COORDINATED MANIPULATOR/SPACECRAFT MOTION CONTROL FOR SPACE ROBOTIC SYSTEMS

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ABSTRACT

This paper studies the coordinated control of space manipulators and their spacecraft. The dynamics of free-flying space robotic systems are written compactly as functions of the system barycentric vectors. A control technique is developed that includes requirements on a spacecraft's position and orientation as well as on its manipulator. A Transposed-Jacobian type controller with inertial feedback is developed and an example is used to demonstrate this technique.

I. INTRODUCTION

Free-flying space robotic systems have been proposed for use in space in which a robotic manipulator is mounted on a spacecraft with a reaction jet attitude control system [1,2]. One control method for such systems calculates the reaction jet forces required to keep the spacecraft stationary while the manipulator achieves minimum time performance [3]. Control schemes that allow the spacecraft to be uncontrolled have been studied to eliminate the use of reaction jet fuel [4-8]. Impacts between the uncontrolled spacecraft and its environment may limit such schemes. Also, the workspace of such systems is restricted due to the existence of dynamic singularities [9]. To achieve an infinite workspace, another control scheme switches between a free-floating mode and one in which the system is treated as a redundant manipulator with a pseudo-inverse Jacobian-based controller [10]. In this scheme, the consumption of reaction jet fuel remains a limitation.

This paper presents a method to control the position and attitude of a system's spacecraft in a coordinated way by using the inherent redundancy in free-flying space robotic systems. The dynamic equations that describe the motion of a rigid free-flying space robotic system are written in a compact form using a Lagrangian formulation. The system center of mass (CM) is chosen to represent the translational degrees-of-freedom (DOF) of the system; barycentric vectors are chosen as kinematic variables. This control scheme has the double advantage of allowing a system's motion to be planned to avoid impacts with its environment, and of maintaining a favorable manipulator configuration during the end-effector's motion. In addition, since a system's spacecraft can be moved, the workspace of its manipulator becomes unlimited. An example is implemented where a Transposed-Jacobian controller with inertial feedback is used.

II. SYSTEM KINEMATICS

First the position and the velocity of an arbitrarily located point of a space manipulator system, as the one shown in Figure 1, are written in terms of the vectorial sums of a minimum number of body-fixed vectors, called barycentric vectors. The body 0 in Figure 1 represents the spacecraft, and the bodies k (k=1,…,N) represent the N manipulator links. The manipulator is in an open-chain configuration so that a system with an N DOF manipulator will have in total N+6 DOF, the additional 6 DOF corresponding to the spacecraft's position and attitude.

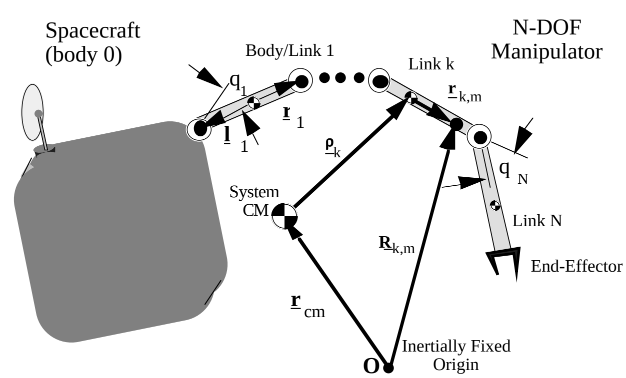

Figure 1. A spatial free-flying manipulator system.

A matrix representation of the kinematics and dynamics is obtained by attaching a reference frame to each center of mass with axes parallel to each body's principal axes. The body inertia matrix expressed in this frame is diagonal.

Left superscripts must be interpreted as "expressed in frame;" a missing left superscript implies a column vector expressed in the inertial frame.

