On Motion Planning of Nonholonomic Mobile Robots

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ABSTRACT

Mobile robots that consist of a mobile platform with one or many manipulators, are of great interest in a number of applications. This paper presents a methodology for generating paths and trajectories for both the mobile platform and the manipulator that will take a system from an initial configuration to a pre-specified final one, without violating the nonholonomic constraint. The generated paths are of polynomial nature and therefore are continuous and smooth. The validity of the methodology is demonstrated using differential-drive and car-like mobile manipulator systems.

Keywords. Nonholonomic systems, path planning, mobile manipulators, pfaffian constraints.

INTRODUCTION

Mobile manipulator systems consist of a mobile platform equipped with manipulators, see Fig. 1. Applications for such systems abound in mining, construction, forestry, planetary exploration and the military.

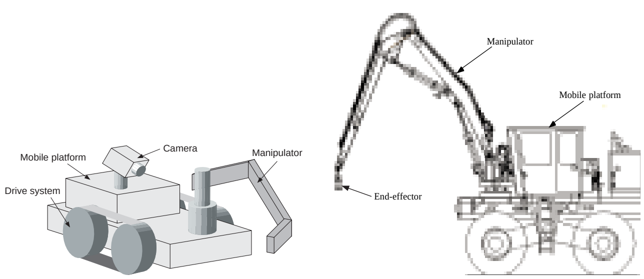

Fig. 1. Mobile manipulator systems.

Moving mobile manipulators systems, present many unique problems that are due to the coupling of holonomic manipulators with nonholonomic bases. Seraji presents a simple on-line approach for motion control of mobile manipulators using augmented Jacobian matrices, [1]. The approach is kinematic and requires additional constraints to be met for the manipulator configuration. The proposed approach can be equally applied to nonholonomic and holonomic mobile robots. Lim and Seraji describe the design and implementation of real-time control system applied on a 7 degree-of-freedom (DOF) arm mounted on a 1-DOF holonomic platform, [2]. The redundant equations are solved using weighted pseudo inverses and geometry based control scheme.

A variety of theoretical and applied control problems have been studied for various classes of nonholonomic control systems. Motion planning problems are concerned with obtaining open loop controls that steer the system from an initial state to a final state without violating the nonholonomic constraints. The algorithm proposed by Lafferiere and Sussmann, is based on expressing the flow resulting from piecewise constant inputs as a formal exponential product expansion involving iterating Lie brackets, [3]. Murray and Sastry used sinusoidal inputs at integrally related frequencies to steer systems that are or can be transferred in power or chained form, [4]. The examples used to illustrate these methods are single platforms or platforms with trailers. A variety of motion planning techniques for nonholonomic systems are described in the book by Li and Canny [5]. A summary of recent developments in the control of nonholonomic systems, including a wealth of references, can be found in Kolmanovsky and McClamroch [6].

This paper focuses on the motion planning problem of mobile manipulator systems, i.e. manipulators attached on mobile platforms. Two commonly available mobile platforms, a car-like and a differentially driven platform, that are subject to nonholonomic constraints, are employed. Here, we present a new methodology for transforming the nonholonomic constraint into a form that can be easily used in computing a path that does not violate the constraint. This transformation allows the generation of smooth and continuous paths for all system control inputs such that the mobile manipulator system is driven from some initial platform-manipulator configuration to any pre-specified final one. Simulation results are included to illustrate the proposed methodology.

KINEMATIC MODELLING

In this paper we study two mobile manipulator systems. The first consists of a two-link manipulator based on a differentially driven mobile platform, while the second consists of the same manipulator based on a car-like mobile platform.

Here, we focus on deriving the kinematics for these systems. To this end, we separate each mobile system into two subsystems, consisting of the mobile platform and the manipulator.

a. Differential Drive Mobile Manipulator

Fig. 2 depicts a differentially driven mobile platform with a manipulator. We first examine the kinematics of the manipulator, as a function of the base motion variables. For simplicity, we consider a planar two-link manipulator arm, as illustrated in Fig. 2. However, the methodology presented is equally applicable to any type of n-jointed mobile platform-mounted manipulator.

