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Explicit dynamics of space free-flyers with multiple manipulators via SPACEMAPLE

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Abstract—This paper focuses on the dynamics of a multiple manipulator space free-flying robot (SFFR) with rigid links and issues relevant to the development of appropriate control algorithms. To develop an explicit dynamics model of such complex systems, the Lagrangian formulation is applied. First, the system kinetic energy is derived based on a developed kinematics approach. Then, through vigorous mathematical analyses, three formats are obtained which describe the contribution of each term of kinetic energy to the equations of motion. Next, explicit derivations of a system's mass matrix, and of the vectors of non-linear velocity terms and generalized forces are introduced for the first time. The obtained dynamics model is very useful for dynamics analyses, design and development of control algorithms for such complex systems. The explicit SFFR dynamics can be implemented either numerically or symbolically. Following the latter approach, the developed symbolic code for dynamics modeling, i.e. SPACEMAPLE, and its verification procedure are described, and issues relevant to the development and computation of dynamics models in control algorithms are briefly discussed. Specific dynamic characteristics of SFFRs compared to fixed-base manipulators are pointed out.

Keywords: Space robotic systems; dynamics; modeling and simulation.

1. INTRODUCTION

Space free-flying robots (SFFR) are space systems that include an actuated satellite base equipped with one or more manipulators. An SFFR whose base actuators are inactive is called a free-floating space robot. Distinct from fixed-based manipulators, the spacecraft (base) of a SFFR responds to dynamic reaction forces due to manipulator motions. In order to control such a system, it is essential to

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consider the dynamic coupling between the manipulators and the base [7–9]. To this end, one should first derive a proper dynamics model for the system, which is to be elaborated in this paper.

Vafa and Dubowsky [1] have described the kinematics and conservation dynamics of a free-floating space manipulator system, using the Virtual Manipulator Approach. No external forces act on the system and so the system center of mass is fixed in inertial space, enabling them to represent a free-floating system by one with a virtual fixed base. Papadopoulos and Dubowsky [2] have employed a barycentric vector approach, to study kinematics and dynamics of a single-arm SFFR in free-floating mode. Taking the center of mass of the whole system as a representative point for the translational motion and using barycentric vectors, which reflect both geometric configuration and mass distribution of the system, results in decoupling the total linear and angular motion from the rest of the equations. This approach was also applied by Papadopoulos and Moosavian [3] to obtain the dynamics and to control a multiple-arm SFFR in free-flying mode. Umetani and Yoshida [4] have presented a Generalized Jacobian Matrix for a free-floating system. Assuming that no external forces are applied on a rigid robotic system with revolute joints, they derive a generalized Jacobian matrix which reflects both momentum conservation laws and kinematic relations. The proposed generalized Jacobian matrix converges to the conventional Jacobian when the base body is relatively massive.

Nakamura *et al.* [5] have studied the mechanics of coordinated object manipulation by multiple robotic arms, taking the object dynamics into consideration. Moosavian and Papadopoulos [6] have developed free-flyer kinematics based on the direct path method and compared that to the barycentric vector approach. Their analysis showed that the direct path method results in equations with simpler terms, and requires significantly less computations for position and velocity analyses. Therefore, it emerges as a more appropriate approach for dynamics modeling of multiple-arm systems, which is studied in this paper.

The focus of this paper is on the dynamics of a multiple-manipulator SFFR based on the direct path kinematics approach. Derivation of the equations of motion results in an explicit derivations of the system's mass matrix, and of the vectors of non-linear velocity terms and generalized forces. Unlike with recursive dynamics formulations, the obtained dynamics model is very useful for dynamics analyses, design studies and the development of control algorithms for SFFRs. The obtained explicit dynamics model of a multiple-manipulator SFFR, published for the first time here, can be implemented either numerically or symbolically. The latter approach was followed, and the developed symbolic code for dynamics modeling, i.e. SPACEMAPLE, and its verification procedure are described here. Dynamics issues important in the development of appropriate control algorithms are also discussed. Specific dynamic characteristics of SFFRs compared to fixed-base manipulators are discussed before concluding the paper.

2. LAGRANGIAN FORMULATION

Since a typical maneuver of a SFFR is of relatively short length and duration, microgravity and dynamic effects due to orbital mechanics are negligible compared to control forces. Therefore, the motion of the system is considered with respect to an in-orbit inertial frame of reference (**XYZ**), and the system potential energy is taken equal to zero. The general Lagrangian formulation for such system yields:

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial T}{\partial \dot{q}_i}\right) - \left(\frac{\partial T}{\partial q_i}\right) = Q_i, \quad i = 1, \ldots, N, \tag{1}$$

where T is the system kinetic energy, N is the system d.o.f., and q_i, $\dot{q}_i$ and Q_i are the ith element of the vector of generalized coordinates, generalized speeds and generalized forces, respectively. To apply (1), and obtain dynamics equations, first the system kinetic energy, T, has to be derived. This can be written as:

$$T = \frac{1}{2}\int_M \dot{\mathbf{R}}_P \cdot \dot{\mathbf{R}}_P \,\mathrm{d}M, \tag{2}$$

where M defines the system distributed mass and $\dot{\mathbf{R}}_P$ is velocity of an arbitrary point P, which can be evaluated based on the direct path kinematics approach for multiple manipulator SFFR with rigid elements, developed in Moosavian and Papadopoulos [6] as:

$P \in Base$: $$\dot{\mathbf{R}}_p^{(0)} = \dot{\mathbf{R}}_{C_0} + \boldsymbol{\omega}_0 \times \mathbf{r}_{p/C_0}, \tag{3a}$$

$P \in Link_i^{(m)}$: $$\dot{\mathbf{R}}_{p_i}^{(m)} = \dot{\mathbf{R}}_{C_0} + \boldsymbol{\omega}_0 \times \mathbf{r}_0^{(m)} + \sum_{k=1}^{i-1} \boldsymbol{\omega}_k^{(m)} \times \left(\mathbf{r}_k^{(m)} - \mathbf{l}_k^{(m)}\right) - \boldsymbol{\omega}_i^{(m)} \times \left(\mathbf{l}_i^{(m)} - \mathbf{r}_{p/C_i^{(m)}}\right), \tag{3b}$$

where $\dot{\mathbf{R}}_{C_0}$ describes the spacecraft center of mass velocity, $\mathbf{r}_{p/C_0}$ describes the position of P with respect to the spacecraft center of mass, vectors $\mathbf{l}_i^{(m)}$, $\mathbf{r}_i^{(m)}$ and so on are body-fixed vectors which describe the position of joints i and $i+1$ with respect to C_i, as seen in Fig. 1, and $\boldsymbol{\omega}_0$ and $\boldsymbol{\omega}_k^{(m)}$ are the angular velocity of the spacecraft and of the kth link of the mth manipulator, respectively. For single d.o.f. joints, the angular velocity of an individual body can be obtained as:

$$\boldsymbol{\omega}_k^{(m)} = \boldsymbol{\omega}_0 + \sum_{i=1}^{k} \dot{\boldsymbol{\theta}}_i^{(m)} \mathbf{z}_i^{(m)} \begin{cases} m = 1, \ldots, n \\ k = 1, \ldots, N_m, \end{cases} \tag{4}$$

where $\mathbf{z}_i^{(m)}$ is a unit vector along the axis of rotation of the ith joint of the mth manipulator, and $\dot{\boldsymbol{\theta}}_i^{(m)}$ is the corresponding joint angle rate.

Substitution of (3) for $\dot{\mathbf{R}}_P$ into (2) yields:

$$T = \frac{1}{2}\int_M (\dot{\mathbf{R}}_{C_0} + \dot{\mathbf{r}}_{C_i} + \boldsymbol{\omega}_i \times \mathbf{r}_{p/C_i}) \cdot (\dot{\mathbf{R}}_{C_0} + \dot{\mathbf{r}}_{C_i} + \boldsymbol{\omega}_i \times \mathbf{r}_{p/C_i}) \,\mathrm{d}M, \tag{5}$$

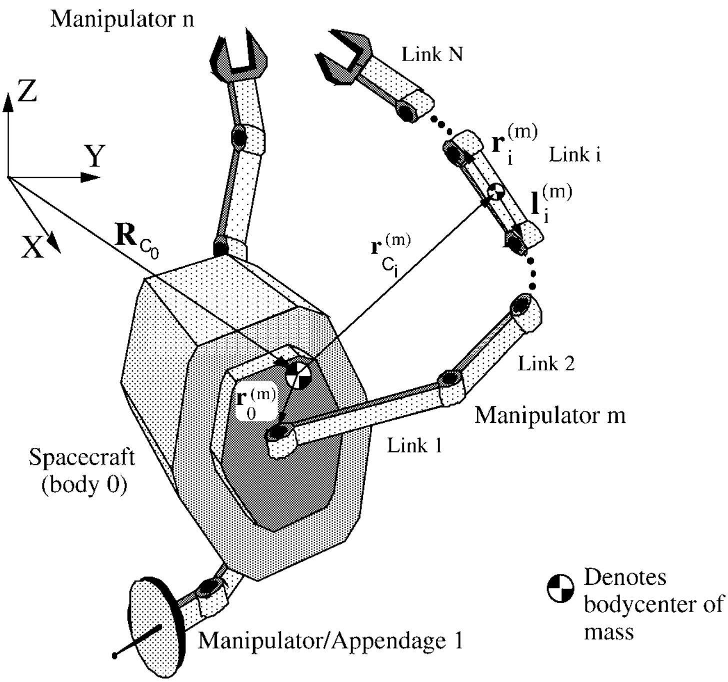

Figure 1. A SFFR system with n manipulators.

