ON THE NATURE OF CONTROL ALGORITHMS FOR SPACE MANIPULATORS

Evangelos Papadopoulos†, Steven Dubowsky††

Department of Mechanical Engineering
Massachusetts Institute of Technology
Cambridge, MA 02139

ABSTRACT

This paper strongly suggests that *nearly any* control algorithm that can be used for fixed-base manipulators can be also employed in the control of free-floating space systems, with a few weak additional conditions, based on the structural similarities between the kinematic and dynamic equations of a free-floating space system and the equations for the same manipulator with a fixed base. Barycenters are used to formulate efficiently the kinematic and dynamic equations of free-floating space manipulators. A control algorithm for a space manipulator system is designed to demonstrate the value of the analysis. The results obtained should encourage the development of a wide variety of control algorithms for free-floating space manipulator systems.

I. INTRODUCTION

The planning and control of the robotic manipulators expected to play important roles in future space missions, pose some problems not found in fixed-base manipulators due to the dynamic coupling between space manipulators and their spacecraft. A number of control techniques for such systems have been proposed. These schemes can be classified in three categories. In the first, reaction jets control spacecraft position and attitude, compensating for any manipulator dynamic forces exerted on the spacecraft. Control laws for earth-bound manipulators can be used in this case, but their utility may be limited because manipulator motions can saturate a reaction jet system. Reaction jets also may consume relatively large amounts of attitude control fuel, limiting the useful life of the system [1,2]. In the second category, reaction wheels or jets control a spacecraft's attitude but not its translation [3,4]. The control of these systems is somewhat more complicated than for the first category, although a technique called the Virtual Manipulator (VM) method can be used to simplify the problem [4-6]. In the third category, free-floating systems have been proposed in order to conserve fuel or electrical power [5-9]. These permit the manipulator's spacecraft to move freely in response to the manipulator motions. These too can be modeled using the VM approach [5,6]. Past control algorithms for free-floating systems have been proposed and their validity demonstrated only on a case by case basis [7-10]. Algorithms for such systems which ignore the kinematics or dynamics of the spacecraft in their formulation have been found to have problems [9,10]. These problems may be attributed to *dynamic singularities* which are not found in earth bound manipulators [11,12].

This paper takes a more fundamental approach to the question of what the characteristics of control algorithms are which may be applied to the motion control of free-floating space manipulators. The results obtained show that nearly any algorithm which can be applied to conventional fixed-base manipulators can be directly applied to free-floating manipulators, with a few weak additional conditions. These results should encourage the development of a wide range of control algorithms for free-floating space manipulator systems.

† Graduate Research Assistant
†† Professor

II. DYNAMICS OF FREE-FLOATING SYSTEMS

This section develops the dynamic equations of a rigid free-floating manipulator system (see Figure 1) using a Lagrangian approach. First, the system kinetic energy is expressed as a function of the generalized coordinates and their velocities.

The body 0 in Figure 1 represents the spacecraft; the bodies k (k=1,...,N) represent the N manipulator links. The manipulator joint angles and velocities are represented by the N×1 column vectors $\mathbf{q}$ and $\dot{\mathbf{q}}$. The spacecraft can translate and rotate in response to the arm movements. The manipulator is assumed to have revolute joints and an open chain kinematic configuration so that, in a system with an N degree-of-freedom (DOF) manipulator, there will be 6+N DOF.

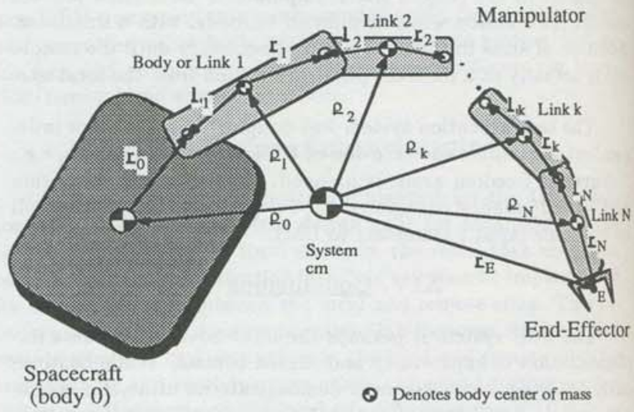

Figure 1. A free-floating space manipulator system.

In the absence of external forces, the system center of mass (*CM*) will be fixed in inertial space and the inertial origin, O, can be chosen to be the *CM*. The system kinetic energy, T, can be written as:

$$T = \frac{1}{2} \sum_{k=0}^{N} \{ \underline{\omega}_k \cdot \underline{\mathbf{I}}_k \cdot \underline{\omega}_k + m_k \, \dot{\underline{\rho}}_k \cdot \dot{\underline{\rho}}_k \} \tag{1}$$

where m_k is the mass of the k^{th} body $\underline{\mathbf{I}}_k$ is its inertia dyadic with respect to its center of mass, and $\dot{\underline{\rho}}_k$ and $\underline{\omega}_k$ are its linear and angular inertial velocities.

It can be shown that T can be written in a more compact form as a function of the N+1 angular velocities as [11,12]:

$$T = \frac{1}{2}\sum_{j=0}^{N}\sum_{i=0}^{N} \underline{\omega}_i \cdot \underline{\mathbf{D}}_{ij} \cdot \underline{\omega}_j \qquad (2)$$

where the $\underline{\mathbf{D}}_{ij}$ terms are inertia dyadics that are functions of the mass and inertia distribution of the space manipulator system, and are given by [11,12]:

$$\underline{\mathbf{D}}_{ij} \equiv \begin{cases} -M\,\{(\underline{\mathbf{l}}_j^* \cdot \underline{\mathbf{r}}_i^*)\,\underline{\mathbf{1}} - \underline{\mathbf{l}}_j^*\,\underline{\mathbf{r}}_i^*\} & i<j \\ \underline{\mathbf{I}}_i + \sum_{k=0}^{N} m_k\,\{(\underline{\mathbf{v}}_{ik} \cdot \underline{\mathbf{v}}_{ik})\,\underline{\mathbf{1}} - \underline{\mathbf{v}}_{ik}\,\underline{\mathbf{v}}_{ik}\} & i=j \\ -M\,\{(\underline{\mathbf{r}}_j^* \cdot \underline{\mathbf{l}}_i^*)\,\underline{\mathbf{1}} - \underline{\mathbf{r}}_j^*\,\underline{\mathbf{l}}_i^*\} & i>j \end{cases} \qquad (3)$$