Holonomic manipulator subsystem. Let ϑ_1 and ϑ_2 represent the joint angles and l_1 and l_2 denote the link lengths of the manipulator arms. The Cartesian coordinates of the end effector E relative to the world frame are given by

$$x_E = x_F + l_1 \cos(\varphi + \vartheta_1) + l_2 \cos(\varphi + \vartheta_1 + \vartheta_2) \quad (1)$$

$$y_E = y_F + l_1 \sin(\varphi + \vartheta_1) + l_2 \sin(\varphi + \vartheta_1 + \vartheta_2) \quad (2)$$

where (x_F, y_F) is the position of mounting point F of the mobile platform and φ is the platform orientation. Eqs. (1)-(2) show that the position of the end-effector depends on the position and the orientation of the mobile platform. This illustrates the fact that mobile manipulators, in contrast to fixed ones, can have an infinite workspace.

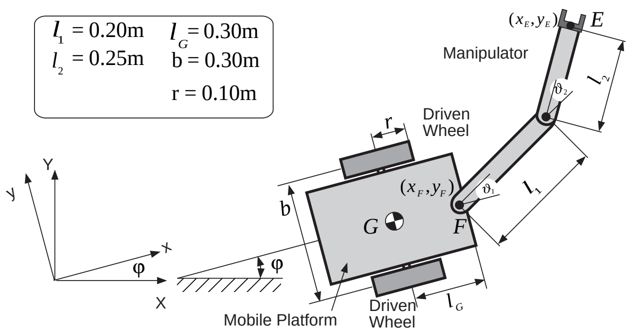

Fig. 2. Mobile manipulator system on a differentially-driven platform.

The direct kinematics equations given by Eqs. (1) and (2) can be inverted. Thus we can find ϑ_1 and ϑ_2 which correspond to a given end-effector position (x_E, y_E) and a given platform position (x_F, y_F) and orientation φ.

The angle ϑ_2 is found by the following expression

$$\vartheta_2 = A\tan 2\{s\vartheta_2, c\vartheta_2\} \quad (3)$$

where s( ) is an abbreviation for sin( ), and c( ) for cos( ). The $s\vartheta_2$ and $c\vartheta_2$ are given by

$$s\vartheta_2 = \pm\left[1 - \frac{(x_E - x_F)^2 + (y_E - y_F)^2 - l_1^2 - l_2^2}{2 l_1 l_2}\right]^{1/2} \quad (4a)$$

$$c\vartheta_2 = \frac{(x_E - x_F)^2 + (y_E - y_F)^2 - l_1^2 - l_2^2}{2 l_1 l_2} \quad (4b)$$

The plus sign in Eq. (4a) corresponds to the elbow-down posture, while the negative sign to the elbow-up posture.

Similarly, the angle ϑ_1 is given by

$$\vartheta_1 = A\tan 2\{s(\varphi + \vartheta_1), c(\varphi + \vartheta_1)\} - \varphi \quad (5)$$

where

$$s(\varphi + \vartheta_1) = \frac{(l_1 + l_2 c\vartheta_2)(y_E - y_F) - l_2 s\vartheta_2 (x_E - x_F)}{(x_E - x_F)^2 + (y_E - y_F)^2} \quad (6a)$$

$$c(\varphi + \vartheta_1) = \frac{(l_1 + l_2 c\vartheta_2)(x_E - x_F) + l_2 s\vartheta_2 (y_E - y_F)}{(x_E - x_F)^2 + (y_E - y_F)^2} \quad (6b)$$

As expected, both manipulator angles are functions of the base coordinates.

The existence of a solution requires that

$$|c\varphi| \le 1 \Rightarrow (x_E - x_F)^2 + (y_E - y_F)^2 \le (l_1 + l_2)^2 \quad (7)$$

If the above inequality is not satisfied, then the target is outside the manipulator reach and thus the mobile platform must move in order to bring the target into the manipulator's workspace. Once the platform moves closer enough to the target, the above constraint will be satisfied and the angles will be given by Eqs. (3) and (5).