which can be simplified to obtain:

$$T = T_0 + T_1 + T_2, \tag{6a}$$

with:

$$T_0 = \frac{1}{2} M(\dot{\mathbf{R}}_{C_0} \cdot \dot{\mathbf{R}}_{C_0}), \tag{6b}$$

$$T_1 = \frac{1}{2}\left\{\boldsymbol{\omega}_0 \cdot \mathbf{I}_0 \cdot \boldsymbol{\omega}_0 + \sum_{m=1}^{n}\sum_{i=1}^{N_m}\left(m_i^{(m)}\dot{\mathbf{r}}_{C_i}^{(m)} \cdot \dot{\mathbf{r}}_{C_i}^{(m)} + \boldsymbol{\omega}_i^{(m)} \cdot \mathbf{I}_i^{(m)} \cdot \boldsymbol{\omega}_i^{(m)}\right)\right\}, \tag{6c}$$

$$T_2 = \dot{\mathbf{R}}_{C_0} \cdot \left(\sum_{m=1}^{n}\sum_{i=1}^{N_m} m_i^{(m)}\dot{\mathbf{r}}_{C_i}^{(m)}\right), \tag{6d}$$

and $\dot{\mathbf{r}}_{C_i}^{(m)}$ describes the velocity of C_i which can be obtained as:

$$\dot{\mathbf{r}}_{C_i}^{(m)} = \boldsymbol{\omega}_0 \times \mathbf{r}_0^{(m)} + \sum_{k=1}^{i-1}\boldsymbol{\omega}_k^{(m)} \times \left(\mathbf{r}_k^{(m)} - \mathbf{l}_k^{(m)}\right) - \boldsymbol{\omega}_i^{(m)} \times \mathbf{l}_i^{(m)} \quad \begin{cases} m = 1, \ldots, n \\ i = 1, \ldots, N_m. \end{cases} \tag{6e}$$

Note that expressions for T are in terms of invariant body-fixed vectors and appropriate transformation matrices for each term must be employed to do the required differentiations in (1).

The vector of generalized coordinates is chosen here as:

$$\mathbf{q} = \left(\mathbf{R}_{C_0}^{\mathrm{T}}, \boldsymbol{\delta}_0^{\mathrm{T}}, \boldsymbol{\theta}^{\mathrm{T}}\right)^{\mathrm{T}}, \tag{7a}$$

which can be arranged as:

$$\mathbf{q} = \left(\mathbf{q}^{(0)^{\mathrm{T}}}, \mathbf{q}^{(1)^{\mathrm{T}}}, \ldots, \mathbf{q}^{(n)^{\mathrm{T}}}\right)^{\mathrm{T}}, \tag{7b}$$

where:

$$\mathbf{q}^{(0)} = \left(\mathbf{R}_{C_0}^{\mathrm{T}}, \boldsymbol{\delta}_0^{\mathrm{T}}\right)^{\mathrm{T}}, \tag{7c}$$

$$\mathbf{q}^{(m)} = \boldsymbol{\theta}^{(m)} = \left(\boldsymbol{\theta}_1^{(m)}, \boldsymbol{\theta}_2^{(m)}, \ldots, \boldsymbol{\theta}_{N_m}^{(m)}\right)^{\mathrm{T}}, \tag{7d}$$

with $\boldsymbol{\delta}_0$ being the spacecraft Euler angles and $\boldsymbol{\theta}_i^{(m)}$ ($i = 1, \ldots, N_m$) describes the mth manipulator joint angles. Instead of corresponding Euler angles, the system dynamics can be formulated on the basis of choosing Euler parameters for orientation representation, Hughes [17]. This selection introduces algebraic constraints to the system, and the Natural Orthogonal Complement Method as presented in Saha and Angeles [18], can be applied to obtain independent system of equations of motion. Using (6) and applying the general Lagrangian formulation, (1), the equations of motion are obtained as:

$$\mathbf{H}(\boldsymbol{\delta}_0, \boldsymbol{\theta})\ddot{\mathbf{q}} + \mathbf{C}\left(\boldsymbol{\delta}_0, \dot{\boldsymbol{\delta}}_0, \boldsymbol{\theta}, \dot{\boldsymbol{\theta}}\right) = \mathbf{Q}(\boldsymbol{\delta}_0, \boldsymbol{\theta}), \tag{8}$$

where the vector of generalized coordinates $\mathbf{q}$ has been already defined in (7), $\mathbf{C}$ is an $N \times 1$ vector which contains all the non-linear velocity terms (in a microgravity environment), and $\mathbf{Q}$ is the $N \times 1$ vector of generalized forces ($N = K + 6$) given by:

$$\mathbf{Q} = \begin{Bmatrix} \mathbf{0}_{6\times 1} \\ \boldsymbol{\tau}_{K\times 1} \end{Bmatrix} + \sum_{p=1}^{i_f} \mathbf{J}_{0,p}^{\mathrm{T}} \mathbf{F}_{0,p} + \sum_{m=1}^{n} \sum_{i=1}^{N_m} \sum_{p=1}^{i_f} \mathbf{J}_{i,p}^{(m)\mathrm{T}} \mathbf{F}_{i,p}^{(m)}, \tag{9}$$

$\mathbf{F}_{0,p}$ is the pth external force/moment applied on the spacecraft, $\mathbf{F}_{i,p}^{(m)}$ is the pth external force/moment applied on the ith body of the mth manipulator, i_f is the number of applied forces/moments on the corresponding body and $\mathbf{J}_{i,p}^{(m)}$ is a Jacobian matrix corresponding to the point of force/moment application. Note that (9) can be obtained based on the definition of generalized forces. This equation can be rearranged so that actuator forces/torques are displayed explicitly. If all external forces except the ones applied on the spacecraft are zero, $\mathbf{Q}$ can be written as:

$$\mathbf{Q} = \mathbf{J}_Q \begin{Bmatrix} {}^0\mathbf{f}_{\mathrm{s}} \\ {}^0\mathbf{n}_{\mathrm{s}} \\ \boldsymbol{\tau}_{K\times 1} \end{Bmatrix}, \tag{10}$$

where ${}^0\mathbf{f}_{\mathrm{s}}$ and ${}^0\mathbf{n}_{\mathrm{s}}$ are the net force and moment applied on the spacecraft, and $\mathbf{J}_Q$ is an $N \times N$ Jacobian matrix. For a well designed system, $\mathbf{J}_Q$ remains non-singular, i.e. any required $\mathbf{Q}$ can be produced by the system's actuators. Next, to obtain an explicit dynamics model of multiple manipulator SFFR, mathematical analyses are presented to help in calculating the mass matrix, the vector of non-linear velocity terms and the generalized forces.

3. MATHEMATICAL PRELIMINARIES

The system kinetic energy [as expressed in (6)], regardless of body specifications, is composed of three typical terms:

$$a_1 = \frac{1}{2} m \dot{\mathbf{r}} \cdot \dot{\mathbf{r}}, \tag{11a}$$

$$a_2 = \frac{1}{2} \boldsymbol{\omega} \cdot \mathbf{I} \cdot \boldsymbol{\omega}, \tag{11b}$$

$$a_3 = \dot{\mathbf{R}}_{C_0} \cdot \sum_k m_k \dot{\mathbf{r}}_k. \tag{11c}$$

So, to differentiate the system kinetic energy according to (1), such terms have to be differentiated. Therefore, preliminary calculations in differentiation of these terms are presented in this section, resulting in three formats which describe the contribution of each term to the equations of motion. These formats, obtained in the Appendix, will be used in deriving the system dynamics model in the following section.

Differentiation of the first term, (11a), yields:

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial a_1}{\partial \dot{q}_i}\right) - \frac{\partial a_1}{\partial q_i} = \left[m \frac{\partial \mathbf{r}}{\partial q_i} \cdot \frac{\partial \mathbf{r}}{\partial q_1} \cdots m \frac{\partial \mathbf{r}}{\partial q_i} \cdot \frac{\partial \mathbf{r}}{\partial q_N} \right] \ddot{\mathbf{q}} + \left[m \frac{\partial \mathbf{r}}{\partial q_i} \cdot \left(\sum_{s=1}^{N} \frac{\partial^2 \mathbf{r}}{\partial q_s \partial q_1} \dot{q}_s \right) \cdots m \frac{\partial \mathbf{r}}{\partial q_i} \cdot \left(\sum_{s=1}^{N} \frac{\partial^2 \mathbf{r}}{\partial q_s \partial q_N} \dot{q}_s \right) \right] \dot{\mathbf{q}}, \tag{12}$$

which describes format-I, defined as the contribution of the first typical term to the equations of motion. Note that $\mathbf{r}$ is differentiated in the inertial frame (see the Appendix).