In the above equation, M is the total system mass, $\underline{\mathbf{1}}$ is the unit dyadic. The vectors $\underline{\mathbf{v}}_{ik}$ (i,k=0,...,N), $\underline{\mathbf{r}}_i^*$ and $\underline{\mathbf{l}}_i^*$ (i=0,...,N) are defined by the barycenters *(BC)* [13,14] of the ith body. First, the body fixed vector $\underline{\mathbf{c}}_i$ is defined as referring to the location of the ith body's barycenter with respect to the body's *CM*. It can be shown that $\underline{\mathbf{c}}_i$ is equal to:

$$\underline{\mathbf{c}}_i = \underline{\mathbf{l}}_i\,\mu_i + \underline{\mathbf{r}}_i\,(1-\mu_{i+1}) \qquad i = 0,...,N \qquad (4)$$

where μ_i represents the mass distribution given by:

$$\mu_i \equiv \begin{cases} 0 & i = 0 \\ \sum_{j=0}^{i-1} \frac{m_j}{M} & i = 1,...,N \\ 1 & i = N+1 \end{cases} \qquad (5)$$

It might be noted that the barycenter of the ith body can be found equivalently by adding a point mass equal to $M\mu_i$ to joint i, and a point mass equal to $M(1-\mu_{i+1})$ to joint i+1, forming an *augmented body* [13,14]. The barycenter is then the center of mass of the augmented body (see Figure 2). Figure 2 also shows the body-fixed vectors $\underline{\mathbf{r}}_i^*$ and $\underline{\mathbf{l}}_i^*$ required by Equation (3) and the vector $\underline{\mathbf{c}}_i^*$, which can be written as:

$$\underline{\mathbf{c}}_i^* = -\underline{\mathbf{c}}_i \qquad (6a)$$

$$\underline{\mathbf{r}}_i^* = \underline{\mathbf{r}}_i - \underline{\mathbf{c}}_i \qquad (6b)$$

$$\underline{\mathbf{l}}_i^* = \underline{\mathbf{l}}_i - \underline{\mathbf{c}}_i \qquad (6c)$$

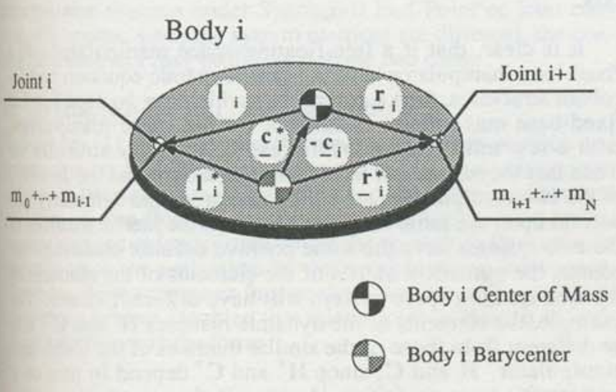

Figure 2. Definition of vectors $\underline{\mathbf{r}}_i^*, \underline{\mathbf{l}}_i^*, \underline{\mathbf{c}}_i^*$.

Finally, the vectors $\underline{\mathbf{v}}_{ik}$ in Equation (3) are defined by:

$$\underline{\mathbf{v}}_{ik} \equiv \begin{cases} \underline{\mathbf{r}}_i^* & i<k \\ \underline{\mathbf{c}}_i^* & k=i \\ \underline{\mathbf{l}}_i^* & i>k \end{cases} \qquad (7)$$

Equation (2) is a compact representation of the system's kinetic energy, but it is convenient to work with a scalar (matrix) form of the equation, using the following notation. Bold lower case symbols represent column vectors; bold upper case symbols represent matrices. Right superscripts are interpreted as "with respect to," left superscripts as "expressed in frame." A missing left superscript implies a column vector expressed in the inertial frame. In addition, we introduce N+1 reference frames, each one attached to the *CM* of each body, with axes parallel to the body's principal axes. Hence, the body inertia matrix expressed in this frame is diagonal.

The system kinetic energy is written in matrix form as follows. The inertial angular velocity of body j, expressed by the vector ω_j, can be written as the sum of the inertial angular velocity of the spacecraft, ω_0, and the inertial angular velocity of body j relative to the spacecraft, ω_j^0:

$$\omega_j = \omega_0 + \omega_j^0 \qquad j = 1,...,N \qquad (8)$$

The angular velocity ω_j^0 can be expressed as a function of the joint angles, $\mathbf{q}$. This is accomplished by defining a 3×3 transformation matrix ${}^{i-1}\mathbf{A}_i(q_i)$ (i=1,...,N), which is a function of q_i (the ith relative joint angle), which transforms a column vector expressed in frame i to a column vector expressed in frame i-1. An additional transformation matrix can be defined as:

$${}^0\mathbf{T}_i\,(q_1,..., q_i) = {}^0\mathbf{A}_1(q_1)...{}^{i-1}\mathbf{A}_i(q_i) \qquad i = 1,...,N \qquad (9)$$