Nonholonomic mobile platform. The platform moves by driving the two independent wheels as shown in Fig. 2. We assume that the speed at which the system moves is low and therefore the two driven wheels do not slip sideways. Hence, the velocity of the platform center of mass, υ_G, is normal to the wheel axis. Its components along the X and Y axes, see Fig. 2, are given by,

$$\dot{x}_G = \upsilon_G \cos\varphi \text{ and } \dot{y}_G = \upsilon_G \sin\varphi \quad (8)$$

Eliminating υ_G from the above equations, we obtain

$$\dot{x}_G \sin\varphi - \dot{y}_G \cos\varphi = 0 \quad (9)$$

Eq. (9) is a nonholonomic constraint and cannot be integrated analytically to result in a constraint between the configuration variables of the platform, namely x_G, y_G and φ. As is well known, the configuration space of the system is three-dimensional (completely unrestricted) while the velocity space is two-dimensional. This constraint, written for the manipulator attachment point F, becomes

$$\dot{x}_F \sin\varphi - \dot{y}_F \cos\varphi + \dot{\varphi}\, l_G = 0 \quad (10)$$

where l_G is the distance between G and F, see Fig. 2.

The mobile platform control variables are the angular velocities of the left and the right wheels, $\dot{\vartheta}_\ell$ and $\dot{\vartheta}_r$ respectively. Its Cartesian velocities $(\dot{x}_F, \dot{y}_F, \dot{\varphi})$ are related to the control variables $(d\vartheta_\ell / dt, d\vartheta_r / dt)$

$$\begin{bmatrix} \dot{x}_F \\ \dot{y}_F \\ \dot{\varphi} \end{bmatrix} = \begin{bmatrix} \frac{r}{2} c\varphi + \frac{l_G r}{b} s\varphi & \frac{r}{2} c\varphi - \frac{l_G r}{b} s\varphi \\ \frac{r}{2} s\varphi - \frac{l_G r}{b} c\varphi & \frac{r}{2} s\varphi + \frac{l_G r}{b} c\varphi \\ -\frac{r}{b} & \frac{r}{b} \end{bmatrix} \begin{bmatrix} \dot{\vartheta}_\ell \\ \dot{\vartheta}_r \end{bmatrix} \quad (11)$$

The two control angular velocities are mapped to three system output velocities. If one eliminates the wheel angular velocities in Eq. (11), Eq. (10) results. Eq. (11) demonstrates the fact that the output velocities are nonzero even if one wheel only is rotating. Furthermore, in contrast to car-like mobile platforms, this platform has the ability to change its orientation on the spot.

b. Car-like Mobile Manipulator

Next, consider a mobile manipulator system whose platform includes a front driven steering wheel and two fixed axis rear wheels, see Fig. 3. The holonomic manipulator subsystem is the same as before and thus Eqs. (1)-(7) describing the manipulator forward kinematics are the same. The rear wheels are parallel to the main axis of the car while the front wheel is used for steering. It is assumed that the wheels do not slip sideways.

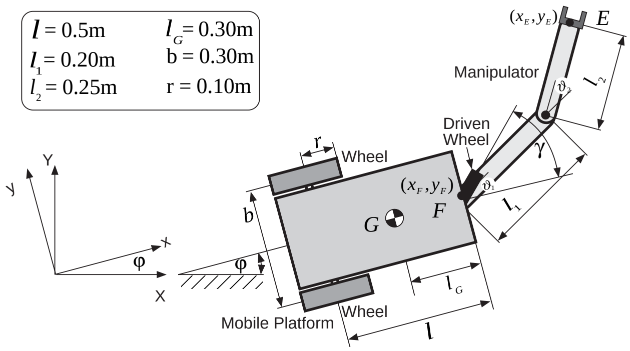

Fig. 3. Car-like mobile manipulator system.

For simplicity, the manipulator is mounted at point F, where the steering wheel is located also. For this point the nonholonomic constraint is written as

$$\dot{x}_F \sin\varphi - \dot{y}_F \cos\varphi + \dot{\varphi}\, l = 0 \tag{12}$$

where $\dot{x}_F$, $\dot{y}_F$ are respectively the X and Y components of the velocity of point F, υ_F, and l is the distance between point F and the rear wheel axis.