Differentiation of the second term, (11b), yields:

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial a_2}{\partial \dot{q}_i}\right) - \frac{\partial a_2}{\partial q_i} = \left[\frac{\partial \boldsymbol{\omega}}{\partial \dot{q}_i} \cdot \mathbf{I} \cdot \frac{\partial \boldsymbol{\omega}}{\partial \dot{q}_1} \cdots \frac{\partial \boldsymbol{\omega}}{\partial \dot{q}_i} \cdot \mathbf{I} \cdot \frac{\partial \boldsymbol{\omega}}{\partial \dot{q}_N} \right] \ddot{\mathbf{q}} + \left[\frac{\partial \boldsymbol{\omega}}{\partial \dot{q}_i} \cdot \mathbf{I} \cdot \frac{\partial \boldsymbol{\omega}}{\partial q_1} + \boldsymbol{\omega} \cdot \mathbf{I} \cdot \frac{\partial^2 \boldsymbol{\omega}}{\partial \dot{q}_i \partial q_1} \cdots \frac{\partial \boldsymbol{\omega}}{\partial \dot{q}_i} \cdot \mathbf{I} \cdot \frac{\partial \boldsymbol{\omega}}{\partial q_N} + \boldsymbol{\omega} \cdot \mathbf{I} \cdot \frac{\partial^2 \boldsymbol{\omega}}{\partial \dot{q}_i \partial q_N} \right] \dot{\mathbf{q}} - \boldsymbol{\omega} \cdot \mathbf{I} \cdot \frac{\partial \boldsymbol{\omega}}{\partial q_i}, \tag{13}$$

which describes format-II. Note that $\boldsymbol{\omega}$ is differentiated in the body frame.

Similarly, considering (11c), we can obtain:

$$\begin{aligned}
\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial a_3}{\partial \dot{q}_i}\right) - \frac{\partial a_3}{\partial q_i} &= \left[\frac{\partial \mathbf{R}_{C_0}}{\partial q_i}\cdot\sum_k m_k \frac{\partial \mathbf{r}_k}{\partial q_1}\cdots\frac{\partial \mathbf{R}_{C_0}}{\partial q_i}\cdot\sum_k m_k \frac{\partial \mathbf{r}_k}{\partial q_N}\right]\ddot{\mathbf{q}} \\
&+ \left[\frac{\partial \mathbf{R}_{C_0}}{\partial q_1}\cdot\sum_k m_k \frac{\partial \mathbf{r}_k}{\partial q_i}\cdots\frac{\partial \mathbf{R}_{C_0}}{\partial q_N}\cdot\sum_k m_k \frac{\partial \mathbf{r}_k}{\partial q_i}\right]\ddot{\mathbf{q}} \\
&+ \left[\frac{\partial \mathbf{R}_{C_0}}{\partial q_i}\cdot\sum_k m_k\left(\sum_{s=1}^{N}\frac{\partial^2 \mathbf{r}_k}{\partial q_1 \partial q_s}\dot{q}_s\right)\cdots\frac{\partial \mathbf{R}_{C_0}}{\partial q_i}\right. \\
&\left.\times\sum_k m_k\left(\sum_{s=1}^{N}\frac{\partial^2 \mathbf{r}_k}{\partial q_N \partial q_s}\dot{q}_s\right)\right]\dot{\mathbf{q}} \\
&+ \left[\left(\sum_{s=1}^{N}\frac{\partial^2 \mathbf{R}_{C_0}}{\partial q_1 \partial q_s}\dot{q}_s\right)\cdot\sum_k m_k\frac{\partial \mathbf{r}_k}{\partial q_i}\cdots\left(\sum_{s=1}^{N}\frac{\partial^2 \mathbf{R}_{C_0}}{\partial q_N \partial q_s}\dot{q}_s\right)\right. \\
&\left.\times\sum_k m_k\frac{\partial \mathbf{r}_k}{\partial q_i}\right]\dot{\mathbf{q}}, \qquad (14)
\end{aligned}$$

which describes format-III.

Next, to obtain the system dynamics model key matrices, the original terms in the system kinetic energy as obtained in (6) are substituted into the corresponding format. Then, following the structure of dynamics model presented in (8), appropriate terms are collected together.

4. EXPLICIT DYNAMICS MODEL

4.1. Mass matrix

To obtain the mass matrix $\mathbf{H}$, according to (8), the acceleration terms in each of the three formats have to be considered. Therefore, H_{ij} is computed by:

- Substituting each term of the system kinetic energy into the corresponding format.
- Finding the coefficients of $\ddot{\mathbf{q}}$ in each format.
- Adding the results, obtained from the three formats, for each term.
- Adding the results, obtained for all of the terms.

Disregarding the details, this procedure eventually yields:

$$H_{ij} = M\frac{\partial \mathbf{R}_{C_0}}{\partial q_i}\cdot\frac{\partial \mathbf{R}_{C_0}}{\partial q_j} + \frac{{}^0\partial \boldsymbol{\omega}_0}{\partial \dot{q}_i}\cdot \mathbf{I}_0 \cdot \frac{{}^0\partial \boldsymbol{\omega}_0}{\partial \dot{q}_j}$$
$$+ \sum_{m=1}^{n}\sum_{k=1}^{N_m}\left(m_k^{(m)}\frac{\partial \mathbf{r}_{C_k}^{(m)}}{\partial q_i}\cdot\frac{\partial \mathbf{r}_{C_k}^{(m)}}{\partial q_j} + \frac{{}^k\partial \boldsymbol{\omega}_k^{(m)}}{\partial \dot{q}_i}\cdot \mathbf{I}_k^{(m)}\cdot\frac{{}^k\partial \boldsymbol{\omega}_k^{(m)}}{\partial \dot{q}_j}\right)$$
$$+ \left(\sum_{m=1}^{n}\sum_{k=1}^{N_m} m_k^{(m)}\frac{\partial \mathbf{r}_{C_k}^{(m)}}{\partial q_i}\right)\cdot\frac{\partial \mathbf{R}_{C_0}}{\partial q_j} + \left(\sum_{m=1}^{n}\sum_{k=1}^{N_m} m_k^{(m)}\frac{\partial \mathbf{r}_{C_k}^{(m)}}{\partial q_j}\right)\cdot\frac{\partial \mathbf{R}_{C_0}}{\partial q_i}, \quad (15)$$

where $\boldsymbol{\omega}_k^{(m)}$ is given by (4) and $\mathbf{r}_{C_k}^{(m)}$ can be substituted from (see Fig. 1):

$$\mathbf{r}_{C_i}^{(m)} = \mathbf{r}_0^{(m)} + \sum_{k=1}^{i-1}\left(\mathbf{r}_k^{(m)} - \mathbf{l}_k^{(m)}\right) - \mathbf{l}_i^{(m)} \begin{cases} m = 1, \ldots, n \\ i = 1, \ldots, N_m. \end{cases} \quad (16)$$

The notation employed here is consistent to Kane and Levinson [10], i.e. a left superscript on partial derivatives refers to the frame in which the differentiation has to be taken, whereas for the inertial frame it is left as blank.

4.2. Vector of non-linear terms

The vector of non-linear velocity terms in (8) can be computed by dropping the acceleration terms in each of the obtained formats. So, C_i is computed following the same procedure as described for computation of the H_{ij}, by considering the coefficients of $\dot{\mathbf{q}}$ and any other term (except those which correspond to $\ddot{\mathbf{q}}$) in each format. This approach yields:

$$\mathbf{C}(\boldsymbol{\delta}_0, \dot{\boldsymbol{\delta}}_0, \boldsymbol{\theta}, \dot{\boldsymbol{\theta}}) = \mathbf{C}_1(\boldsymbol{\delta}_0, \dot{\boldsymbol{\delta}}_0, \boldsymbol{\theta}, \dot{\boldsymbol{\theta}})\dot{\mathbf{q}} + \mathbf{C}_2(\boldsymbol{\delta}_0, \dot{\boldsymbol{\delta}}_0, \boldsymbol{\theta}, \dot{\boldsymbol{\theta}}), \quad (17a)$$

where:

$$\mathrm{C}_{1_{ij}} = M\frac{\partial \mathbf{R}_{C_0}}{\partial q_i}\cdot\left(\sum_{s=1}^{N}\frac{\partial^2 \mathbf{R}_{C_0}}{\partial q_s \partial q_j}\dot{q}_s\right) + \frac{{}^0\partial \boldsymbol{\omega}_0}{\partial \dot{q}_i}\cdot \mathbf{I}_0\cdot\frac{{}^0\partial \boldsymbol{\omega}_0}{\partial q_j} + \boldsymbol{\omega}_0\cdot \mathbf{I}_0\cdot\frac{{}^0\partial^2 \boldsymbol{\omega}_0}{\partial \dot{q}_i \partial q_j}$$
$$+ \frac{\partial \mathbf{R}_{C_0}}{\partial q_i}\cdot\sum_{m=1}^{n}\sum_{k=1}^{N_m}\left(m_k^{(m)}\sum_{s=1}^{N}\frac{\partial^2 \mathbf{r}_{C_k}^{(m)}}{\partial q_s \partial q_j}\dot{q}_s\right)$$
$$+ \left(\sum_{s=1}^{N}\frac{\partial^2 \mathbf{r}_{C_k}^{(m)}}{\partial q_s \partial q_i}\dot{q}_s\right)\cdot\sum_{m=1}^{n}\sum_{k=1}^{N_m}\left(m_k^{(m)}\frac{\partial \mathbf{r}_{C_k}^{(m)}}{\partial q_j}\right)$$
$$+ \sum_{m=1}^{n}\sum_{k=1}^{N_m}\left(m_k^{(m)}\frac{\partial \mathbf{r}_{C_k}^{(m)}}{\partial q_i}\cdot\left(\sum_{s=1}^{N}\frac{\partial^2 \mathbf{r}_{C_k}^{(m)}}{\partial q_s \partial q_j}\dot{q}_s\right)\right.$$
$$\left. + \frac{{}^k\partial \boldsymbol{\omega}_k^{(m)}}{\partial \dot{q}_i}\cdot \mathbf{I}_k^{(m)}\cdot\frac{{}^k\partial \boldsymbol{\omega}_k^{(m)}}{\partial q_j} + \boldsymbol{\omega}_k^{(m)}\cdot \mathbf{I}_k^{(m)}\cdot\frac{{}^k\partial^2 \boldsymbol{\omega}_k^{(m)}}{\partial \dot{q}_i \partial q_j}\right), \quad (17b)$$

and:

$$\mathrm{C}_{2_i} = -\left(\boldsymbol{\omega}_0 \cdot \mathbf{I}_0 \cdot \frac{{}^0\partial \boldsymbol{\omega}_0}{\partial q_i} + \sum_{m=1}^{n} \sum_{k=1}^{N_m} \boldsymbol{\omega}_k^{(m)} \cdot \mathbf{I}_k^{(m)} \cdot \frac{{}^k\partial \boldsymbol{\omega}_k^{(m)}}{\partial q_i}\right). \tag{17c}$$

Note that expressing the angular velocity as a function of the Euler rates, vector $\mathbf{C}_2$ can be combined with the first term of (17a). Then, the vector of non-linear velocity terms can be written as:

$$\mathbf{C}(\boldsymbol{\delta}_0, \dot{\boldsymbol{\delta}}_0, \boldsymbol{\theta}, \dot{\boldsymbol{\theta}}) = \breve{\mathbf{C}}(\boldsymbol{\delta}_0, \dot{\boldsymbol{\delta}}_0, \boldsymbol{\theta}, \dot{\boldsymbol{\theta}})\dot{\mathbf{q}}. \tag{18}$$

This is a representation of non-linear velocity terms which is preferred in the development of adaptive control algorithms.

4.3. Vector of generalized forces

As described in (10), if all external forces except the ones applied on the spacecraft are zero, the vector of generalized forces $\mathbf{Q}$ is written as:

$$\mathbf{Q} = \mathbf{J}_Q \begin{Bmatrix} {}^0\mathbf{f}_\mathrm{s} \\ {}^0\mathbf{n}_\mathrm{s} \\ \boldsymbol{\tau}_{K\times 1} \end{Bmatrix} = \begin{Bmatrix} \mathbf{0}_{6\times 1} \\ \boldsymbol{\tau}_{K\times 1} \end{Bmatrix} + \mathbf{J}_0^\mathrm{T} \begin{Bmatrix} {}^0\mathbf{f}_\mathrm{s} \\ {}^0\mathbf{n}_\mathrm{s} \end{Bmatrix}. \tag{19}$$

Assuming that ${}^0\mathbf{f}_\mathrm{s}$ and ${}^0\mathbf{n}_\mathrm{s}$ are applied at the spacecraft center of mass, $\mathbf{J}_0$ is defined as:

$$\begin{Bmatrix} {}^0\dot{\mathbf{R}}_{C_0} \\ {}^0\boldsymbol{\omega}_0 \end{Bmatrix} = \mathbf{J}_0 \dot{\mathbf{q}}. \tag{20a}$$

Then, $\mathbf{J}_0$ can be obtained as:

$$\mathbf{J}_0 = \begin{bmatrix} \mathbf{T}_0^\mathrm{T} & \mathbf{0}_{3\times 3} & \mathbf{0}_{3\times 3} \\ \mathbf{0}_{3\times 3} & \mathbf{S}_0 & \mathbf{0}_{3\times K} \end{bmatrix}_{6\times N}, \tag{20b}$$

where $\mathbf{S}_0(\boldsymbol{\delta}_0)$ is a 3×3 matrix, see Meirovitch [11], relating the spacecraft angular velocity to the corresponding Euler rates as:

$${}^0\boldsymbol{\omega}_0 = \mathbf{S}_0(\boldsymbol{\delta}_0)\dot{\boldsymbol{\delta}}_0. \tag{20c}$$

Therefore, $\mathbf{J}_Q$ is obtained as:

$$\mathbf{J}_Q = \begin{bmatrix} \mathbf{T}_0 & \mathbf{0}_{3\times 3} & \mathbf{0}_{3\times K} \\ \mathbf{0}_{3\times 3} & \mathbf{S}_0^\mathrm{T} & \mathbf{0}_{3\times K} \\ \mathbf{0}_{K\times 3} & \mathbf{0}_{K\times 3} & \mathbf{1}_{K\times K} \end{bmatrix}_{N\times N}, \tag{21}$$

which can be substituted into (19) to obtain $\mathbf{Q}$. This completes the derivation of the dynamics model for a multiple-arm SFFR with rigid elements. Note that computation of the obtained dynamics equations can be done either by numerical or symbolical programming tools. Obtaining the system response using analytical expressions was employed in developing SPACEMAPLE and is discussed later in this paper.

5. DYNAMICS SPECIFICATIONS IN SPACE

Unlike fixed-base manipulators, in space robotic systems any motion of a single link creates a reactional motion of the whole system. In free-floating mode, where no external force is applied on the system, the motion is dynamically constrained, i.e. total linear and angular momentum of the system is conserved. Also, the Jacobian matrix as obtained in Moosavian and Papadopoulos [6] becomes mass dependent. In other words, the inertial linear velocity of an arbitrary point P and angular velocity of the corresponding body is affected by the mass distribution over the entire system. Surprisingly, this correlation between arms and the free base also affects the relative motion of the end-effector with respect to the base. This is due to the fact that joint angles and rates are dynamically coupled, even though the relative motion can be expressed in terms of a fixed-base-type Jacobian. To observe specific characteristics of space robotic systems, in a more vigorous investigation, elements of the dynamics model for a fixed-base manipulator can be compared to those of a space robotic system as follows.

For a fixed-base serial manipulator, as shown in Asada and Slotine [12], the mass matrix $\mathbf{H}$, and the vector of non-linear velocity terms $\mathbf{C}$, can be obtained as:

$$\mathbf{H} = \sum_{i=1}^{N} \left(m_i \mathbf{J}_L^{(i)\mathrm{T}} \mathbf{J}_L^{(i)} + \mathbf{J}_A^{(i)\mathrm{T}}\, {}^0\mathbf{I}_i^{CM_i} \mathbf{J}_A^{(i)} \right), \tag{22a}$$

$$C_i = \sum_{j=1}^{N} \sum_{k=1}^{N} m_{ijk} \dot{q}_k \dot{q}_j, \tag{22b}$$

where:

$$\mathbf{J}_L^{(i)} = \left\lfloor \left(\left[{}^0\mathbf{z}_1 \right]^{\times} {}^0\mathbf{P}_{CM_i}^1 \right) \cdots \left(\left[{}^0\mathbf{z}_i \right]^{\times} {}^0\mathbf{P}_{CM_i}^i \right) \mathbf{0}_{3\times 1} \cdots \mathbf{0}_{3\times 1} \right\rfloor_{3\times N}, \tag{23a}$$

$$\mathbf{J}_A^{(i)} = \left\lfloor {}^0\mathbf{z}_1 \cdots {}^0\mathbf{z}_i \mathbf{0}_{3\times 1} \cdots \mathbf{0}_{3\times 1} \right\rfloor_{3\times N}, \tag{23b}$$

$$m_{ijk} = \frac{\partial H_{ij}}{\partial q_k} - \frac{1}{2} \frac{\partial H_{jk}}{\partial q_i}, \tag{23c}$$

and:

$${}^0\mathbf{P}_{CM_i}^j = {}^0\mathbf{T}_j \left({}^j\mathbf{P}_{CM_i}^j \right), \tag{23d}$$

m_i is the ith link mass, ${}^0\mathbf{I}_i^{CM_i}$ is its inertia matrix with respect to the center of mass expressed in the fixed frame, ${}^0\mathbf{z}_i$ is a unit vector along the ith joint axis expressed in the fixed frame, ${}^{j}\mathbf{P}^{j}_{CM_i}$ is the position vector of the ith center of mass with respect to the origin of the jth frame as seen in that frame, and ${}^{0}\mathbf{T}_j$ is the rotation matrix between the jth frame and the fixed one. It can be shown that the obtained H_{ij} and C_i for a fixed-base manipulator are functions of specific set of mass parameters as:

$$H_{ij} = h_{ij}(\tilde{m}_k, \ldots, \tilde{m}_N) h'_{ij}(\theta_1, \ldots, \theta_N) \quad k = \max(i, j), \tag{24a}$$

$$C_i = f_i(\tilde{m}_i, \ldots, \tilde{m}_N) f'_i(\theta_1, \ldots, \theta_N, \dot{\theta}_1, \ldots, \dot{\theta}_N), \tag{24b}$$

where h_{ij}, h'_{ij}, f_i, and f'_i are functions of the given arguments, $\tilde{m}_i$ denotes ith link mass properties (both mass and inertia), and $\boldsymbol{\theta}_i$ is the ith joint variable. As it is seen, mass properties have a backward propagation effect on the dynamics model. In other words, mass properties of link 'i' do not appear in the $\mathbf{H}$ elements which correspond to posterior joint variables, i.e. $i + 1, \ldots, N$. For instance, mass properties of the first link only appear in H_{11} and C_1. On the contrary, for space manipulators in the free-floating mode, this is no longer true and every element of the dynamics model is affected by mass properties of all links. Therefore, any deviation in the estimation of mass parameters has a more drastic effect on the performance of model-based control algorithms in space. This makes dynamics modeling of such systems a very important stage for the development of appropriate control algorithms in space.