${}^0\mathbf{T}_i$ transforms a column vector expressed in frame i to a column vector in frame 0. Finally, $\mathbf{T}_0$ is the transformation matrix from the spacecraft frame to the inertial frame and is a function of the spacecraft's attitude, expressed by the Euler parameters ε and η, $\mathbf{T}_0 = \mathbf{T}_0(\varepsilon, \eta)$, see [15]. Using these transformation matrices, the inertial velocity ω_j^0 of link j relative to the spacecraft can be written as:

$$\omega_j^0 = \mathbf{T}_0 \sum_{i=1}^{j} {}^0\mathbf{T}_i\,{}^i\mathbf{u}_i\,\dot{q}_i = \mathbf{T}_0\,{}^0\mathbf{F}_j\,\dot{\mathbf{q}} \qquad j = 1,...,N \qquad (10)$$

where ${}^i\mathbf{u}_i$ is the unit column vector in frame i parallel to the axis of revolution through joint i, and ${}^0\mathbf{F}_j$ is a 3×N matrix, function of the joint angles $\mathbf{q}$ only and given by:

$${}^0\mathbf{F}_j(\mathbf{q}) \equiv [{}^0\mathbf{T}_1\,{}^1\mathbf{u}_1,\ {}^0\mathbf{T}_2\,{}^2\mathbf{u}_2, ...,\ {}^0\mathbf{T}_j\,{}^j\mathbf{u}_j,\ \mathbf{0}\,] \qquad j = 1,...,N \qquad (11)$$

where $\mathbf{0}$ is a 3×(N-j) zero element matrix. It is easy to show that the inertia matrices $\mathbf{D}_{ij}$ that correspond to the inertia dyadics given by equation (3) can be expressed with respect to the spacecraft frame of reference as:

$$\mathbf{D}_{ij} = \mathbf{T}_0\,{}^0\mathbf{D}_{ij}\,\mathbf{T}_0^T \qquad i, j = 1,...,N \qquad (12)$$

${}^0\mathbf{D}_{ij}$ is formed according to Equation (3) with all vectors expressed in the base frame. This proves that ${}^0\mathbf{D}_{ij}$ is a function of $\mathbf{q}$ only. Also due to Equation (3), ${}^0\mathbf{D}_{ij} = {}^0\mathbf{D}_{ji}^T$. For convenience define:

$${}^0\mathbf{D}_j \equiv \sum_{i=0}^{N} {}^0\mathbf{D}_{ij} \qquad j = 0,...,N \qquad {}^0\mathbf{D} \equiv \sum_{j=0}^{N} {}^0\mathbf{D}_j \qquad (13a)$$

$$ {}^0\mathbf{D}_q \equiv \sum_{j=1}^{N} {}^0\mathbf{D}_j {}^0\mathbf{F}_j \qquad {}^0\mathbf{D}_{qq} \equiv \sum_{j=1}^{N} \sum_{i=1}^{N} {}^0\mathbf{F}_i^T \, {}^0\mathbf{D}_{ij} \, {}^0\mathbf{F}_j \quad (13b) $$

where all the above are functions of $\mathbf{q}$ only. The terms in Equation (13b) depend upon the manipulator mass and inertia properties, while the terms in equation (13a), in addition depend upon the spacecraft inertia.

Using Equations (8-13), the matrix form of Equation (2) can be written as:

$$ T = \frac{1}{2} {}^0\omega_0^T \, {}^0\mathbf{D} \, {}^0\omega_0 + \frac{1}{2} {}^0\omega_0^T \, {}^0\mathbf{D}_q \dot{\mathbf{q}} + \frac{1}{2} \dot{\mathbf{q}}^T \, {}^0\mathbf{D}_q^T \, {}^0\omega_0 + \frac{1}{2} \dot{\mathbf{q}}^T \, {}^0\mathbf{D}_{qq} \dot{\mathbf{q}} \quad (14) $$

where ${}^0\omega_0$ is the spacecraft angular velocity expressed in its frame. Note that T is a function of ${}^0\omega_0$, $\dot{\mathbf{q}}$ and $\mathbf{q}$ only. This observation suggests that if ${}^0\omega_0$ can be expressed as a function of $\mathbf{q}$ and $\dot{\mathbf{q}}$ only, then the spacecraft attitude coordinates are *ignorable*, that is they do not appear in the expression for T, see [16]. This is exactly the case with free-floating space manipulators.

The system angular momentum can be written as [11,12]:

$$ \mathbf{l}_{cm} = \mathbf{T}_0 \sum_{j=0}^{N} \sum_{i=0}^{N} {}^0\mathbf{D}_{ij} \, {}^0\omega_j = \mathbf{T}_0 \left({}^0\mathbf{D} \, {}^0\omega_0 + {}^0\mathbf{D}_q \dot{\mathbf{q}} \right) \quad (15) $$

In the absence of external torques, the system angular momentum is constant. The first two terms in Equation (14) vanish assuming that the system initial angular momentum is zero. Equation (15) then yields the spacecraft inertial angular velocity as a function of the inertial angular velocities relative to the spacecraft frame:

$$ {}^0\omega_0 = - {}^0\mathbf{D}^{-1} \, {}^0\mathbf{D}_q \dot{\mathbf{q}} \quad (16) $$

Note that inversion of ${}^0\mathbf{D}$ is always possible because it is a symmetric positive definite matrix that represents the inertia of the free-floating system about its *CM*. Substituting ${}^0\omega_0$ in Equation (9) and after some algebraic manipulation, T results in:

$$ T = \frac{1}{2} \dot{\mathbf{q}}^T \mathbf{H}^*(\mathbf{q}) \dot{\mathbf{q}} \quad (17) $$

where $\mathbf{H}^*(\mathbf{q})$ is the system inertia matrix, given by:

$$ \mathbf{H}^*(\mathbf{q}) \equiv {}^0\mathbf{D}_{qq} - {}^0\mathbf{D}_q^T \, {}^0\mathbf{D}^{-1} \, {}^0\mathbf{D}_q \quad (18) $$

Again, the fact that ${}^0\mathbf{D}_{ij} = {}^0\mathbf{D}_{ji}^T$ was used. The expression for T given by Equation (17) is identically equal to the system *Routhian*, see [16], and is thus the appropriate Lagrangian for this system. Equation (17) shows that T is a function of $(\mathbf{q}, \dot{\mathbf{q}})$, the manipulator joint angles and velocities.