It can be shown that the vector from the inertially fixed origin **O** to an arbitrary point m on body k, $\mathbf{R}_{k,m}$ is given by [11]:

$$\mathbf{R}_{k,m} = \mathbf{r}_{cm} + \sum_{i=0}^{N} \mathbf{v}_{ik,m} \tag{1}$$

where the vectors $\mathbf{v}_{ik,m}$ are given by [11]:

$$\mathbf{v}_{ik,m} \equiv \mathbf{v}_{ik} + \delta_{im}\mathbf{r}_{k,m} \tag{2}$$

where δ_{im} is a Kronecker delta, and $\mathbf{v}_{ik}$ are barycentric vectors defined by [9,11]:

$$\mathbf{v}_{ik} \equiv \left\{ \begin{array}{ll} \mathbf{r}_i^* = \mathbf{r}_i - \mathbf{c}_i & i<k \\ \mathbf{c}_i^* = -\mathbf{c}_i & i=k \\ \mathbf{l}_i^* = \mathbf{l}_i - \mathbf{c}_i & i>k \end{array} \right\} \tag{3a}$$

with:

$$\mathbf{c}_i = \mathbf{l}_i \sum_{j=0}^{i-1} \frac{m_j}{M} + \mathbf{r}_i \left(1 - \sum_{j=0}^{i} \frac{m_j}{M} \right) \qquad i = 0,\ldots,N \tag{3b}$$

where $\mathbf{r}_{cm}$, $\mathbf{r}_{k,m}$, $\mathbf{l}_i$, $\mathbf{r}_i$, are defined in Figure 1, m_i is the mass of body i, and M is the total system mass.

The velocity of a point m in body k, $\dot{\mathbf{R}}_{k,m}$, is:

$$\dot{\mathbf{R}}_{k,m} = \dot{\mathbf{r}}_{cm} - \sum_{i=0}^{N} (\mathbf{v}_{ik,m})^{\times} \boldsymbol{\omega}_i \tag{4}$$

where $\boldsymbol{\omega}_i$ is the angular velocity of the i^{th} body, and the $(^{\times})$ denotes the cross-product skew-symmetric matrix, see [13]. Equation (4) can be expressed as function of the manipulator joint rates $\dot{\mathbf{q}} = d/dt[q_1,q_2,\ldots,q_N]^T$, by writing the angular velocity of the k^{th} body as:

$$\boldsymbol{\omega}_k = \boldsymbol{\omega}_0 + \mathbf{T}_0\, {}^0\mathbf{F}_k\, \dot{\mathbf{q}} \qquad k = 1,\ldots,N \tag{5}$$

where $\boldsymbol{\omega}_0$ is the spacecraft angular velocity, $\mathbf{T}_0$ is a 3×3 transformation matrix that describes the orientation $\boldsymbol{\Theta}$ of the spacecraft frame with respect to the inertial frame and ${}^0\mathbf{F}_k$ is a 3×N matrix given by:

$${}^0\mathbf{F}_k \equiv \left[{}^0\mathbf{T}_1\, {}^1\mathbf{u}_1, \ldots, {}^0\mathbf{T}_k\, {}^k\mathbf{u}_k, \mathbf{0} \right] \qquad k = 1,\ldots,N \tag{6}$$

The vector ${}^i\mathbf{u}_i$ is the unit column vector in frame i parallel to the revolute axis through joint i, ${}^0\mathbf{T}_i$ is a 3×3 transformation matrix that describes the orientation of the i^{th} frame with respect to the spacecraft frame, and **0** is a 3×(N-k) zero element matrix.

Based on Equations (4) and (5), the linear and angular velocity of point m in body k, $\dot{\mathbf{x}}_{k,m}$ can be written as:

$$\dot{\mathbf{x}}_{k,m} = \begin{bmatrix} \dot{\mathbf{R}}_{k.m} \\ \boldsymbol{\omega}_k \end{bmatrix} = \mathbf{J}^+_{k,m}\, \dot{\mathbf{z}}_0 \tag{7}$$

where the independent system velocities $\dot{\mathbf{z}}_0$ are defined as:

$$\dot{\mathbf{z}}_0 \equiv \begin{bmatrix} {}^0\dot{\mathbf{r}}_{cm} \\ {}^0\boldsymbol{\omega}_0 \\ \dot{\mathbf{q}} \end{bmatrix} \tag{8}$$

