{J}_1^{(m)} = -\left[\mathbf{T}_0\,{}^0\mathbf{r}_0^{(m)} + \sum_{k=1}^{i-1} \left[\mathbf{T}_k^{(m)}\left({}^k\mathbf{r}_k^{(m)} - {}^k\mathbf{l}_k^{(m)}\right)\right] - \mathbf{T}_i^{(m)}\left({}^i\mathbf{l}_i^{(m)} - {}^i\mathbf{r}_{p/C_i^{(m)}}\right)\right]^{\times} \tag{27b}$$

$$\mathbf{J}_2^{(m)} = -\sum_{k=1}^{i-1} \left[\mathbf{T}_k^{(m)}\left({}^k\mathbf{r}_k^{(m)} - {}^k\mathbf{l}_k^{(m)}\right)\right]^{\times} \mathbf{E}_k^{(m)} + \left[\mathbf{T}_i^{(m)}\left({}^i\mathbf{l}_j^{(m)} - {}^i\mathbf{r}_{p/C_i^{(m)}}\right)\right]^{\times} \mathbf{E}_i^{(m)} \tag{27c}$$

$$\mathbf{J}_3^{(m)} = \mathbf{E}_i^{(m)} \tag{27d}$$

and the definitions given for different terms in Eq. (15), are applicable here, too.

Similar to the above, $\mathbf{J}_p^{(0)}$ can be obtained for the one corresponding to point P on the spacecraft, based on Eqs. (12), and (24a),

$$\mathbf{J}_p^{(0)} = \begin{bmatrix} \mathbf{1}_{3\times 3} & \mathbf{J}_1^{(0)} & \mathbf{0} \\ \mathbf{0}_{3\times 3} & \mathbf{1}_{3\times 3} & \mathbf{0} \end{bmatrix}_{6\times N} \tag{28}$$

where

$$\mathbf{J}_1^{(0)} = -\left[\mathbf{T}_0^0 \mathbf{r}_{p/C_0}\right]^{\times} \tag{29}$$

IV. DISCUSSION AND COMPARISONS

In this section the two approaches developed for kinematics analysis of SFFR with rigid multiple manipulators are compared. However, as revealed by the above formulations, the barycentric vector approach (BVA) is developed based on

a. Taking the center of mass of the whole system as a representative point for the system's translational motion.
b. Using a set of body-fixed barycentric vectors which reflect both geometric configuration and mass distribution of the system.

On the other hand, the direct path method (DPM) is developed based on

a. Taking a point on the base body (preferably its CM) as the representative point for the translation of the system.
b. Using a set of body-fixed geometric vectors.

Comparing the results obtained for position analysis, Eqs. (8) compared to Eqs. (21), it can be seen that the direct path approach results in single summations, yielding more compact relationships. Note that presence of double summations in Eqs. (8) means that all system links are contributing in defining the position of any arbitrary point P. This is due to the fact that by taking the center of mass of the whole system as a representative point for the translation of the system, the mass distribution of the entire system (represented in Eq. (3)) has to be taken into account in writing position relationships. The difference between the two approaches is more considerable for computing velocities, Eqs. (11) compared to Eqs. (24), as this leads to a big difference between the resulting Jacobian matrices, Eqs. (15) compared to Eqs. (27). Note that the complexity of Jacobian matrix is important because many control algorithms require its computation; these algorithms can be implemented more easily using the direct path approach.

To obtain quantitative comparison results for the two approaches, a 14-DOF free-flying system is considered, see Figure 3. The spacecraft includes three open chain appendages, two of which are 3-DOF manipulators, while the third is a 2-DOF communication antenna. A complete description of the system can be found in ref. 13. The required multiplications and additions for obtaining the position of a point P, $\mathbf{R}_{p_i}^{(m)}$, on the first ($i = 1$) and the last link ($i = 3$) of the first manipulator ($m = 1$), are compared in Table I. It is seen that the required computations for the BVA are almost twice as many as those of the DPM, when a point P belongs to the