It is easy to show that the system inertia matrix, $\mathbf{H}^*$, is an N×N positive definite symmetric inertia matrix, which depends on $\mathbf{q}$ and the system properties. All elements of $\mathbf{H}^*$ are functions of the manipulator joint angles q_i (i=1,...,N) only, since the total system inertia with respect to its *CM*, ${}^0\mathbf{D}$, and ${}^0\mathbf{D}_j$ are functions of only the q_i's and not of the spacecraft attitude. This proves that the system inertia matrix $\mathbf{H}^*$ has the same structural properties as the inertia matrices that correspond to fixed-base manipulators.

The above equations are important because they show how to construct the system inertia matrix $\mathbf{H}^*$ efficiently. The steps needed to accomplish this task are: first, compute all the ${}^0\mathbf{v}_{ik}$ vectors according to Equations (4)-(7) and (9). Second, compute the ${}^0\mathbf{D}_{ij}$ inertia matrices, according to Equation (3), using the ${}^0\mathbf{v}_{ik}$ in the place of the $\mathbf{v}_{ik}$. Third, find the ${}^0\mathbf{F}_i$ matrices according to Equation (11) and, finally, find the inertia matrix $\mathbf{H}^*$. Equations (17) and (18) are very useful in understanding the dynamics of free-floating systems and we will refer to them again.

In the absence of gravity, the potential energy of a rigid system is zero, and the system's dynamic equations are given by:

$$ \frac{d}{dt} \left\{ \frac{\partial T}{\partial \dot{\mathbf{q}}} \right\} - \frac{\partial T}{\partial \mathbf{q}} = \tau \quad (19) $$

where τ is the generalized force vector which, in this case, is equal to the torque vector $[\, \tau_1, \tau_2, ..., \tau_N \,]^T$.

Applying Equation (19) to the kinetic energy given by Equation (17) results in a set of N dynamic equations of the form:

$$ \mathbf{H}^*(\mathbf{q}) \ddot{\mathbf{q}} + \mathbf{C}^*(\mathbf{q}, \dot{\mathbf{q}}) \dot{\mathbf{q}} = \tau \quad (20) $$

where $\mathbf{H}^*(\mathbf{q})$, is the system inertia matrix defined by Equation (18) and $\mathbf{C}^*(\mathbf{q}, \dot{\mathbf{q}}) \dot{\mathbf{q}}$ contains the nonlinear Coriolis and centrifugal terms. Note that the system dynamic equation is a function only of the joint variables, and not of the spacecraft attitude or position variables. This results from the fact that the system kinetic energy does not depend on spacecraft attitude nor on its angular or linear velocity, when the initial angular momentum is zero and the system is free of external torques. The spacecraft's contribution to the system's kinetic energy, T, enters in through the inertia matrices ${}^0\mathbf{D}_{0i}$ (i=0,...,N), which depend on its mass m_0 and inertia $\mathbf{I}_0$.

III. THE NATURE OF CONTROL ALGORITHMS FOR FREE-FLOATING SYSTEMS

It is well known that one needs three basic elements (or some combination of them) in order generally to control a fixed-base manipulator. These are, first, an invertible representation of manipulator kinematics, which can be in the form of a closure equation or a Jacobian. Most control algorithms use the latter. Second, one needs a set of dynamic equations which describe the response of manipulator joint angles to actuator torques or forces. Third, a control algorithm must use sensory information and calculate the required torques or forces to achieve a desired task.

It is clear, that if a free-floating space manipulator and a fixed-base manipulator have the same dynamic equation and Jacobian structures, then a control law which can be used for that fixed-base manipulator is suitable for the space manipulator, with a few mild limitations discussed later. By structure we mean that the matrices of the dynamic equations and the Jacobian of the two manipulators have the same order and symmetry and depend upon the same variables. Further, the inertia matrices of the two systems have the same positive definite character. Of course, the numerical values of the elements of the matrices of the free-floating space system will have different values. For example, the elements of the dynamic matrices $\mathbf{H}^*$ and $\mathbf{C}^*$, will be different from those of the similar matrices of the fixed-base manipulator, $\mathbf{H}$ and $\mathbf{C}$, since $\mathbf{H}^*$ and $\mathbf{C}^*$ depend in part on a spacecraft's mass properties. As a result, the same torque vector τ will produce different joint accelerations in the two systems. However, we are interested here in the structure of the dynamic equations and not in numerical values of the inertia matrix elements. Also, since the applicability of fixed-base controllers does not depend upon the existence of gravity, it can be neglected for the purposes of this comparison.

We will compare the structures of the dynamic and kinematic equations of free-floating manipulators to the ones for fixed-base manipulators and show that based on the above argument it should be possible to develop a free-floating space manipulator control algorithm based on nearly any algorithm used for fixed-base manipulators, provided that some weak conditions hold. Two types of motion control will be considered. The first, called Spacecraft End-Point Motion Control, is the form of control in which the manipulator end-point is commanded to move to a location fixed to its own spacecraft, or when a simple joint motion is commanded, such as when the manipulator is to be driven at its stowed position. The second, called Inertial End-Point Motion Control, is when the manipulator end-point is commanded to move with respect to inertial space.

A. Spacecraft End-Point Motion Control.

The comparison between this form of control for a free-floating manipulator and a fixed-base manipulator is rather straightforward. It has been shown, see Equation (20), that the minimum number of equations describing the dynamics of the N+6 DOF space system is N for an N DOF manipulator, the same as for a fixed-base N DOF manipulator. It is also been proven above that the space system inertia matrix, $\mathbf{H}^*$, depends only on the manipulator's joint variables, $\mathbf{q}$, and is a symmetric matrix. These are also the properties of the inertia matrix for the fixed-base system. It can be shown also that the $\mathbf{H}^*$ is positive definite, as $\mathbf{H}$. Finally, since $\mathbf{C}^*$ is derived from $\mathbf{H}^*$, it will have the same form as the fixed-base $\mathbf{C}$ which is derived from $\mathbf{H}$. Hence, the dynamic equations of both systems have the same structure as defined above.