The Jacobian $\mathbf{J}^+_{k,m}$ is given by:

$$\mathbf{J}^+_{k,m}(\boldsymbol{\Theta},\mathbf{q}) = \mathrm{diag}(\mathbf{T}_0,\mathbf{T}_0)\, {}^0\mathbf{J}^+_{k,m}(\mathbf{q}) \tag{9}$$

with:

$${}^0\mathbf{J}^+_{k,m}(\mathbf{q}) = \begin{bmatrix} \mathbf{1} & {}^0\mathbf{J}_{11k.m} & {}^0\mathbf{J}_{12k.m} \\ \mathbf{0} & \mathbf{1} & {}^0\mathbf{J}_{22k.m} \end{bmatrix} \tag{10a}$$

$${}^0\mathbf{J}_{11k,m} \equiv -\sum_{i=0}^{N} [{}^0\mathbf{T}_i\, {}^i\mathbf{v}_{ik,m}]^{\times}$$

$${}^0\mathbf{J}_{12k,m} \equiv -\sum_{i=1}^{N} [{}^0\mathbf{T}_i\, {}^i\mathbf{v}_{ik,m}]^{\times}\, {}^0\mathbf{F}_i$$

$${}^0\mathbf{J}_{22k,m} \equiv {}^0\mathbf{F}_k \tag{10b}$$

where **1** is the unity 3×3 matrix and **0**, the zero 3×3 matrix. ${}^0\mathbf{J}_{11k,m}$ is a 3×3 matrix, and ${}^0\mathbf{J}_{12k,m}$ and ${}^0\mathbf{J}_{22k,m}$ are 3×N matrices. All matrices in Equations (10) depend on the system configuration $\mathbf{q}$, only. The size of ${}^0\mathbf{J}^+_{k,m}$ is 6×(N+6), so even when N=6, it is a non-square matrix, due to the redundant nature of the system. Note that these Jacobians are *basic Jacobians*, that is Jacobians independent of the particular parameter set used to describe the end-effector orientation [12]. Kinematic equations related to the particular orientation representation also must be used. If the 321 sequence of Euler angles is used to represent the orientation $\boldsymbol{\Theta}_\kappa$ of the k^{th} body, then the following holds, [13]:

$$\dot{\boldsymbol{\Theta}}_k = \frac{d}{dt}[\theta_{1,k},\theta_{2,k},\theta_{3,k}]^T = \mathbf{S}_k^{-1}(\boldsymbol{\Theta}_k)\boldsymbol{\omega}_k \qquad k = 0,\ldots,N \tag{11}$$

where $\mathbf{S}_k$ is an invertible matrix except at some isolated points and given by:

$$\mathbf{S}_k(\boldsymbol{\Theta}_k) = \begin{bmatrix} \cos\theta_{2k}\cos\theta_{3k} & -\sin\theta_{1k} & 0 \\ \cos\theta_{2k}\sin\theta_{3k} & \cos\theta_{1k} & 0 \\ -\sin\theta_{2k} & 0 & 1 \end{bmatrix} \tag{12}$$

A four parameter representation of the orientation can be used if representation singularities can be a problem, [13]. Equations similar to those above can be written for the end-effector by noting that for the end-point, body k = N and point m = E. The subscripts N and E are dropped for simplicity and the resulting expressions are written here for future reference:

$$\dot{\mathbf{x}} = \mathbf{J}^{+}\,\dot{\mathbf{z}}_0 = \mathrm{diag}(\mathbf{T}_0,\mathbf{T}_0)\begin{bmatrix} \mathbf{1} & {}^0\mathbf{J}_{11} & {}^0\mathbf{J}_{12} \\ \mathbf{0} & \mathbf{1} & {}^0\mathbf{J}_{22} \end{bmatrix}\dot{\mathbf{z}}_0 \quad (13)$$

Note that the rank of $\mathbf{J}^{+}$ and of $\mathbf{J}^{+}_{k,m}$ is always six, because its first six columns always contain six independent column vectors. This reflects the fact that any position or orientation can be reached by moving the spacecraft alone.