The differential kinematics of the car-like mobile platform are described by the following equations

$$\dot{x}_F = \upsilon_F \cos(\varphi+\gamma) = \omega\, r \cos(\varphi+\gamma) \tag{13a}$$

$$\dot{y}_F = \upsilon_F \sin(\varphi+\gamma) = \omega\, r \sin(\varphi+\gamma) \tag{13b}$$

$$\dot{\varphi} = \frac{\upsilon_F}{l}\sin\gamma = \frac{\omega\, r}{l}\sin\gamma \tag{13c}$$

where γ is the steering angle, $\upsilon_F = \omega r$ is the velocity due to the driving wheel at F, ω is the front wheel angular rate, and r is its radius. Eqs. (13) can be written as

$$\begin{bmatrix}\dot{x}_F\\ \dot{y}_F\\ \dot{\varphi}\end{bmatrix} = \begin{bmatrix}\cos(\varphi+\gamma) & 0\\ \sin(\varphi+\gamma) & 0\\ 1/l \cdot \sin\gamma & 0\end{bmatrix}\begin{bmatrix}\upsilon_F\\ \dot{\gamma}\end{bmatrix} \tag{14}$$

Eq. (14) maps the two input velocities, υ_F and $\dot{\gamma}$, to the three output velocities, $\dot{x}_F$, $\dot{y}_F$ and $\dot{\varphi}$. If one eliminates the input velocities, the nonholonomic constraint given by Eq. (12) results.

Inspection of Eq. (14) reveals that one of its columns is zero. Because of this, if the mobile platform is not moving, then neither the position nor the orientation of the platform can change using the steering wheel.

PATH PLANNING

A mobile system is especially useful when the manipulator task is outside the manipulator reach. Therefore, in this section we assume that this is always the case, in other words that inequality (7) is not satisfied for a given target.

A prerequisite for the successful use of a mobile manipulator system is the availability of a planning methodology that can generate a feasible path for driving the end-effector to the desired coordinates without violating the system nonholonomic constraint. However, in many applications, it is required that the platform position and orientation is pre-specified for a number of reasons. Such reasons include the particular site geometry or ground morphology, the ease of controlling or supervising the manipulator, the maximization of the system's manipulability or force output, etc.

Given that in general the system configuration includes the position and orientation of both the platform and the manipulator, then a central problem that must be tackled is described as follows. Find driven joint paths for both the mobile platform and its manipulator that can drive the system from some initial configuration to a final pre-specified one. Of course, it is very desirable to generate smooth paths that can be obtained using simple and computationally inexpensive methods.

To solve this problem, we focus our attention in finding a path for the mobile platform, which connects its initial configuration $(x_F^{\,in}, y_F^{\,in}, \varphi^{in})$ to the final one, $(x_F^{\,fin}, y_F^{\,fin}, \varphi^{fin})$. This problem is not trivial, due to the fact that in our disposal we have only two controls, and at the same time we must satisfy the nonholonomic constraint, and achieve a change in the three dimensional configuration space. Next, we develop a planning methodology for the two mobile platforms we consider in this paper.

a. Differentially Driven Mobile Platform

The nonholonomic constraint of the differentially-driven mobile platform given by Eq. (10) is scleronomic and thus it can be written in the Pfaffian form

$$P(x_F, y_F, \varphi)dx_F + Q(x_F, y_F, \varphi)dy_F + R(x_F, y_F, \varphi)d\varphi = 0 \tag{15}$$

where,

$$P(x_F, y_F, \varphi) = \sin\varphi,\ Q(x_F, y_F, \varphi) = -\cos\varphi,\ R(x_F, y_F, \varphi) = l_G$$

Eq. (15) is not easy to work with for planning purposes. However, Pfaffian equations can be transformed into the following form, [7],

$$du + v\, dw = 0 \tag{16}$$

that is much easier to work with for planning purposes. In Eq. (16), u, v, w are properly selected functions of platform position and orientation, x_F, y_F, and φ.

Should it be possible to transform Eq. (15) into Eq. (16), the following equations must hold

$$P = \frac{\partial u}{\partial x_F} + v\frac{\partial w}{\partial x_F},\ \ Q = \frac{\partial u}{\partial y_F} + v\frac{\partial w}{\partial y_F},\ \ R = \frac{\partial u}{\partial \varphi} + v\frac{\partial w}{\partial \varphi} \tag{17}$$