If the spacecraft becomes very large, m_0 and I_0 approach infinity, and $\mathbf{H}^*$ and $\mathbf{C}^*$ converge to $\mathbf{H}$ and $\mathbf{C}$. This should be expected, because a very large spacecraft will not react to the manipulator's motions and the system will behave essentially as a fixed-base system. Also, the order of the system remains fixed, equal to N, irrespective of the size of m_0 and $\mathbf{I}_0$. Finally, since the motion of the space manipulator is controlled with respect to its own base, then the Jacobian relating its joint angle and the end-effector velocities is identical to that of the fixed-base manipulator, called $\mathbf{J}$. The above observations hold equally for the simple joint control problem where a $\mathbf{J}$ is not required.

Thus we conclude that nearly any control algorithm that can be used for fixed-base manipulators can also be used for space manipulator systems under Spacecraft End-Point or joint control. Of course, since the system matrices are different, the control gain matrices may be different in the two cases.

B. Inertial End-Point Motion Control.

References [11,12] show that the inertial end-point position and orientation, $\mathbf{x}$, are not unique functions of the manipulator joint angles, $\mathbf{q}$, because of the effect of a spacecraft's orientation and position. A free-floating space manipulator system is an under-constrained redundant system where the spacecraft final attitude depends upon the path taken by the manipulator in joint space. However, these references further show that it is still possible to construct a system Jacobian that relates joint motions $\dot{\mathbf{q}}$ to end-point velocities $\dot{\mathbf{x}}$ in the form:

$$\dot{\mathbf{x}} = [\,\dot{\mathbf{r}}_E\,, \omega_E\,]^T = \mathbf{J}^*\,\dot{\mathbf{q}} \tag{21}$$

$$\mathbf{J}^* = \mathrm{diag}(\mathbf{T}_0, \mathbf{T}_0)\,{}^0\mathbf{J}^*(\mathbf{q}) \tag{22}$$

where $\dot{\mathbf{r}}_E$ is the end-point inertial velocity, ω_E is the end-point inertial angular velocity, and ${}^0\mathbf{J}^*(\mathbf{q})$ is a 6×N Jacobian which is a function of both the manipulator configuration, $\mathbf{q}$, and of the system mass and inertia properties. If N=6, then ${}^0\mathbf{J}^*(\mathbf{q})$ becomes a square matrix. $\mathbf{T}_0$ depends on the spacecraft attitude, which can be measured or estimated as shown in [11]. It is unnecessary to use spacecraft attitude where the inertial motion is measured with respect to the spacecraft frame, such as in references [7,8]. In that case the Jacobian required in equation (21) is simply ${}^0\mathbf{J}^*(\mathbf{q})$.

It is well known that the Jacobian, $\mathbf{J}$, of a fixed-base manipulator is a 6×N matrix that depends on $\mathbf{q}$ and the link lengths of the manipulator. $\mathbf{J}^*$ or ${}^0\mathbf{J}^*(\mathbf{q})$ has the same dimensions as $\mathbf{J}$ and also depends on $\mathbf{q}$ as well as on the ${}^0\mathbf{v}_{ik}$ vectors, scaled by the ${}^0\mathbf{D}_{ij}$ (i,j=0,...,N) inertia matrices. This means that the free-floating system differential kinematics, although complicated, have the same structure as the kinematics of the same manipulator with a fixed base, as defined above. Indeed, if a spacecraft's mass and inertia, m_0 and $\mathbf{I}_0$, are large, $\mathbf{T}_0$ approaches a constant matrix and $\mathrm{diag}(\mathbf{T}_0, \mathbf{T}_0)\,{}^0\mathbf{J}^*(\mathbf{q})$ results in the normal fixed base manipulator Jacobian. Mass and inertia dependencies vanish.

However, one important difference is that the workspace of the free-floating system is reduced compared to that of the fixed-base manipulator [4,6]. In addition, the free-floating system workspace can be divided in two regions, the *Path Independent Workspace (PIW)* and the *Path Dependent Workspace (PDW)* [11,12]. If the end-point path has points in the PDW, the manipulator may become *dynamically singular*, i.e., its Jacobian $\mathbf{J}^*$ or ${}^0\mathbf{J}^*(\mathbf{q})$ becomes singular, although it may not be *kinematically singular*, meaning alignment of axes or points at the workspace boundaries. A workspace location may be singular or not depending on the *path* or *history* of the motion. If the end-point path belongs entirely in the PIW, no dynamic singularities occur. This leads to the additional condition that the controller must be able to overcome or avoid these singularities.

From the above discussion we concluded that the structure of the kinematics of a free-floating manipulator under Inertial End-Point Control are the same to the fixed-base manipulator case, with the additional conditions that the spacecraft attitude may need to be estimated or measured and dynamic singularities must be avoided. Further, since the dynamics for this case are identical to those discussed above for Spacecraft End-Point Control, they have the same structure as a fixed-base system. Thus it follows that nearly any control algorithm that can be used for fixed-base manipulators can also be used for space manipulator systems under Inertial End-Point Control, provided, of course, that the appropriate matrices are used. For example, laws like resolved rate, resolved acceleration, impedance control or computed torque can be used in space if one uses the appropriate Jacobian and inertia matrix. If these matrices are exactly known, then, as in the fixed-base manipulator case, there is *no* need for end-point sensing control. The controller can rely entirely upon information provided internally by the system. However, end-point sensing may be needed for space manipulator systems when the uncertainty in the system parameters is so large that the resulting real-time errors are unacceptable. This is also true for fixed-base systems.