III. SYSTEM DYNAMICS

Here the equations of motion for the system shown in Fig. 1, are written using a Lagrangian approach. The potential energy due to gravity is zero and since the system is assumed to be rigid, the potential energy due to flexibility is also zero. We just need an expression for its kinetic energy, in order to form the system Lagrangian. It can be shown that the kinetic energy can be compactly written in the form [11]:

$$T = \frac{1}{2}\,\dot{\mathbf{z}}_0^{T}\,\mathbf{H}^{+}(\mathbf{q})\,\dot{\mathbf{z}}_0 \quad (14)$$

where $\mathbf{H}^{+}(\mathbf{q})$ is a (N+6)×(N+6) positive definite symmetric inertia matrix, given by:

$$\mathbf{H}^{+}(\mathbf{q}) = \begin{bmatrix} M\mathbf{1} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & {}^0\mathbf{D}(\mathbf{q}) & {}^0\mathbf{D}_{\mathbf{q}}(\mathbf{q}) \\ \mathbf{0} & {}^0\mathbf{D}_{\mathbf{q}}(\mathbf{q})^{T} & {}^0\mathbf{D}_{\mathbf{qq}}(\mathbf{q}) \end{bmatrix} \quad (15)$$

where $\mathbf{1}$ is the unit 3×3 matrix, $\mathbf{0}$ a zero matrix of appropriate size, and M is the total system mass. ${}^0\mathbf{D}$ is the 3×3 system inertia matrix with respect to the system CM, and as such it is a positive definite symmetric matrix. ${}^0\mathbf{D}_{\mathbf{q}}$ is a 3×N matrix and ${}^0\mathbf{D}_{\mathbf{qq}}$ is an N×N matrix. These are given by:

$${}^0\mathbf{D}_j \equiv \sum_{i=0}^{N} {}^0\mathbf{D}_{ij} \quad (j = 0,\ldots,N), \qquad {}^0\mathbf{D} \equiv \sum_{j=0}^{N} {}^0\mathbf{D}_j \quad (16a)$$

$${}^0\mathbf{D}_{\mathbf{q}} \equiv \sum_{j=1}^{N} {}^0\mathbf{D}_j\,{}^0\mathbf{F}_j, \qquad {}^0\mathbf{D}_{\mathbf{qq}} \equiv \sum_{j=1}^{N} {}^0\mathbf{F}_i^{T}\,{}^0\mathbf{D}_j\,{}^0\mathbf{F}_j \quad (16b)$$

where the inertia matrices ${}^0\mathbf{D}_{ij}$ are functions of the barycentric vectors [11]:

$${}^0\mathbf{D}_{ij} \equiv \begin{Bmatrix} -M\{({}^0\mathbf{l}_j^{*T}\,{}^0\mathbf{r}_i^{*})\mathbf{1} - {}^0\mathbf{l}_j^{*}\,{}^0\mathbf{r}_i^{*T}\} & i<j \\ {}^0\mathbf{I}_i + \sum_{k=0}^{N} m_k\{({}^0\mathbf{v}_{ik}^{T}\,{}^0\mathbf{v}_{ik})\mathbf{1} - {}^0\mathbf{v}_{ik}\,{}^0\mathbf{v}_{ik}^{T}\} & i=j \\ -M\{({}^0\mathbf{r}_j^{*T}\,{}^0\mathbf{l}_i^{*})\mathbf{1} - {}^0\mathbf{r}_j^{*}\,{}^0\mathbf{l}_i^{*T}\} & i>j \end{Bmatrix} \quad (17)$$

The generalized forces required in the Lagrangian approach can be found using the principle of virtual work. It can be shown that if $\boldsymbol{\tau}$ is an N×1 torque vector acting at the manipulator joints, and ${}^0\mathbf{f}_{k,m}$ and ${}^0\mathbf{n}_{k,m}$ are the force and torque acting on point m of body k, as measured from the spacecraft, and ${}^0\mathbf{J}^{+}_{k,m}$ is a Jacobian given by Eq. (10), then the (N+6)×1 vector of generalized forces, $\mathbf{Q}$, is given by:

$$\mathbf{Q} \equiv \begin{bmatrix} \mathbf{0} \\ \mathbf{0} \\ \boldsymbol{\tau} \end{bmatrix} + \sum_{k=0}^{N}\sum_{m} \{{}^0\mathbf{J}^{+}_{k,m}(\mathbf{q})\}^{T} \begin{bmatrix} {}^0\mathbf{f}_{k.m} \\ {}^0\mathbf{n}_{k.m} \end{bmatrix} \quad (18)$$