Next we construct the differential equations that u, v, w, must satisfy. To this end, we set

$$P' = \frac{\partial Q}{\partial \varphi} - \frac{\partial R}{\partial y_F},\ \ Q' = \frac{\partial R}{\partial x_F} - \frac{\partial P}{\partial \varphi},\ \ R' = \frac{\partial P}{\partial y_F} - \frac{\partial Q}{\partial x_F} \tag{18}$$

and, after we substitute Eq. (17) into Eq. (18), we get

$$P' = \frac{\partial v}{\partial \varphi}\frac{\partial w}{\partial y_F} - \frac{\partial v}{\partial y_F}\frac{\partial w}{\partial \varphi} \tag{19a}$$

$$Q' = \frac{\partial v}{\partial x_F}\frac{\partial w}{\partial \varphi} - \frac{\partial v}{\partial \varphi}\frac{\partial w}{\partial x_F} \tag{19b}$$

$$R' = \frac{\partial v}{\partial y_F}\frac{\partial w}{\partial x_F} - \frac{\partial v}{\partial x_F}\frac{\partial w}{\partial y_F} \tag{19c}$$

Multiplying these equations, first by $\partial w / \partial x_F$, $\partial w / \partial y_F$, and $\partial w / \partial \varphi$ respectively and adding the results yields the following equation

$$P'\frac{\partial w}{\partial x_F} + Q'\frac{\partial w}{\partial y_F} + R'\frac{\partial w}{\partial \varphi} = 0 \tag{20}$$

Next, multiplying Eqs. (19a)-(19c) by $\partial v / \partial x_F$, $\partial v / \partial y_F$, and $\partial v / \partial \varphi$ respectively, and adding the results, we get

$$P'\frac{\partial v}{\partial x_F} + Q'\frac{\partial v}{\partial y_F} + R'\frac{\partial v}{\partial \varphi} = 0 \tag{21}$$

From the above analysis, we conclude that both v and w satisfy the same first order partial differential equation.

Finally, multiplying Eqs. (19a, b, c) by $P - \partial u / \partial x_F$, $Q - \partial u / \partial y_F$ and $R - \partial u / \partial \varphi$ respectively, adding and using Eqs. (17) and (20) we get

$$\begin{aligned} &\left(P - \frac{\partial u}{\partial x_F}\right)P' + \left(Q - \frac{\partial u}{\partial y_F}\right)Q' + \left(R - \frac{\partial u}{\partial \varphi}\right)R' = \\ &\qquad v\left(P'\frac{\partial w}{\partial x_F} + Q'\frac{\partial w}{\partial y_F} + R'\frac{\partial w}{\partial \varphi}\right) = 0 \end{aligned} \tag{22}$$

Therefore, u satisfies the following differential equation

$$P'\frac{\partial u}{\partial x_F} + Q'\frac{\partial u}{\partial y_F} + R'\frac{\partial u}{\partial \varphi} = PP' + QQ' + RR' \tag{23}$$

The right hand side in the above equation does not vanish, because the condition of integrability is not satisfied. If the system was holonomic, then u obviously would satisfy the same equation as v and w.

We have identified the differential equations that each of u, v, and w satisfy, namely Eqs. (23), (21) and (20). Next, we solve Eq. (20) and find w in the case of the nonholonomic constraint given by Eq. (12), for which

$$P' = \sin\varphi, \quad Q' = -\cos\varphi, \quad R' = 0 \tag{24}$$

Eq. (20) can be solved by constructing the following system of equations, [8],

$$\frac{dx_F}{P'} = \frac{dy_F}{Q'} = \frac{d\varphi}{R'} \tag{25}$$

and integrating them. The solution of Eq. (20) is any function of the two independent integrals of Eqs. (25). Two such integrals are the following

$$\alpha(x_F, y_F, \varphi) = x_F \cdot \cos\varphi + y_F \cdot \sin\varphi = k_1 \tag{26a}$$

$$\beta(x_F, y_F, \varphi) = \varphi = k_2 \tag{26b}$$

The solution for w can be any function of α and β. For simplicity we choose

$$w = \beta \tag{27}$$

Eq. (26b) represents a relationship among x_F, y_F, and φ, for which Eq. (15) due to Eqs. (17) becomes

$$P(x_F, y_F, \varphi)dx_F + Q(x_F, y_F, \varphi)dy_F + R(x_F, y_F, \varphi)d\varphi =$$

$$= \frac{\partial u}{\partial x_F}dx_F + \frac{\partial u}{\partial y_F}dy_F + \frac{\partial u}{\partial \varphi}d\varphi = du \tag{28a}$$

that is, when Eq. (27) holds, the term

$$P \cdot dx_F + Q \cdot dy_F + R \cdot d\varphi \tag{28b}$$

is a perfect differential. Now, we can use the relationship $\beta(x_F, y_F, \varphi) = k_2$ to remove φ and $d\varphi$ from the differential given by Eq. (28b). The resulting expression is also a perfect differential, namely $dh(x_F, y_F, \beta)$

$$dh(x_F, y_F, \beta) = \sin\beta \cdot dx_F - \cos\beta \cdot dy_F = 0 \tag{29}$$