C. Differences between free-floating and fixed-base manipulators.

So far, we have focused our analysis on the similarities between fixed-base and free-floating systems and have shown that it is possible to develop space control algorithms based on nearly any algorithm used for fixed-base manipulators. Now we discuss some of the practical implementation points of space manipulator control.

1. Terrestrial fixed-base manipulator Jacobians depend on the joint angles **q**, only. In space, the system Jacobians also depend on spacecraft orientation, see Equation (22). This orientation can be calculated, as shown in [11,12], or measured on-line by additional sensors. No such procedure is needed for fixed-base systems.
2. In general, the knowledge of kinematic parameters, such as link lengths, is enough for fixed-base manipulator control purposes. In space, this is not true. The Jacobian of free-floating space system depends on its dynamic properties, such as the masses and inertias of its spacecraft, and on its manipulator's link lengths. In addition, system dynamics are more complicated and depend on products of inertias which increase the error in obtaining the mass matrix. External sensing or on or off-line parameter identification, can be very important for space systems.
3. Singularities are functions of the kinematic structure of the terrestrial fixed-base manipulator only. In space, dynamic singularities exist that depend on the mass and inertia distribution [11,12]. A point in the space system workspace can be singular or not depending on the path taken to reach it. These singularities represent physical limitations and must be avoided. Terrestrial and space workspace sizes and structures are not the same.
4. It is not possible to map desired Cartesian workspace points to a unique set of desired joint angles **q** for free-floating systems in space, as can be done for fixed-base manipulators because infinite sets of joint angles correspond, in general, to any workspace point. Which of these sets of joint angles will actually result when the end-point reaches the desired workspace point depends on the path taken to reach this point. This characteristic of space systems excludes one early manipulator control algorithm, the "point to point" control, see [17].

The above analysis confirms that, with some weak conditions, nearly any control algorithm that can be used in fixed-base systems, can also be used in free-floating systems. This is demonstrated below by applying a control algorithm developed for fixed based manipulators to a space system under inertial end-point control.

IV. A PLANAR EXAMPLE

Here the relatively simple, fixed-base algorithm called the Transpose Jacobian Control by Craig [18], is applied to the simple, planar, free-floating space manipulator system shown in Figure 3, whose parameters are given in Table I. As shown in [13], the system Jacobian in Equation (22) is given by:

$$\mathbf{J}^* = \begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix} {}^0\mathbf{J}^*(\mathbf{q}) \tag{23a}$$

where

$${}^0\mathbf{J}^*(\mathbf{q}) =$$

$$\frac{1}{D}\begin{bmatrix} -(\beta s_1+\gamma s_{12})D_0 & \beta s_1 D_2-\gamma s_{12}(D_0+D_1) \\ -\alpha(D_1+D_2)+(\beta c_1+\gamma c_{12})D_0 & -(\alpha+\beta c_1)D_2+\gamma c_{12}(D_0+D_1) \end{bmatrix} \tag{23b}$$

and $s_1 \equiv \sin(q_1)$, $c_{12} \equiv \cos(q_1+q_2)$ etc. The inertia scalar sums D, D_0, D_1 and D_2 are defined in the Appendix by Equation (A8) and $\alpha \equiv {}^0r_0^*$, $\beta \equiv {}^1r_1^*$ and $\gamma \equiv {}^2c_2^* + r_2$. The lengths α, β, γ are manipulator link lengths scaled by the mass ratios (m_i/M).

Since each D_i and D are functions of **q**, the Jacobian elements are more complicated functions of the **q** than their fixed-base counterparts. This Jacobian, $\mathbf{J}^*$, should be compared to the fixed-base manipulator Jacobian **J** which is given by:

$$\mathbf{J}(\mathbf{q}) = \begin{bmatrix} -(l_1+r_1)s_1-(l_2+r_2)s_{12} & -(l_2+r_2)s_{12} \\ (l_1+r_1)c_1+(l_2+r_2)c_{12} & (l_2+r_2)c_{12} \end{bmatrix} \tag{24}$$

It can be seen that $\mathbf{J}^*$ and **J** have the same structure.

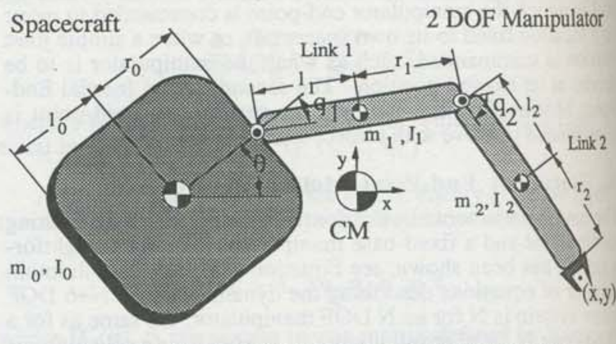

Figure 3. A planar free-floating manipulator system.

Table I. The system parameters.

Body	l_i (m)	r_i (m)	m_i (Kg)	I_i (Kg m^2)
0	.5	.5	40	6.667
1	.5	.5	4	0.333
2	.5	.5	3	0.250

The system inertia matrix is found according to the analysis presented above (see Appendix A for details). The result is:

$$\mathbf{H}^* = \begin{bmatrix} {}^0d_{11}+2\,{}^0d_{12}+{}^0d_{22}-\frac{(D_1+D_2)^2}{D} & {}^0d_{12}+{}^0d_{22}-\frac{D_2(D_1+D_2)}{D} \\ {}^0d_{12}+{}^0d_{22}-\frac{D_2(D_1+D_2)}{D} & {}^0d_{22}-\frac{D_2^2}{D} \end{bmatrix} \tag{25}$$