Since ${}^0\boldsymbol{\omega}_0$ are not proper generalized velocities, Lagrange's equations cannot be used readily to write the equations of motion. A quasi-Lagrangian approach, or a combination of Lagrangian and momentum methods, can be used [11,13]. The result is given here without proof, see [11]:

$$\mathbf{H}^{+}(\mathbf{q})\,\ddot{\mathbf{z}}_0 + \mathbf{C}^{+}(\mathbf{q}, {}^0\boldsymbol{\omega}_0, \dot{\mathbf{q}}) = \mathbf{Q} \quad (19)$$

where $\mathbf{C}^{+}$ contains the nonlinear terms of the equations of motion. Equation (19) represents N+6 equations that describe the motion of a free-flying manipulator system under the effect of external forces and torques and internal actuator torques. The generalized forces $\mathbf{Q}$ are decomposed into the unknown disturbance forces, $\mathbf{Q}_d$, and the control forces, $\mathbf{Q}_c$:

$$\mathbf{Q} = \mathbf{Q}_c + \mathbf{Q}_d \quad (20)$$

The control forces include the spacecraft's thruster forces, ${}^0\mathbf{f}_S$, and torques, ${}^0\mathbf{n}_S$, and the manipulator's joint torques, $\boldsymbol{\tau}$. Forces can be generated by thruster actuators, while torques on the spacecraft can be generated by thruster actuators, momentum gyros or reaction wheels. To simplify the notation, the subscript S is used to represent the CM of the spacecraft, which corresponds to k=0, m=CM, see Equations (9), (10). Then, the control forces can be written as:

$$\mathbf{Q}_c = \mathbf{J}_q^{T}\begin{bmatrix} {}^0\mathbf{f}_S \\ {}^0\mathbf{n}_S \\ \boldsymbol{\tau} \end{bmatrix} = \begin{bmatrix} \mathbf{1} & {}^0\mathbf{J}_{11.S} & {}^0\mathbf{J}_{12.S} \\ \mathbf{0} & \mathbf{1} & {}^0\mathbf{J}_{22.S} \\ \mathbf{0} & \mathbf{0} & \mathbf{1} \end{bmatrix}^{T}\begin{bmatrix} {}^0\mathbf{f}_S \\ {}^0\mathbf{n}_S \\ \boldsymbol{\tau} \end{bmatrix} \quad (21)$$

Note that $\mathbf{J}_q$ is square and always invertible.

IV. COORDINATED MOTION CONTROL

The similarity between Equation (19) and the equations of motion that correspond to a fixed-based manipulator leads to an investigation of whether control laws that are applicable in the latter case also can be used in the control of space robotic systems. However, two differences between the two situations must be pointed out. The first is that an appropriate representation of a spacecraft's attitude is needed, e.g. Euler angles, see Equation (11), or Euler parameters. The second difference is that, due to a spacecraft's mobility, a space robotic system is inherently redundant. This redundancy can be used to achieve additional tasks; here it is used to control a spacecraft's location and attitude by augmenting the system output. This has the advantage that the location and attitude of the spacecraft can be controlled to follow some prescribed plan, and hence, impacts can be avoided. In addition, by planning a spacecraft's motion, the end effector can reach a point while its manipulator assumes some desired configuration. For example, this scheme may allow a manipulator to be in a configuration suitable for applying some forces or to avoid singularities.

To control both the spacecraft and the manipulator in a coordinated way, a relation between the system independent velocities, $\dot{\mathbf{z}}_0$, and the output velocities is needed. To this end, Equation (7) is combined to Equation (11) and written for the spacecraft's CM, which corresponds to k=0 and m=CM, see Equation (7). Subsequently, Equation (13) is combined to Equation (11) as written for the end-effector, and the result is:

$$\dot{\mathbf{z}}_1 \equiv [\dot{\mathbf{r}}_E^T, \dot{\boldsymbol{\theta}}_E^T, \dot{\mathbf{R}}_S^T, \dot{\boldsymbol{\theta}}_S^T]^T = \mathbf{J}_z \dot{\mathbf{z}}_0 \qquad (22)$$