6. GENERATION OF THE SYMBOLIC CODE: SPACEMAPLE

As mentioned earlier, computation of the obtained dynamics equations can be done either numerically or symbolically. The latter is chosen here to develop a symbolic code called SPACEMAPLE. However, to compare the two programming approaches, the required steps in numerical computation of the obtained dynamics is first reviewed. To this end, the preparation of few sample terms, i.e. ${}^{k}\partial\boldsymbol{\omega}_k^{(m)}/\partial q_i$ and ${}^{k}\partial\boldsymbol{\omega}_k^{(m)}/\partial \dot{q}_i$, for numerical computation is discussed. In a similar way, other terms in H_{ij}, C_i and $\mathbf{J}_Q$ can be obtained, and programmed in the corresponding environment.

First, preliminary calculations for numerical computer programming of ${}^{k}\partial\boldsymbol{\omega}_k^{(m)}/\partial q_i$ and ${}^{k}\partial\boldsymbol{\omega}_k^{(m)}/\partial \dot{q}_i$ is presented. Following the arrangement of (7) for the vector of generalized coordinates, the angular velocity of the kth link of the mth manipulator expressed in its own body-fixed frame, ${}^{k}\boldsymbol{\omega}_k^{(m)}$, can be obtained as:

$$\begin{aligned} {}^{k}\boldsymbol{\omega}_k^{(m)} &= {}^{k-1}\mathbf{T}_k^{(m)\mathrm{T}}\, {}^{k-2}\mathbf{T}_{k-1}^{(m)\mathrm{T}} \cdots {}^{0}\mathbf{T}_1^{(m)\mathrm{T}} \mathbf{S}_0 \dot{\boldsymbol{\delta}}_0 \\ &\quad + \sum_{s=1}^{k-1} \left({}^{k-1}\mathbf{T}_k^{(m)\mathrm{T}}\, {}^{k-2}\mathbf{T}_{k-1}^{(m)\mathrm{T}} \cdots {}^{s}\mathbf{T}_{s+1}^{(m)\mathrm{T}} \dot{\boldsymbol{\theta}}_s^{(m)}\, {}^{s}\mathbf{z}_s^{(m)} \right) + \dot{\boldsymbol{\theta}}_k^{(m)}\, {}^{k}\mathbf{z}_k^{(m)}, \end{aligned} \tag{25}$$

where $\mathbf{S}_0$ has been already defined in (20c), ${}^{i-1}\mathbf{T}_i^{(m)}$ is rotation matrix between the ith body-fixed frame and the previous frame, and ${}^{i}\mathbf{z}_i^{(m)} \equiv (0, 0, 1)^{\mathrm{T}}$ is a unit vector along axis of rotation of the ith joint of the mth manipulator expressed in its own body-fixed frame. Therefore, one obtains:

$$\frac{{}^{k}\partial \boldsymbol{\omega}_k^{(m)}}{\partial q_i^{(p)}} = \begin{cases} \mathbf{s}_1 & \text{if } p = 0 \\ \mathbf{0} & \text{if } (p \neq 0 \text{ and } p \neq m) \\ \mathbf{s}_2 & \text{if } (p = m \text{ and } i < k) \\ \mathbf{0} & \text{if } (p = m \text{ and } i > k) \\ \mathbf{s}_3 & \text{if } (p = m \text{ and } i = k), \end{cases} \tag{26}$$

where:

$$\mathbf{s}_1 = {}^{k-1}\mathbf{T}_k^{(m)\mathrm{T}}\, {}^{k-2}\mathbf{T}_{k-1}^{(m)\mathrm{T}} \cdots {}^{0}\mathbf{T}_1^{(m)\mathrm{T}} \frac{\partial \mathbf{S}_0}{\partial q_i^{(0)}} \dot{\boldsymbol{\delta}}_0, \tag{27a}$$

$$\begin{aligned} \mathbf{s}_2 = {} & {}^{k-1}\mathbf{T}_k^{(m)\mathrm{T}} \cdots {}^{i}\mathbf{T}_{i+1}^{(m)\mathrm{T}} \left(\frac{\partial\, {}^{i-1}\mathbf{T}_i^{(m)\mathrm{T}}}{\partial q_i^{(m)}} \right) {}^{i-2}\mathbf{T}_{i-1}^{(m)\mathrm{T}} \cdots {}^{0}\mathbf{T}_1^{(m)\mathrm{T}} \mathbf{S}_0 \dot{\boldsymbol{\delta}}_0 \\ & + \sum_{s=1}^{i-1} \left({}^{k-1}\mathbf{T}_k^{(m)\mathrm{T}} \cdots {}^{i}\mathbf{T}_{i+1}^{(m)\mathrm{T}} \left(\frac{\partial\, {}^{i-1}\mathbf{T}_i^{(m)\mathrm{T}}}{\partial q_i^{(m)}} \right) {}^{i-2}\mathbf{T}_{i-1}^{(m)\mathrm{T}} \cdots {}^{s}\mathbf{T}_{s+1}^{(m)\mathrm{T}} \dot{\boldsymbol{\theta}}_s^{(m)}\, {}^{s}\mathbf{z}_s^{(m)} \right), \end{aligned} \tag{27b}$$

$$\begin{aligned} \mathbf{s}_3 = {} & \frac{\partial\, {}^{k-1}\mathbf{T}_k^{(m)\mathrm{T}}}{\partial q_k^{(m)}} {}^{k-2}\mathbf{T}_{k-1}^{(m)\mathrm{T}} \cdots {}^{0}\mathbf{T}_1^{(m)\mathrm{T}} \mathbf{S}_0 \dot{\boldsymbol{\delta}}_0 \\ & + \sum_{s=1}^{k-1} \left(\frac{\partial\, {}^{k-1}\mathbf{T}_k^{(m)\mathrm{T}}}{\partial q_k^{(m)}} {}^{k-2}\mathbf{T}_{k-1}^{(m)\mathrm{T}} \cdots {}^{s}\mathbf{T}_{s+1}^{(m)\mathrm{T}} \dot{\boldsymbol{\theta}}_s^{(m)}\, {}^{s}\mathbf{z}_s^{(m)} \right). \end{aligned} \tag{27c}$$

Similarly, we can obtain:

$$\frac{{}^{k}\partial \boldsymbol{\omega}_k^{(m)}}{\partial \dot{q}_i^{(p)}} = \begin{cases} \mathbf{s}_1^* & \text{if } p = 0 \\ \mathbf{0} & \text{if } (p \neq 0 \text{ and } p \neq m) \\ \mathbf{s}_2^* & \text{if } (p = m \text{ and } i < k) \\ \mathbf{0} & \text{if } (p = m \text{ and } i > k) \\ \mathbf{s}_3^* & \text{if } (p = m \text{ and } i = k), \end{cases} \tag{28}$$

where:

$$\mathbf{s}_1^* = {}^{k-1}\mathbf{T}_k^{(m)\mathrm{T}}\, {}^{k-2}\mathbf{T}_{k-1}^{(m)\mathrm{T}} \cdots {}^{0}\mathbf{T}_1^{(m)\mathrm{T}} \mathbf{S}_0 \frac{\partial \dot{\boldsymbol{\delta}}_0}{\partial \dot{q}_i^{(0)}}, \tag{29a}$$

$$\mathbf{s}_2^* = {}^{k-1}\mathbf{T}_k^{(m)\mathrm{T}}\, {}^{k-2}\mathbf{T}_{k-1}^{(m)\mathrm{T}} \cdots {}^{i}\mathbf{T}_{i+1}^{(m)\mathrm{T}}\, {}^{i}\mathbf{z}_i^{(m)}, \tag{29b}$$

$$\mathbf{s}_3^* = {}^{k}\mathbf{z}_k^{(m)}. \tag{29c}$$

Note that $\mathbf{S}_0$ is a function of $\boldsymbol{\delta}_0$ and ${}^{i-1}\mathbf{T}_i^{(m)}$ is just a function of $\mathbf{q}_i^{(m)}$. Therefore, $\partial^{i-1}\mathbf{T}_i^{(m)\mathrm{T}}/\partial q_i^{(m)}$, $\partial \mathbf{S}_0/\partial q_i^{(0)}$ and $\partial \dot{\boldsymbol{\delta}}_0/\partial \dot{q}_i^{(0)}$ can be calculated analytically, and substituted into (27) and (29). Other terms in H_{ij}, C_i and $\mathbf{J}_Q$ can also be calculated in a similar way. The obtained results can then be programmed in a numerical environment, to quantify the system dynamics.