The system inertia matrix, $\mathbf{H}^*$, is a 2×2 symmetric matrix whose elements are functions of the joint angles q_1 and q_2. Note that D represents the inertia of the whole system with respect to its *CM* and thus, is always a positive number. The above matrix can be seen to have the same structure as the fixed-base inertia matrix **H**, whose elements are given by:

$$\mathbf{H} = \begin{bmatrix} h_{11} & h_{12} \\ h_{12} & h_{22} \end{bmatrix} \tag{26a}$$

$$h_{11} = I_1+m_1l_1^2+m_2(l_1+r_1)^2+2m_2l_2(l_1+r_1)\cos(q_2)+I_2+m_2l_2^2$$

$$h_{12} = h_{21} = m_2l_2(l_1+r_1)\cos(q_2)+I_2+m_2l_2^2$$

$$h_{22} = I_2+m_2l_2^2 \tag{26b}$$

At the limit, when both m_0 and I_0 approach infinity, it is easy to see that $\beta \to l_1+r_1$, $\gamma \to l_2+r_2$, i.e., they approach the manipulator link lengths, $m_0/M \to 1$, $m_1/M \to 0$, $m_2/M \to 0$, $D_0/D \to 1$, $D_1/D \to 0$ and $D_2/D \to 0$; T_0 becomes a constant transformation from the manipulator base frame to the inertial frame, usually the unit matrix; and finally, $\mathbf{J}^* \to \mathbf{J}$, the fixed-base manipulator Jacobian, and $\mathbf{H}^* \rightarrow \mathbf{H}$, the mass matrix of the fixed-base manipulator, as given by Equations (24) and (26).

One can select any control algorithm that can be used for fixed-base manipulators, using the two matrices $\mathbf{H}^*$ and $\mathbf{J}^*$, depending on the manipulator task. In this case, the Transpose Jacobian Control was used, augmented by a velocity feedback term for increased stability margins. The end-point position and velocity, $\mathbf{x}$ and $\dot{\mathbf{x}}$, can be calculated or measured directly. Assuming we measure $\mathbf{x}$ and $\dot{\mathbf{x}}$, the control law is:

$$\tau = \mathbf{J}^{*T} \{ \mathbf{K}_p (\mathbf{x}_{des} - \mathbf{x}) - \mathbf{K}_d \dot{\mathbf{x}} \} \tag{27}$$

where $\mathbf{x}_{des}$ is the inertial desired point location. The matrices $\mathbf{K}_p$ and $\mathbf{K}_d$ are diagonal. Note that this algorithm drives the end-point to the desired location, but does not specify a path. If the control gains are large enough, then the motion of the end-point will be a straight line. The torque vector is non-zero till the ($\mathbf{x}_{des}$ - $\mathbf{x}$) and $\dot{\mathbf{x}}$ are zero, or till the vector in the brackets in Equation (27) is in the null space of $\mathbf{J}^{*T}$. For the purpose of this example, the end-point path will be restricted to the PIW part of the workspace, and hence dynamic singularities will be avoided.

Figure 4 shows the motion of the end-point from the initial location (1,0) to the final (0.8,0.8). The control gain matrices are $\mathbf{K}_p$ = diag(5,5) and $\mathbf{K}_d$ = diag(15,15). The end-point path, shown with a heavy line, is almost a straight line and converges to the desired location. Shown also is the end-point path that results when the control law given by Equation (27) uses the fixed-base Jacobian given by Equation (24). In this case, the end-point diverges from the straight line because it does not resolve the error term correctly. Depending on the situation, the use of the fixed-base Jacobian can create stability problems [9].

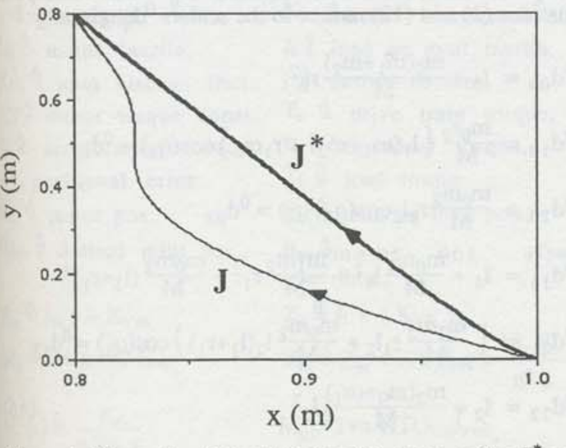

Figure 4. The end-point path in inertial space, using $\mathbf{J}^*$ and $\mathbf{J}$.

Figure 5 shows the spacecraft attitude θ and the joint angles as a function of time, during the end-point motion depicted in Figure 4, when $\mathbf{J}^*$ is used. Note that although the spacecraft attitude changes for about 35°, the manipulator end-point converges to the desired inertial location as it would do if its base were fixed.

We have assumed that the end-point position and velocity are measured. However, if we know the system parameters exactly, they can both be calculated. To do this, one has to measure the joint angles $\mathbf{q}$ and calculate the spacecraft attitude using the conservation of momentum equation [11]. Then the manipulator would rely entirely on internally provided information and would not need end-point sensing. However, it may be very difficult to obtain the correct values for all the system dynamic parameters and any errors will be magnified due to the products involved in the computations. Thus, it is not the physics of the problem that forces the use of end-point control, but rather the incomplete information about the dynamics of the plant. This fact is also true for fixed-base manipulators, although in this case the uncertainty is less severe because the system matrices depend on fewer parameters.

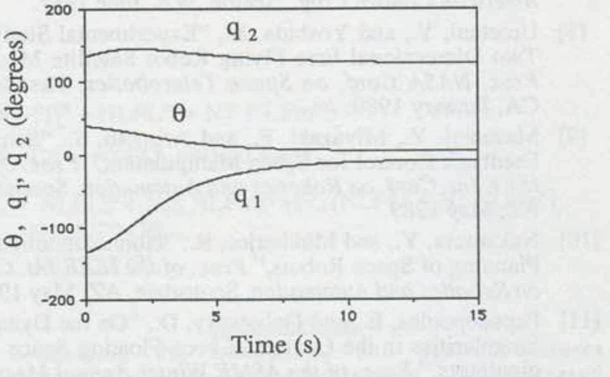

Figure 5. The spacecraft attitude θ and the joint angles q_1 and q_2 as a function of time.