where $\dot{\mathbf{z}}_1$ is the output velocities vector and $\mathbf{J}_z$ is a 12×12 Jacobian matrix if N=6, given by:

$$\mathbf{J}_z = \mathrm{diag}\left(\mathbf{T}_0, \mathbf{S}_E^{-1}\mathbf{T}_0, \mathbf{T}_0, \mathbf{S}_S^{-1}\mathbf{T}_0\right) \begin{bmatrix} \mathbf{1} & {}^0\mathbf{J}_{11} & {}^0\mathbf{J}_{12} \\ \mathbf{0} & \mathbf{1} & {}^0\mathbf{J}_{22} \\ \mathbf{1} & {}^0\mathbf{J}_{11.S} & {}^0\mathbf{J}_{12.S} \\ \mathbf{0} & \mathbf{1} & \mathbf{0} \end{bmatrix} \qquad (23)$$

Note that $\dot{\mathbf{z}}_1$ contains the spacecraft and end-effector linear velocity and orientation rates. Neglecting the non-physical representation singularities, $\mathbf{J}_z$ is an invertible 12×12 matrix, unless the manipulator is kinematically singular.

The equations of motion (19) and the Jacobian given by Equation (23) can be used to implement various motion control techniques in a way similar to Khatib's "operational" approach, where a Jacobian relating generalized joint velocities to operational velocities is used to design controllers in the cartesian space [12]. Here, a Transposed-Jacobian type controller with inertial feedback is designed to demonstrate this approach. The equations of motion in the $\mathbf{z}_1$ domain can be found by substituting Equation (22) into Equation (19) to obtain the form:

$$\tilde{\mathbf{H}}\, \ddot{\mathbf{z}}_1 + \tilde{\mathbf{C}} = (\mathbf{J}_z^{-1})^T \mathbf{Q}_c \qquad (24)$$

where $\tilde{\mathbf{C}}$ contains the nonlinear terms and $\tilde{\mathbf{H}}$ is given by:

$$\tilde{\mathbf{H}} = (\mathbf{J}_z^{-1})^T \mathbf{H}^+ \mathbf{J}_z^{-1} \qquad (25)$$

This inertia matrix is positive definite if $\mathbf{J}_z$ is nonsingular. An error $\mathbf{e}$ is defined as:

$$\mathbf{e} \equiv \mathbf{z}_{1,des} - \mathbf{z}_1 \qquad (26)$$

where $\mathbf{z}_1$ is provided by inertial feedback, and $\mathbf{z}_{1,des}$ is the desired inertial point. It is assumed here that inertial measurements of the position and orientation of the spacecraft and of the end-effector are available. By taking the input $\mathbf{Q}_c$ to be:

$$\mathbf{Q}_c = \mathbf{J}_z^T \{ \tilde{\mathbf{H}} (\mathbf{K}_p \mathbf{e} + \mathbf{K}_d \dot{\mathbf{e}} + \ddot{\mathbf{z}}_{1,des}) + \tilde{\mathbf{C}} \} \qquad (27)$$

where $\mathbf{K}_p$, and $\mathbf{K}_d$ are positive definite diagonal matrices, the error dynamics become:

$$\ddot{\mathbf{e}} + \mathbf{K}_d \dot{\mathbf{e}} + \mathbf{K}_p \mathbf{e} = \mathbf{0} \qquad (28)$$

and therefore the error converges asymptotically to zero. Equation (27) is a modification of the operational space controller, [12]. If high enough gains are used, a simpler Transposed Jacobian controller can be employed, [14]:

$$\mathbf{Q}_c = \mathbf{J}_z^T (\mathbf{K}_p \mathbf{e} + \mathbf{K}_d \dot{\mathbf{e}}) \qquad (29)$$