Although numerical derivation seems a cumbersome procedure, it would be the only choice if symbolical programming tools were not available. Note that for the numerical development of the dynamic properties of mechanical manipulators, existing recursive algorithms can be followed. To solve direct dynamics, these algorithms utilize iterative routines for inverse dynamics, and joint forces and torques inputs. For instance, a computer code has been developed and employed for a dynamical study of the first element launch (FEL) configuration of the Space Station Freedom by Grewal and Modi [13]. Similarly, Anderson and Duan [14] have implemented parallel computational algorithms for dynamics of multiple rigid-body systems. For further details, one can see a comparison of different methods for developing the dynamics of rigid-body systems presented by Ju and Mansour [15]. Here, our focus is on the computation of the explicit dynamics model based on Lagrange formulation. However, by means of symbolical tools, each term can be analytically calculated in a computer program. For instance, (25) can be directly programmed to represent ${}^k\boldsymbol{\omega}_k^{(m)}$. Then, $\partial^k\boldsymbol{\omega}_k^{(m)}/\partial q_i$ and $\partial^k\boldsymbol{\omega}_k^{(m)}/\partial \dot{q}_i$ will be analytically calculated in a single step, rather than going through different options in (26) and (28). Furthermore, using mathematical identities and factorization techniques, the result can be simplified to reduce the resulting analytical expressions.

According to the above reasons, derivation of the developed dynamics equations of motion has been programmed in a symbolic environment (MAPLE) for a multiple-manipulator SFFR with rigid elements in a general configuration. The code (SPACEMAPLE) yields the mass matrix $\mathbf{H}$, the vector of non-linear velocity terms $\mathbf{C}$, the Jacobian matrix $\mathbf{J}_Q$ used in describing the vector of generalized forces, the Jacobian matrix $\mathbf{J}_c$ which describes the task space (employed in different control algorithms) and its time derivative $\dot{\mathbf{J}}_c$, each one as an analytical function of generalized coordinates/speeds.

The program, as depicted in Fig. 2, is initiated by determining the system general configuration, i.e. number of manipulators/appendages, number of links for each one, and d.o.f. for the spacecraft (i.e. 3 for planar motions or 6 for three-dimensional maneuvers). Then, mass properties and geometric parameters for each element of the system are specified. These parameters can be substituted by numerical quantities or left as parameters. The latter results in longer expressions, while the first one yields more concise results, particularly when some components of geometric vectors or inertia matrices are zero. In fact, in most studies the dynamics has to be modeled for a specific system, and then employed in simulation and control investigations. Usually, for these investigations, the simulation routine has to be run tens of times. Therefore, it is preferable to substitute the system parameters by their

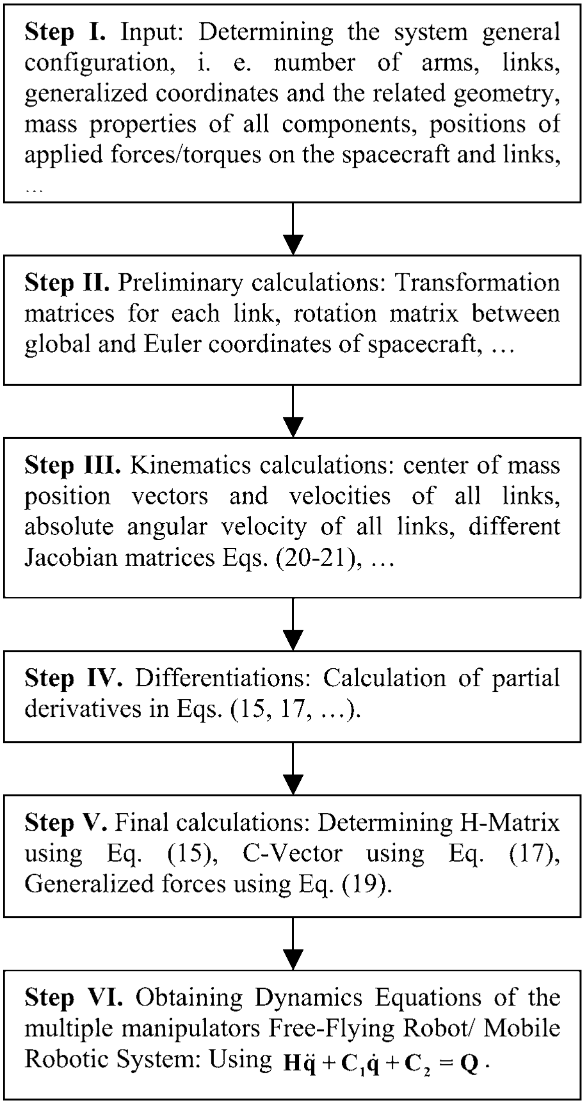

Figure 2. Basic steps of SPACEMAPLE.

values at the very beginning and make it more concise. The cost is just running SPACEMAPLE once some changes in the system parameters have to be made.

To simplify the obtained analytical expressions, at each intermediate step, mathematical tools and factorization techniques available in MAPLE are used. The result of this fairly refined code is an analytical dynamics model of any specified multiple manipulator SFFR with rigid elements in terms of generalized coordinates/speeds. The resulting models were used in simulation and control investigations, as presented in Moosavian and Rastegari [16].

Table 1.
Typical results of verification procedure for SPACEMAPLE

Row	**C**	**H**				
		First column	Second column	Third column	Fourth column	Fourteenth column
1	−0.13E−14	0.0	0.0	0.0	0.11E−13	0.0
2	0.18E−14	0.0	0.0	0.0	0.71E−14	0.0
3	−0.47E−14	0.0	0.0	0.0	0.0	0.0
4	−0.27E−14	0.11E−13	0.71E−14	0.0	0.14E−13	0.56E−16
5	−0.27E−14	0.18E−14	0.0	−0.14E−13	0.18E−14	0.28E−16
6	0.18E−14	0.10E−14	−0.18E−14	0.0	−0.36E−14	0.0
7	0.0	0.0	−0.78E−15	0.0	−0.18E−14	0.0
8	−0.44E−15	−0.83E−15	0.36E−14	−0.67E−15	0.36E−14	0.0
9	0.0	−0.67E−15	0.44E−15	−0.22E−15	0.0	0.0
10	−0.13E−14	0.11E−14	0.89E−15	−0.89E−15	0.18E−14	0.0
11	0.17E−15	0.0	−0.18E−14	−0.44E−15	−0.38E−14	0.0
12	0.39E−15	0.0	−0.39E−15	0.11E−15	−0.78E−15	0.0
13	0.28E−16	0.0	0.0	0.0	−0.28E−16	−0.69E−17
14	−0.35E−16	0.0	0.0	0.0	0.56E−16	0.28E−16

Undoubtedly, before using the obtained models via SPACEMAPLE the code had to be verified, which was performed in a vigorous way. In brief, SPACEMAPLE was used for fixed-base systems which represent limiting cases of space robotic systems (letting spacecraft mass go to infinity). The output results were verified by comparisons to those obtained by hand calculations. However, since in these limiting cases most of the terms in the dynamics equations vanish, the model must be verified also in a general case, i.e. for a multiple-manipulator space robotic system. This was done by developing another simpler code at a very fundamental level, and comparing the numerical results of the two for a large number of cases in different systems and configurations. The simpler code computes the system kinetic energy, using (6), and substitutes the result directly into the equations of motion, (1). Obviously, such code yields very long equations of motion, compared to the very compact ones of SPACEMAPLE. However, the simplicity of this code makes it fairly reliable, so that it can be employed as a yardstick for the verification procedure. In fact, this was a very helpful approach in identifying minor mistakes at various levels and verifying SPACEMAPLE at the end.