By integrating Eq. (29) we obtain

$$h(x_F, y_F, \beta) = x_F \cdot \sin\beta - y_F \cdot \cos\beta = const. \tag{30}$$

If in $h(x_F, y_F, \beta)$ the variable φ is re-inserted by removing the constant β, then $h(x_F, y_F, \beta)$ becomes $u(x_F, y_F, \varphi)$

$$u(x_F, y_F, \varphi) = x_F \cdot \sin\varphi - y_F \cdot \cos\varphi \tag{31}$$

We thus have u and w. The expression for v is found using any of Eqs. (17). By choosing the third one, we get

$$v(x_F, y_F, \varphi) = l_G - x_F \cdot \cos\varphi - y_F \cdot \sin\varphi \tag{32}$$

To conclude, the nonholonomic constraint described by Eq. (15) can be transformed to the equivalent form given by Eq. (16), provided that

$$u(x_F, y_F, \varphi) = x_F \cdot \sin\varphi - y_F \cdot \cos\varphi \tag{31}$$

$$v(x_F, y_F, \varphi) = l_G - x_F \cdot \cos\varphi - y_F \cdot \sin\varphi \tag{32}$$

$$w(x_F, y_F, \varphi) = \varphi \tag{33}$$

Eqs. (31)-(33) constitute a transformation $(x_F, y_F, \varphi) \to (u, v, w)$, which is defined at every point of the configuration space of the system. If we write Eqs. (31)-(33) in matrix form, then we have

$$\begin{bmatrix} \sin\varphi & -\cos\varphi & 0 \\ \cos\varphi & \sin\varphi & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x_F \\ y_F \\ \varphi \end{bmatrix} = \begin{bmatrix} u \\ l_G - v \\ w \end{bmatrix} \tag{34}$$

where the determinant of the above matrix is always -1. Therefore Eqs. (31)-(33) constitute a global diffeomorphism in the configuration space.

It is interesting to observe that Eqs. (31) and (32) can be written also as

$$\begin{aligned} \begin{bmatrix} u \\ l_G - v \end{bmatrix} &= \begin{bmatrix} \sin\varphi & -\cos\varphi \\ \cos\varphi & \sin\varphi \end{bmatrix} \begin{bmatrix} x_F \\ y_F \end{bmatrix} = \\ &= \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} \cos\varphi & \sin\varphi \\ \sin\varphi & -\cos\varphi \end{bmatrix} \begin{bmatrix} x_F \\ y_F \end{bmatrix} \end{aligned} \tag{35}$$

where the first matrix corresponds to a reflection with respect to a 45^0 slope line while the second corresponds to a reflection with respect to a $\varphi/2$ line slope.

So far, we have achieved to transform the nonholonomic constraint to the equivalent form given by Eq. (16). Next, note that if we choose functions f and g as follows

$$w = f(t) \tag{36}$$

$$u = g(w) \quad (37)$$

$$v = -\frac{du}{dw} = -g'(w) \quad (38)$$

then Eq. (16) is satisfied identically. Therefore, the planning problem reduces to the selection of functions f and g such that they satisfy the initial and final configuration variables.