Note that if a singularity is encountered, the controllers given by Equations (27) and (29) will result in errors but will not fail computationally. If some small disturbance acts on the system, a small steady state error is expected, because these controllers are basically PD controllers. Finally, the reaction jet forces and torques and the joint torques can be found by inverting Equation (21). Assuming that Equation (29) is used, these are given by:

$$\begin{bmatrix} {}^0\mathbf{f}_S \\ {}^0\mathbf{n}_S \\ \boldsymbol{\tau} \end{bmatrix} = (\mathbf{J}_q^T)^{-1} \mathbf{J}_z^T (\mathbf{K}_p \mathbf{e} + \mathbf{K}_d \dot{\mathbf{e}}) \qquad (30)$$

The inversion of $\mathbf{J}_q$ is possible since this Jacobian is always nonsingular. Equation (30) is the final result that permits coordinated control of both the spacecraft and its manipulator, based on inertial measurements of the spacecraft and end-effector locations and orientations. If no such measurements are available, the error $\mathbf{e}$ can be estimated by integrating the equations of motion in real time, but then errors due to model uncertainties will be introduced. This method of motion control is demonstrated with an example.

V. EXAMPLE

As an example of the algorithm outlined in the previous section, a controller was designed that is capable of simultaneously controlling a spacecraft's motion as well as its manipulator's end-effector motion. The example system is planar and consists of a two DOF manipulator on a three DOF spacecraft, see Figure 2. Its kinematic, mass and inertia properties are given in Table I.

Table I. System parameters for the example.

Body	l_i (m)	r_i (m)	m_i (Kg)	I_i (Kg m^2)
0	.5	.5	40	6.667
1	.5	.5	4	0.333
2	.5	.5	3	0.250

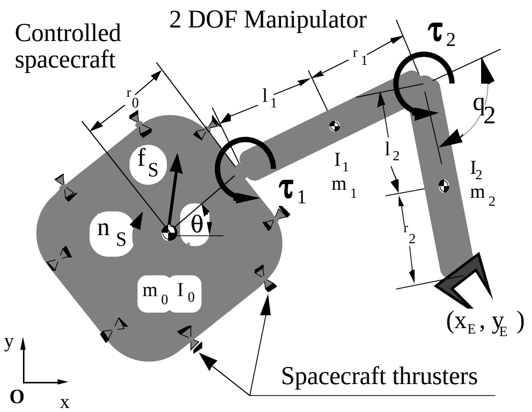

Figure 2. A two-DOF manipulator on a three-DOF free-flying spacecraft.

The spacecraft is assumed to be equipped with reaction jets that can provide forces and torques proportional to the commanded control input. In this case the independent coordinates vector, $\mathbf{z}_0$, and the vector of coordinates to be controlled, $\mathbf{z}_1$, are:

$$\mathbf{z}_0 = [{}^0x_{cm},{}^0y_{cm},\theta,q_1,q_2]^T, \quad \mathbf{z}_1 = [x_S,y_S,\theta,x_E,y_E]^T \tag{31}$$

where ${}^0x_{cm}$ and ${}^0y_{cm}$ are the system CM coordinates with respect to an inertia frame, θ is the spacecraft's attitude, q_1,q_2 are the manipulator joint angles, x_S,y_S are the spacecraft's CM coordinates and finally x_E,y_E are the end-effector's coordinates. $\mathbf{J}_q$ is always nonsingular and in this example, $\mathbf{J}_z$ is nonsingular unless $\sin(q_2) = 0$, which corresponds to a kinematically singular manipulator.

The controller described by Equation (30) is used to calculate the required reaction jet forces and torques, ${}^0\mathbf{f}_S$ and ${}^0n_S$, and the manipulator joint torques, $\boldsymbol{\tau}$. It allows the specification of a desired trajectory for both the spacecraft and the manipulator. Hence, the motion of the whole system can be coordinated. If the given trajectory is not feasible, the desired motion will not be achieved, but the controller will try instead to come as close as possible to the specified trajectory.