Table 1 shows typical results of this verification procedure for SPACEMAPLE, i.e. the difference between obtained results of SPACEMAPLE and those of the simple code, for a 14-d.o.f. space robotic system (with three manipulators/appendages) shown in Fig. 3. As it is seen, the differences between obtained vectors of non-linear velocity terms (**C**) and a few sample columns of two mass matrices (**H**) are zero, either exactly or approximately (order of 10^{-13} and lower which are due to truncations). Although these results correspond to a single random set of generalized coordinates/speeds (with non-zero entries), the differences are of the

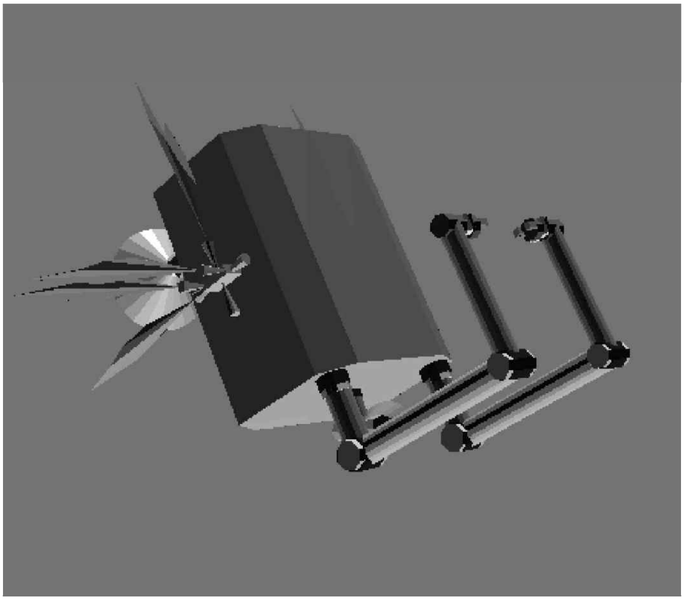

Figure 3. A three manipulator/appendage space free-flyer considered for the verification procedure.

same order for several other trials. Therefore, it can be concluded that the developed SPACEMAPLE code is free of errors.

7. CONCLUSIONS

The general Lagrangian formulation was applied to obtain an explicit dynamics model of a multiple manipulator SFFR, which is very useful for dynamics analyses, design studies and the development of model-based control algorithms for such complex systems. To model the system dynamics, mathematical analyses showed the existence of three key type of terms and three different formats were developed to apply on these. Separate calculations of the mass matrix, vector of non-linear velocity terms and generalized forces were presented, and the obtained results were assembled to obtain the dynamics model.

It was also shown that for a fixed-base manipulator, mass properties have a backward propagation effect on the elements of the mass matrix and the vector of non-linear velocity terms, while these elements are affected by the mass properties of all links for a space manipulator. Therefore, any deviation in the estimation of mass parameters has a more drastic effect on the performance of model-based control algorithms in space. Computation of the obtained dynamics can be done either by numerical or symbolical programming tools. It was shown that calculation of each term for numerical programming usually divides into several branches, while by means of the symbolical tools, each term can be analytically calculated. Also, using different mathematical identities and factorization techniques, the result can be simplified to reduce the obtained analytical expressions. Therefore, derivation of the dynamics equations has been programmed symbolically as SPACEMAPLE for a general multiple-manipulator space robotic system with rigid elements. Developing another simpler code and comparing the numerical results of the two in numerous cases vigorously verified the code. Research institutes interested in obtaining SPACEMAPLE, should send their requests to the authors.

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APPENDIX: THREE FORMATS USED IN MATHEMATICAL ANALYSES

As discussed in Section 3, the system kinetic energy is composed of three typical terms, which have to be differentiated according to (1). Differentiation of these terms, is presented in this Appendix to obtain three formats as used in deriving the system dynamics model.

Considering the first typical term, as given in (11a) and repeated here:

$$a_1 = \frac{1}{2} m \dot{\mathbf{r}} \cdot \dot{\mathbf{r}}. \tag{11a}$$

Its differentiation with respect to $\dot{q}_i$ as an arbitrary generalized speed is obtained as:

$$\frac{\partial a_1}{\partial \dot{q}_i} = m \frac{\partial \dot{\mathbf{r}}}{\partial \dot{q}_i} \cdot \dot{\mathbf{r}}. \tag{A.1}$$

It should be noted that for the implementation of the following formulation, $\mathbf{r}$ has to be differentiated in the inertial frame.[1] Then, $\dot{\mathbf{r}} = \mathrm{d}\mathbf{r}/\mathrm{d}t$ can be calculated as:

$$\dot{\mathbf{r}} = \sum_{s=1}^{N} \frac{\partial \mathbf{r}}{\partial q_s} \dot{q}_s, \tag{A.2}$$

which yields:

$$\frac{\partial \dot{\mathbf{r}}}{\partial \dot{q}_i} = \frac{\partial \mathbf{r}}{\partial q_i}. \tag{A.3}$$

Substitution of (A.3) into (A.1) yields:

$$\frac{\partial a_1}{\partial \dot{q}_i} = m \frac{\partial \mathbf{r}}{\partial q_i} \cdot \dot{\mathbf{r}}, \tag{A.4}$$

[1] If $\mathbf{r}$ is not differentiated in the inertial frame, then:

$$\dot{\mathbf{r}} = {}^{\mathrm{B}}\dot{\mathbf{r}} + \boldsymbol{\omega}_{\mathrm{B}} \times \mathbf{r}, \tag{A.1a}$$

where ${}^{\mathrm{B}}\dot{\mathbf{r}}$ is time differentiation of $\mathbf{r}$ when expressed in a frame B which has an angular velocity of $\boldsymbol{\omega}_{\mathrm{B}}$ with respect to the inertial frame, and can be computed as

$${}^{\mathrm{B}}\dot{\mathbf{r}} = \sum_{s=1}^{N} \frac{{}^{\mathrm{B}}\partial \mathbf{r}}{\partial q_s} \dot{q}_s + \frac{{}^{\mathrm{B}}\partial \mathbf{r}}{\partial t}. \tag{A.1b}$$

Note that a left superscript on partial derivatives denotes the frame in which the differentiation has to be taken. Therefore, unless ${}^{\mathrm{B}}\partial \mathbf{r}/\partial t = \mathbf{0}$, it can be seen that:

$$\frac{{}^{\mathrm{B}}\partial \dot{\mathbf{r}}}{\partial q_s} \neq \frac{{}^{\mathrm{B}}\partial \mathbf{r}}{\partial q_s}, \tag{A.1c}$$

which necessitates the condition of differentiating $\mathbf{r}$ in the inertial frame, for writing (A.2) and (A.3).

which can be differentiated with respect to time, to obtain:

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial a_1}{\partial \dot{q}_i}\right) = m\frac{\partial \dot{\mathbf{r}}}{\partial q_i}\cdot\dot{\mathbf{r}} + m\frac{\partial \mathbf{r}}{\partial q_i}\cdot\ddot{\mathbf{r}}. \tag{A.5}$$

Also, a_1 can be differentiated with respect to q_i as an arbitrary generalized coordinate:

$$\frac{\partial a_1}{\partial q_i} = m\frac{\partial \dot{\mathbf{r}}}{\partial q_i}\cdot\dot{\mathbf{r}}. \tag{A.6}$$

Therefore, based on (A.5) and (A.6), we can writte:

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial a_1}{\partial \dot{q}_i}\right) - \frac{\partial a_1}{\partial q_i} = m\frac{\partial \mathbf{r}}{\partial q_i}\cdot\ddot{\mathbf{r}}, \tag{A.7}$$

where $\ddot{\mathbf{r}}$ can be obtained as:

$$\ddot{\mathbf{r}} = \sum_{s=1}^{N}\left\{\frac{\partial}{\partial q_s}\left(\sum_{t=1}^{N}\frac{\partial \mathbf{r}}{\partial q_t}\dot{q}_t\right)\dot{q}_s + \frac{\partial \mathbf{r}}{\partial q_s}\ddot{q}_s\right\}. \tag{A.8}$$

Substitution of (A.8) into (A.7), and further simplifications, yield:

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial a_1}{\partial \dot{q}_i}\right) - \frac{\partial a_1}{\partial q_i} &= \left[m\frac{\partial \mathbf{r}}{\partial q_i}\cdot\frac{\partial \mathbf{r}}{\partial q_1}\cdots m\frac{\partial \mathbf{r}}{\partial q_i}\cdot\frac{\partial \mathbf{r}}{\partial q_N}\right]\ddot{\mathbf{q}} \\ &\quad + \left[m\frac{\partial \mathbf{r}}{\partial q_i}\cdot\left(\sum_{s=1}^{N}\frac{\partial^2 \mathbf{r}}{\partial q_s \partial q_1}\dot{q}_s\right)\cdots m\frac{\partial \mathbf{r}}{\partial q_i}\cdot\left(\sum_{s=1}^{N}\frac{\partial^2 \mathbf{r}}{\partial q_s \partial q_N}\dot{q}_s\right)\right]\dot{\mathbf{q}},\end{aligned} \tag{A.9}$$

which describes format-I, given as (12), where $\mathbf{r}$ has to be differentiated in the inertial frame.