For example, because of Eq. (33), f is any function of time that satisfies the initial and final orientation φ. Such functions can be polynomials, splines, or any other continuous and smooth time function. Function g is constructed in the same way, using Eq. (31) for computing the initial and final values for u. Finally, v is computed using Eq. (38). Once u, v, and w have been found, platform coordinates are computed by inverting Eq. (34). This results in the required history of platform locations and orientations that if followed, they will drive the platform to the desired final location and orientation, without violating the nonholonomic constraint.

b. Car-like Mobile Platform

Eq. (12) describes the nonholonomic constraint in the case of the car-like mobile platform. This equation has exactly the same form with Eq. (10). Therefore by applying the same methodology we find that Eq. (12) is equivalent to

$$du + v\,dw = 0 \quad (16)$$

with,

$$u(x_F, y_F, \varphi) = x_F \cdot \sin\varphi - y_F \cdot \cos\varphi \quad (39)$$

$$v(x_F, y_F, \varphi) = l - x_F \cdot \cos\varphi - y_F \cdot \sin\varphi \quad (40)$$

$$w(x_F, y_F, \varphi) = \varphi \quad (41)$$

Eqs. (39)-(41) constitute a global diffeomorphism and they have the same meaning as Eqs. (31)-(33), i.e. they describe two reflections and a parallel translation. Again, if we choose functions *f* and *g* according to Eqs. (36)-(38), then we can compute a path for the mobile platform that does not violate the constraint.

SIMULATION RESULTS

To illustrate the methodology described in the previous section, we employ the mobile systems shown in Figs. 2 and 3. The main task for each system is to have the end-effector reach a desired target point with coordinates (x_E, y_E). Since the system is redundant, we can satisfy more requirements. For example, we can also specify the desired platform location and orientation, and solve for the required manipulator configuration using the holonomic inverse relations, Eqs. (3) and (5).

In this example, we require instead that the end-effector arrives at the target at some desired configuration, for example at one which maximizes its force output or the manipulability criterion. Setting ϑ_2 restricts point F on the platform to be on a circle in Fig. 4, with radius

$$R = \sqrt{(l_1 + l_2 \cos\vartheta_2^{fin})^2 + l_2^2 \sin^2\vartheta_2^{fin}} \quad (42)$$

For some desired orientation of the mobile platform, φ^{fin}, the exact location of point F can be chosen such that it minimizes the total path length from the initial to the final destination. To this end, we construct paths that start at the initial configuration and end on the circular locus shown in Fig 4. The path selected is the one with minimum length. By doing so, both the final position and the path of the platform are known. Hence, Eqs. (5) and (6) can be used to calculate the final value of the angle ϑ_1. Clearly, this approach requires that we are able to solve the core problem of computing the joint histories needed to drive the system to a desired final configuration $(x_E^{fin}, y_E^{fin}, \varphi^{fin}, \vartheta_1^{fin}, \vartheta_2^{fin})$.

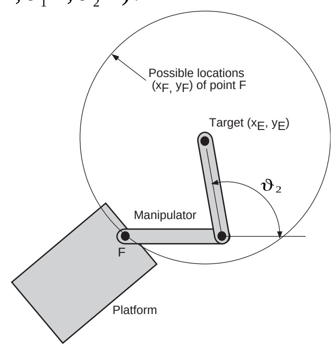

Fig. 4. Platform possible final positions.

In order to find a path for the platform using the method presented above we select functions f and g in Eqs. (36)-(38) to be respectively fifth and third order polynomials of time

$$f(t) = a_5 t^5 + a_4 t^4 + a_3 t^3 + a_2 t^2 + a_1 t + a_0$$

$$g(w) = b_3 w^3 + b_2 w^2 + b_1 w + b_0$$

where the coefficients above are calculated such that f and g satisfy the initial and final conditions of the motion. These conditions include positions, velocities and for the platform orientation $f = \varphi$, accelerations, too.

For the simulation, the total time was chosen equal to 6s (for reasonable average velocities), while the initial configuration is given by $(x_E^{in}, y_E^{in}, \varphi^{in}, \vartheta_1^{in}, \vartheta_2^{in}) = (0.5m, 0.5m, -90°, -30°, -60°)$. Using Eq. (1) and (2) we calculate the initial position of the platform, which is $(x_F^{in}, y_F^{in}) = (0.85m, 0.67m)$ in both system cases. The final configuration for the manipulator assumes a desired angle $\vartheta_2^{fin} = 135°$ (isotropic point). For a final orientation $\varphi^{fin} = 60^0$, using a trial and error procedure we find the path of minimum length and the final position of the platform, which is $(x_F^{fin}, y_F^{fin}) = (1.86m, 1.89m)$ in both system cases. As stated above the final value of ϑ_1 is calculated using Eqs. (5) and (6) and is $\vartheta_1^{fin} = -102.5^0$. The final system configuration is then given by $(x_E^{fin}, y_E^{fin}, \varphi^{fin}, \vartheta_1^{fin}, \vartheta_2^{fin}) = (2m, 2m, 60°, -102.5°, 135°)$.