In the simulations that follow, the inertially fixed frame is set at the position of the system CM at the initial time, when $(\theta,q_1,q_2) = (21^\circ,-58^\circ,60.3^\circ)$ and the end-effector is at (2.0, 0.0). In this coordinate frame, the end-effector is commanded to move to point (1.5, 1.5). The spacecraft is commanded to move to $(\theta,x_S,y_S) = (15^\circ,0.5,0.5)$. The gains used are $\mathbf{K}_p$ = diag(30,30,30,30,30) and $\mathbf{K}_d$ = diag(60,60,60,60,60). Figure 3 shows the motion of the system in inertial space. The end-effector converges along a straight line to the desired point, while the spacecraft assumes the desired location and attitude. Note that if the spacecraft were fixed at its initial location, the end-effector would have reached point B in an almost singular configuration. If the spacecraft were free-floating and its reaction jets were turned off, then point B could not have been reached from point A following the straight line path, due to the existence of dynamic singularities, see [9,11]. In contrast to these schemes, coordinated control permits the end-effector to reach point B in a favorable configuration, away from singularities.

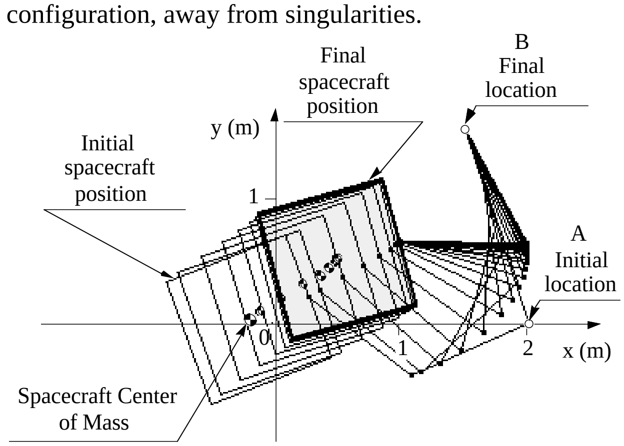

Figure 3. Coordinated spacecraft/manipulator motion of the example system.

Figure 4 shows the error in the spacecraft's position and attitude during the end-effector motion. These errors are eliminated in about 12 sec.

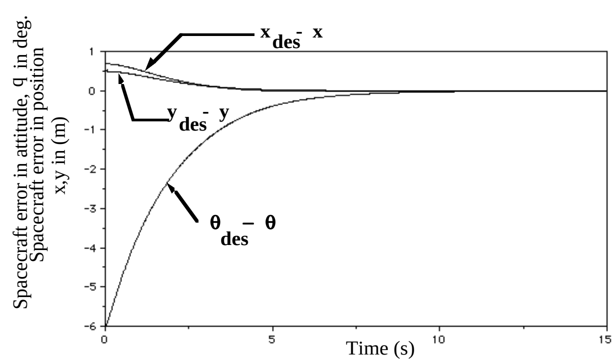

Figure 4. Motion of the spacecraft during the maneuver shown in Figure 3.

Figure 5 shows the reaction jet forces required to move the spacecraft during the first five seconds of the maneuver.

Since the error converges asymptotically to zero and no disturbances act on the system, the reaction jet forces approach asymptotically to zero as soon as the motion stops.

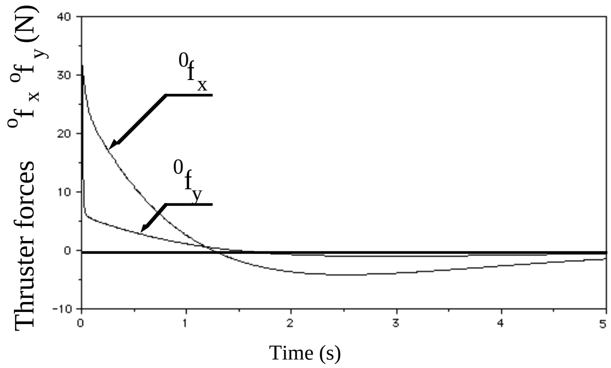

Figure 5. Thruster forces required during the maneuver shown in Figure 3.

VI. CONCLUSIONS

This paper presents the dynamics of free-flying space robotic systems as compactly written functions of the system's barycentric vectors. Coordinated control of both a spacecraft and its manipulator is based on augmenting the control requirements to include the location and attitude of a system's spacecraft. This scheme can be used to avoid impacts of a robotic system with its environment or to maintain a favorable manipulator configuration. A Transposed-Jacobian type controller with inertial feedback demonstrates these techniques.

VII. ACKNOWLEDGMENTS

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