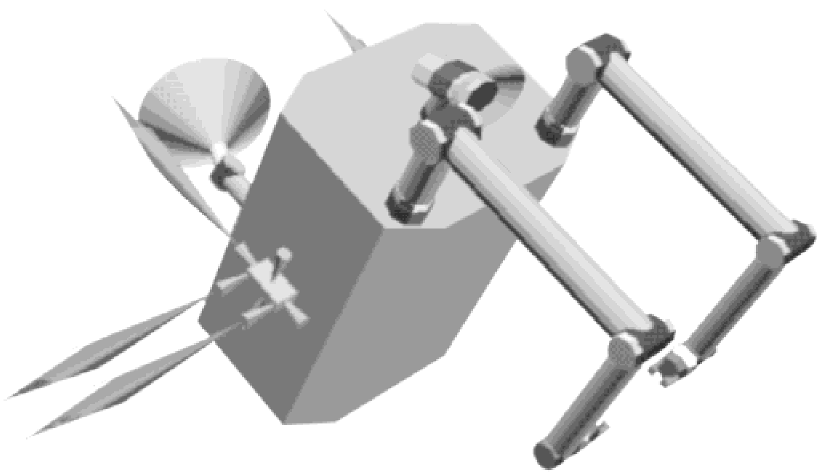

Figure 3. A spatial three-manipulator/appendage space free-flyer.

Table I. Comparing the required computations for position analysis, i.e., obtaining $R_{p_i}^{(1)}$ based on Eqs. (8b) and (21b).

	$i=1$		$i=3$	
	Mult.	Add.	Mult.	Add.
BVA	81	84	81	84
DPM	18	21	36	39

third link. The difference is more significant when a point on the first link is considered. In such a case, the required computations for the DPM substantially decrease (multiplications and additions of 18 and 21, respectively) while the BVA requires the same effort as before (i.e., 81 multiplications and 84 additions). Therefore, as a result, the required computations for the BVA are almost four times greater than those of the DPM in this case. Note that here the required computations for calculating the barycentric vectors, given by Eqs. (6) and (7), are not considered. Taking these computations into account makes the difference even more pronounced. The required computations for obtaining the velocity of a point P, according to Eqs. (11) and (24), are displayed in Table II. These results ratify that the DPM requires significantly less computations, e.g., 135 multiplications and 111 additions for BVA are compared to 30 and 27 for DPM, respectively.

It should be mentioned that the barycentric vector approach is one which may be very useful in developing dynamics equations and control laws in certain cases. In fact, it results in decoupling of the total linear and angular motion from the rest of the equations, when no external forces and torques are applied on the system. This is very helpful in studying attitude dynamics of large space systems, decoupled from the orbital mechanics, or the dynamics and control of free-floating robotic systems (i.e., the ones in which the spacecraft thrusters are turned off). However, according to the above discussion, the direct path approach results in more compact equations in kinematics and consequently in dynamics. This approach results in a larger number of dynamics equations with simpler terms which have clearer physical interpretation, see ref. 13. Since the system dynamics cannot be reduced when external forces act on the system, this approach becomes more appropriate in the case of multiple arm systems, subjected to spacecraft thruster forces or other external forces and torques. Note that to develop model-based algorithms for controlling free-floating systems, the dynamics model obtained based on the direct path approach will have to be reduced by mathematical techniques such as the orthogonal complement methods.[a] However, if the barycentric vector approach is used, no such methods are needed and the equations can be directly reduced using a modified Lagrangian, see ref. 7.

Table II. Comparing the required computations for velocity analysis, i.e., obtaining $R_{p_i}^{(1)}$ based on Eqs. (11b) and (24b).

	$i=1$		$i=3$	
	Mult.	Add.	Mult.	Add.
BVA	135	111	135	111
DPM	30	27	60	51

V. CONCLUSIONS

This paper studied the kinematics of a multiple manipulator space free-flying robot (SFFR). Two basic approaches for kinematics modelling of a rigid multibody space robotic system were developed, and the obtained results were compared from a computational point of view. Taking the center of mass of the whole system as a representative point for the translational motion of the system, and using a set of body-fixed vectors which reflect both geometric configuration and mass distribution of the system, are characteristics of the so-called *barycentric vector approach*. This approach eventually results in decoupling the total linear and angular motion from the rest of the dynamics equations, when no external forces/torques are applied on the system. On the other hand, taking a point on the base body as the representative point for the translational motion of the system (preferably the center of mass of the spacecraft), characterizes the so-called *direct path method*. This approach results in a larger number of dynamics equations with simpler terms with clearer physical meaning. A 14-DOF space free-flying system was considered for a quantitative comparison between the two approaches. It was shown that the direct path method requires significantly less computations for position and velocity analyses. Therefore, as a result, using the direct path approach is more appropriate when dealing with multiple arm

[a] For a description of the natural orthogonal complement method see ref. 12.

systems, and especially in the presence of external forces/torques.

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