Fig. 5 depicts snapshots of the motion of the differentially-driven system, while Fig. 6 depicts snapshots for the car-like mobile system. In general, both systems move in similar ways. However the differential-drive system starts with a turn on-the-spot and then proceeds to the target, while the car-like system, first moves forward and then toward the target. This behavior is due to its design which does not allow on-the-spot maneuvers.

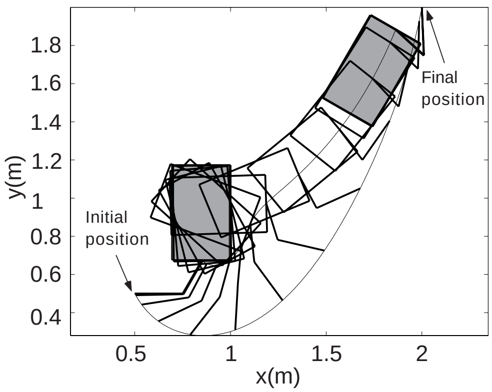

Fig. 5. Motion animation of the differential-drive mobile manipulator.

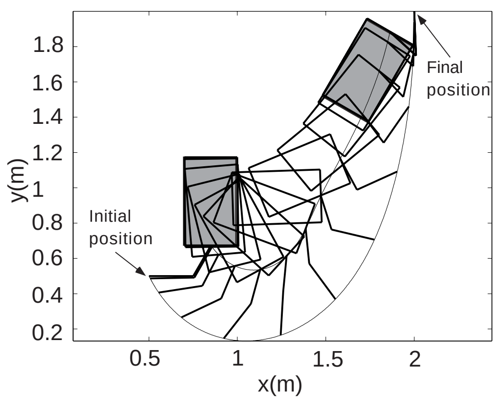

Fig. 6. Motion animation of the car-like drive mobile manipulator.

Figs. 7 and 8 depict platform and manipulator joint velocities that correspond to the two mobile systems. It can be observed that all trajectories are smooth and that both mobile robotic systems start and stop smoothly.

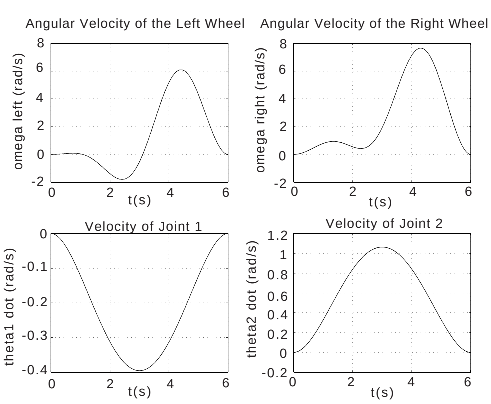

Fig. 7. Input velocities of the differentially driven mobile manipulator.

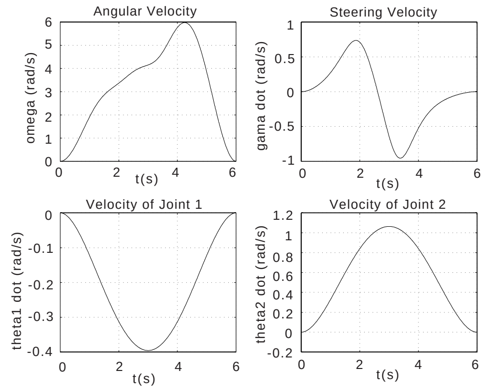

Fig. 8. Input velocities of the car-like mobile manipulator.

CONCLUSIONS

This paper proposed a planning methodology for mobile manipulator systems consisting of a mobile platform and a manipulator. This methodology uses an appropriate transformation of variables to generate paths for both the mobile platform and the manipulator that will take a mobile system from an initial platform-manipulator configuration to a pre-specified final one, without violating the nonholonomic constraint. The generated paths can be constructed using a variety of functions including polynomials, splines, etc. The resulting paths are continuous and smooth. The validity of the methodology was demonstrated using a differential-drive and a car-like mobile manipulator system.

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