Next, considering the second term as given in (11b):

$$a_2 = \frac{1}{2}\boldsymbol{\omega}\cdot\mathbf{I}\cdot\boldsymbol{\omega}. \tag{11b}$$

Its differentiation with respect to q_i as an arbitrary generalized coordinate is:

$$\frac{\partial a_2}{\partial q_i} = \boldsymbol{\omega}\cdot\mathbf{I}\cdot\frac{\partial \boldsymbol{\omega}}{\partial q_i}, \tag{A.10}$$

where $\boldsymbol{\omega}$ is differentiated in the body frame. Also, differentiation of a_2 with respect to $\dot{q}_i$ as an arbitrary generalized speed, is obtained as:[2]

$$\frac{\partial a_2}{\partial \dot{q}_i} = \boldsymbol{\omega}\cdot\mathbf{I}\cdot\frac{\partial \boldsymbol{\omega}}{\partial \dot{q}_i}, \tag{A.11}$$

[2]Considering (A.1a and b), since $\partial\boldsymbol{\omega}/\partial t = \mathbf{0}$ and $\boldsymbol{\omega}\times\boldsymbol{\omega} = \mathbf{0}$, it is preferable to implement all differentiations related to a_2 in an appropriate body frame. Therefore, angular velocity of any individual body ($\boldsymbol{\omega}$) is differentiated in the corresponding body frame, where $\mathbf{I}$ is a constant.

which can be differentiated with respect to time, to obtain:

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial a_2}{\partial \dot{q}_i}\right) = \dot{\boldsymbol{\omega}} \cdot \mathbf{I} \cdot \frac{\partial \boldsymbol{\omega}}{\partial \dot{q}_i} + \boldsymbol{\omega} \cdot \mathbf{I} \cdot \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial \boldsymbol{\omega}}{\partial \dot{q}_i}\right). \tag{A.12}$$

Then, $\dot{\boldsymbol{\omega}}$ can be computed as:

$$\dot{\boldsymbol{\omega}} = \sum_{s=1}^{N} \left\{ \frac{\partial \boldsymbol{\omega}}{\partial q_s}\dot{q}_s + \frac{\partial \boldsymbol{\omega}}{\partial \dot{q}_s}\ddot{q}_s \right\}. \tag{A.13}$$

Also:

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial \boldsymbol{\omega}}{\partial \dot{q}_i}\right) = \sum_{s=1}^{N} \left\{ \frac{\partial^2 \boldsymbol{\omega}}{\partial \dot{q}_i \partial q_s}\dot{q}_s + \frac{\partial^2 \boldsymbol{\omega}}{\partial \dot{q}_i \partial \dot{q}_s}\ddot{q}_s \right\} = \sum_{s=1}^{N} \frac{\partial^2 \boldsymbol{\omega}}{\partial \dot{q}_i \partial q_s}\dot{q}_s. \tag{A.14}$$

Substitution of (A.13) and (A.14) into (A.12), and subtracting (A.10) from the result, after further simplifications, yield:

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial a_2}{\partial \dot{q}_i}\right) - \frac{\partial a_2}{\partial q_i} = &\left[\frac{\partial \boldsymbol{\omega}}{\partial \dot{q}_i} \cdot \mathbf{I} \cdot \frac{\partial \boldsymbol{\omega}}{\partial \dot{q}_1} \cdots \frac{\partial \boldsymbol{\omega}}{\partial \dot{q}_i} \cdot \mathbf{I} \cdot \frac{\partial \boldsymbol{\omega}}{\partial \dot{q}_N}\right]\ddot{\mathbf{q}} \\ &+ \left[\frac{\partial \boldsymbol{\omega}}{\partial \dot{q}_i} \cdot \mathbf{I} \cdot \frac{\partial \boldsymbol{\omega}}{\partial q_1} + \boldsymbol{\omega} \cdot \mathbf{I} \cdot \frac{\partial^2 \boldsymbol{\omega}}{\partial \dot{q}_i \partial q_1} \cdots \frac{\partial \boldsymbol{\omega}}{\partial \dot{q}_i} \cdot \mathbf{I} \cdot \frac{\partial \boldsymbol{\omega}}{\partial q_N}\right. \\ &\left. + \boldsymbol{\omega} \cdot \mathbf{I} \cdot \frac{\partial^2 \boldsymbol{\omega}}{\partial \dot{q}_i \partial q_N}\right]\dot{\mathbf{q}} - \boldsymbol{\omega} \cdot \mathbf{I} \cdot \frac{\partial \boldsymbol{\omega}}{\partial q_i},\end{aligned} \tag{A.15}$$

which describes format-II, given as (13), and can be considered as contribution of the second term to the equations of motion. Note that $\boldsymbol{\omega}$ is differentiated in a body frame in which $\mathbf{I}$ is considered as a constant dyad.

Finally, considering the third typical term of the system kinetic energy as defined in (11c):

$$a_3 = \dot{\mathbf{R}}_{C_0} \cdot \sum_k m_k \dot{\mathbf{r}}_k. \tag{11c}$$

Its differentiation with respect to q_i is:

$$\frac{\partial a_3}{\partial q_i} = \frac{\partial \dot{\mathbf{R}}_{C_0}}{\partial q_i} \cdot \left(\sum_k m_k \dot{\mathbf{r}}_k\right) + \dot{\mathbf{R}}_{C_0} \cdot \left(\sum_k m_k \frac{\partial \dot{\mathbf{r}}_k}{\partial q_i}\right), \tag{A.16}$$

and its differentiation with respect to $\dot{q}_i$ can be obtained as:

$$\frac{\partial a_3}{\partial \dot{q}_i} = \frac{\partial \mathbf{R}_{C_0}}{\partial q_i} \cdot \left(\sum_k m_k \dot{\mathbf{r}}_k\right) + \dot{\mathbf{R}}_{C_0} \cdot \left(\sum_k m_k \frac{\partial \mathbf{r}_k}{\partial q_i}\right), \tag{A.17}$$

where all derivatives are computed in the inertial frame. Then, (A.17) can be differentiated with respect to time, which yields:

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial a_3}{\partial \dot{q}_i}\right) = \frac{\partial \dot{\mathbf{R}}_{C_0}}{\partial q_i} \cdot \left(\sum_k m_k \dot{\mathbf{r}}_k\right) + \frac{\partial \mathbf{R}_{C_0}}{\partial q_i} \cdot \left(\sum_k m_k \ddot{\mathbf{r}}_k\right) + \ddot{\mathbf{R}}_{C_0} \cdot \left(\sum_k m_k \frac{\partial \mathbf{r}_k}{\partial q_i}\right) + \dot{\mathbf{R}}_{C_0} \cdot \left(\sum_k m_k \frac{\partial \dot{\mathbf{r}}_k}{\partial q_i}\right), \qquad \text{(A.18)}$$

Therefore, subtracting (A.16) from (A.18) yields:

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial a_3}{\partial \dot{q}_i}\right) - \frac{\partial a_3}{\partial q_i} = \frac{\partial \mathbf{R}_{C_0}}{\partial q_i} \cdot \left(\sum_k m_k \ddot{\mathbf{r}}_k\right) + \ddot{\mathbf{R}}_{C_0} \cdot \left(\sum_k m_k \frac{\partial \mathbf{r}_k}{\partial q_i}\right), \qquad \text{(A.19)}$$

where $\ddot{\mathbf{r}}_k$ and $\ddot{\mathbf{R}}_{C_0}$ can be written in terms of generalized coordinates, and their rates as given in (A.8). Substitution of these vectors by appropriate expressions and further simplifications lead to:

$$\begin{aligned}
\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial a_3}{\partial \dot{q}_i}\right) - \frac{\partial a_3}{\partial q_i} = {} & \left[\frac{\partial \mathbf{R}_{C_0}}{\partial q_i} \cdot \sum_k m_k \frac{\partial \mathbf{r}_k}{\partial q_1} \cdots \frac{\partial \mathbf{R}_{C_0}}{\partial q_i} \cdot \sum_k m_k \frac{\partial \mathbf{r}_k}{\partial q_N}\right]\ddot{\mathbf{q}} \\
& + \left[\frac{\partial \mathbf{R}_{C_0}}{\partial q_1} \cdot \sum_k m_k \frac{\partial \mathbf{r}_k}{\partial q_i} \cdots \frac{\partial \mathbf{R}_{C_0}}{\partial q_N} \cdot \sum_k m_k \frac{\partial \mathbf{r}_k}{\partial q_i}\right]\ddot{\mathbf{q}} \\
& + \left[\frac{\partial \mathbf{R}_{C_0}}{\partial q_i} \cdot \sum_k m_k \left(\sum_{s=1}^{N} \frac{\partial^2 \mathbf{r}_k}{\partial q_1 \partial q_s}\dot{q}_s\right) \cdots \frac{\partial \mathbf{R}_{C_0}}{\partial q_i} \right. \\
& \left. \times \sum_k m_k \left(\sum_{s=1}^{N} \frac{\partial^2 \mathbf{r}_k}{\partial q_N \partial q_s}\dot{q}_s\right)\right]\dot{\mathbf{q}} \\
& + \left[\left(\sum_{s=1}^{N} \frac{\partial^2 \mathbf{R}_{C_0}}{\partial q_1 \partial q_s}\dot{q}_s\right) \cdot \sum_k m_k \frac{\partial \mathbf{r}_k}{\partial q_i} \cdots \left(\sum_{s=1}^{N} \frac{\partial^2 \mathbf{R}_{C_0}}{\partial q_N \partial q_s}\dot{q}_s\right) \right. \\
& \left. \times \sum_k m_k \frac{\partial \mathbf{r}_k}{\partial q_i}\right]\dot{\mathbf{q}},
\end{aligned} \qquad \text{(A.20)}$$

which describes format-III given as (14), where $\mathbf{R}_{C_0}$ and $\mathbf{r}_k$ have to be differentiated in the inertial frame.

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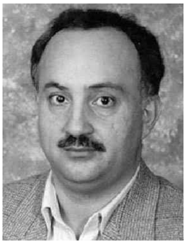

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