Following the above procedure, any control algorithm that employs the system $\mathbf{H}^*$ and $\mathbf{J}^*$ can be designed. However, control methods that depend on the cancellation of terms, like the computed torque methods, require the exact system inertia matrix $\mathbf{H}^*$, and thus emphasis must be placed in its computation.

V. CONCLUSIONS

A fundamental study has been performed of the characteristics of control algorithms which may be applied to the motion control of space manipulators. The results obtained show that nearly any control algorithm which can be applied to conventional terrestrial fixed-base manipulators, with a few additional conditions, can be directly applied to free-floating space manipulators. We hope that the results encourage the development of more effective control algorithms for free-floating space manipulator systems.

ACKNOWLEDGMENTS

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APPENDIX A

For the planar two link system, shown in Figure 3, the two coordinates of the end-effector, x and y, are assumed to be controlled by the two manipulator joint angles, q_1 and q_2. The end-effector orientation is not controlled for this two DOF system (N=2), hence Equation (21) for this system is simply:

$$\dot{\mathbf{x}} = \dot{\mathbf{r}}_E = \frac{d}{dt}[x, y]^T = \mathbf{J}^* \dot{\mathbf{q}} \tag{A1}$$

where $\mathbf{J}^*$ is given by Equation (23).

In the following, the construction of the system inertia matrix will be demonstrated. First express all $\mathbf{v}_{ik}$ in Equation (7) in the frame of the ith body, according to Equations (4), (5) and (6). For the sake of simplicity assume that all $\mathbf{r}_i$ and $\mathbf{l}_i$ are parallel to the x axis of the ith frame. Hence, only the x-component of the ${}^i\mathbf{v}_{ik}$ is non-zero and given by:

$$\begin{aligned}
{}^0r_0^* &= \frac{1}{M} r_0 m_0 \\
{}^0c_0^* &= -\frac{1}{M} r_0(m_1+m_2) \\
{}^0l_0^* &= -\frac{1}{M} r_0(m_1+m_2) - l_0 \\
{}^1r_1^* &= \frac{1}{M}\{r_1(m_0+m_1)+l_1 m_0\} \\
{}^1c_1^* &= \frac{1}{M}(l_1 m_0 - r_1 m_2) \\
{}^1l_1^* &= -\frac{1}{M}\{l_1(m_1+m_2)+r_1 m_2\} \\
{}^2r_2^* &= \frac{1}{M} l_2(m_0+m_1) + r_2 \\
{}^2c_2^* &= \frac{1}{M} l_2(m_0+m_1) \\
{}^2l_2^* &= -\frac{1}{M} l_2 m_2
\end{aligned} \tag{A2}$$

where the total mass of the system, M, is given by:

$$M = m_0 + m_1 + m_2 \tag{A3}$$

For this example, the transformation matrix from the spacecraft frame to the inertial frame, $\mathbf{T}_0$, is given by:

$$\mathbf{T}_0(\theta) = \mathrm{Rot}(\theta) = \begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix} \tag{A4}$$

where θ denotes the spacecraft attitude, as shown in Figure 3. Only the planar sub-part of the transformation matrices is used for simplicity. The transformation matrices ${}^0\mathbf{T}_i$ are found using Equation (9):

$$ {}^0\mathbf{T}_1 = \mathrm{Rot}(q_1) \qquad {}^0\mathbf{T}_2 = \mathrm{Rot}(q_1)\,\mathrm{Rot}(q_2) \tag{A5}$$

For the planar case, the inertia matrices ${}^0\mathbf{D}_{ij}$ which correspond to Equations (3) and (12) reduce to the scalars ${}^0d_{ij}$, given by:

$$\begin{aligned}
{}^0d_{00} &= I_0 + \frac{m_0(m_1+m_2)}{M} r_0^2 \\
{}^0d_{10} &= \frac{m_0 r_0}{M}\{l_1(m_1+m_2) + r_1 m_2\}\cos(q_1) = {}^0d_{01} \\
{}^0d_{20} &= \frac{m_0 m_2}{M} r_0 l_2 \cos(q_1+q_2) = {}^0d_{02} \\
{}^0d_{11} &= I_1 + \frac{m_0 m_1}{M} l_1^2 + \frac{m_1 m_2}{M} r_1^2 + \frac{m_0 m_2}{M}(l_1+r_1)^2 \\
{}^0d_{21} &= \left\{\frac{m_1 m_2}{M} r_1 l_2 + \frac{m_0 m_2}{M} l_2(l_1+r_1)\right\}\cos(q_2) = {}^0d_{12} \\
{}^0d_{22} &= I_2 + \frac{m_2(m_0+m_1)}{M} l_2^2
\end{aligned} \tag{A6}$$

Both ${}^i\mathbf{u}_i$ (i=1,2) in Equation (11) are equal to $[0\ 0\ 1]^T$; the ${}^0\mathbf{F}_i$ matrices reduce to:

$$ {}^0\mathbf{F}_1 = [1\ \ 0] \qquad {}^0\mathbf{F}_2 = [1\ \ 1] \tag{A7}$$

For simplicity, drop the left superscripts from ${}^0\mathbf{D}_j$ and set:

$$D_j \equiv \sum_{i=0}^{2} {}^0d_{ij} \quad j = 0,1,2 \qquad D \equiv D_0 + D_1 + D_2 \tag{A8}$$

The system inertia matrix is assembled using Equations (A6), (A7), (A8) and reduces to the following:

$$\mathbf{H}^*(\mathbf{q}) = \sum_{i=1}^{2}\sum_{j=1}^{2} {}^0\mathbf{F}_i^T \left({}^0d_{ij} - \frac{D_i D_j}{D}\right) {}^0\mathbf{F}_j \tag{A9}$$

Its explicit form is given as Equation (25).