A METHOD FOR ESTIMATING THE MASS PROPERTIES OF A MANIPULATOR BY MEASURING THE REACTION MOMENTS AT ITS BASE

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ABSTRACT

Emulating on earth the weightlessness of a manipulator free floating in space requires knowledge of the manipulator's mass properties. Knowing the manipulator's mass properties, the gravitationally induced forces and moments can be estimated and compensated. A method for calculating these properties by measuring the reaction forces and moments at the base of the manipulator is described. A manipulator is mounted on a six-degree-of-freedom force sensor and the reaction forces and moments at its base are measured for different positions of the links of the manipulator as well as for different orientations of its base. A procedure is developed to calculate from these measurements some combinations of the mass properties of the manipulator. The mass properties identified are not sufficiently complete for computed torque and other dynamic control techniques, but do allow compensation for the gravitational load on the links of the manipulator, and for simulation of weightless conditions on a *space emulator*. The algorithm has been experimentally demonstrated on a PUMA 260, and used to measure nine independent combinations of the sixteen mass parameters of the base and three proximal links of the manipulator.

I. INTRODUCTION

An experimental *space emulator* has been developed for emulating space conditions in the laboratory. Using this system the motion of a manipulator free floating in space, or attached to a small satellite can be experimentally demonstrated. In order to emulate space conditions the mass properties of the manipulator must be known.

The space emulator operates under admittance control. A force sensor measures the dynamic reaction forces at the base of the manipulator, and using this measurement, the space emulator controls the motion of the base to be the same as if the manipulator were free floating in space. The force sensor measurement is the sum of the gravitational and dynamic forces. As the position of the links and the orientation of the base of the manipulator change, the gravitational load also changes. Knowing the manipulator's mass properties, the gravitational forces can be estimated and subtracted from the forces and moments measured at the base, so that any motions of the base will correspond only to the dynamic forces that would exist in real space conditions.

The space emulator can also be used to identify the mass properties of a manipulator. Statically the gravitational forces and moments at the base of the manipulator are measured and the position of the links of the manipulator and the orientation of the base of the manipulator to the vertical are known. Using these measurements the mass properties of the manipulator can be calculated.

[1] In this paper *position* implies *position* and *orientation*, and *force* implies both *force* and *torque*.

In general, estimation of the mass properties of the links of a manipulator are needed for three broad classes of problems:

(a) Dynamic modelling of the inertial parameters of a manipulator for feedforward, computed torque and other dynamic control algorithms.
(b) Static modelling of the mass properties of the links for compensation of the gravitational load on the joints of the manipulator.
(c) Static modelling of the mass properties of the links for compensation of the gravitational load on the base of the manipulator for space emulation systems.

Several methods have been developed for measurement of the mass properties of a manipulator. The mass properties of the load at the end effector can be measured by using a wrist force/torque sensor [9]. Armstrong, Khatib and Burdick [4] measured the mass properties of a PUMA 560 robot by counterbalancing the disassembled parts; this method is tedious and the cross terms of the inertia matrix cannot be obtained in this way. Ann, Atkeson, and Hollerbach [2] and Atkeson, An, and Hollerbach [3] developed an algorithm to determine the inertial properties of a manipulator from measurement of the joint force/torque[1] along with the joint position, velocity and acceleration. Their method does not use any additional sensor hardware (joint torque is estimated from measurement of the motor current), but is limited to manipulators with low joint friction such as direct drive arms. Slotine [10] developed an adaptive control algorithm for the estimation of the mass properties. These last two methods are unable to determine all of the mass properties, but the missing terms have no effect on the dynamic control of the arm.

The method presented in this paper calculates the mass properties of each of the links of a manipulator by measuring the reaction forces at the base of the manipulator as the links are moved. This method is being used for gravity compensation of a manipulator mounted on a Space Vehicle Emulation System [6]. Not all of the mass properties are identified, but the missing terms have no effect on the gravitational load on the emulator force sensor. This method requires that the manipulator be mounted on the six degree-of-freedom force sensor. This force sensor is a necessary part of the Space Vehicle Emulation System.

II. THE SPACE VEHICLE EMULATION SYSTEM

In the future, robotic manipulators will be increasingly used in applications in which the base of the manipulator is not fixed, but is free floating [1]. Such applications include manipulators carried by spacecraft for satellite repair. The case of the free floating manipulator is essentially the special case of a manipulator mounted on a compliant base in which the compliance has become infinite. The control of such mobile manipulators would be more difficult than that of industrial manipulators, which are generally attached securely to relatively rigid and stationary bases. Algorithms for the control of free floating manipulators have been developed [7], [3], [13], [14], [15]. The Space Vehicle Emulation System has been built as a test facility to demonstrate and experimentally evaluate these solutions.

The Space Vehicle Emulation System consists of a manipulator mounted on an emulator system [8], [11]. The experimental manipulator is a PUMA 260 with a custom joint controller, and the emulator system comprises a six degree of freedom Stewart mechanism, six degree of freedom force sensor, and computer controller. The experimental manipulator and the emulator system are controlled by individual DEC PDP-11/73's, and can be coordinated using a communication link between the two computers. A sketch of the system is shown in Figure 1.

The Stewart mechanism and force sensor can, respectively, move and measure forces in any of the three translational directions (x,y,z), and any of the the three rotational directions, $(\theta_x,\theta_y,\theta_z)$ possible in general three dimensional space. The platform stands approximately 3 feet of the ground in its home position. The major hardware elements of the Space Vehicle Emulation System are shown in Figure 2, and a photograph of the System is shown in Figure 3.

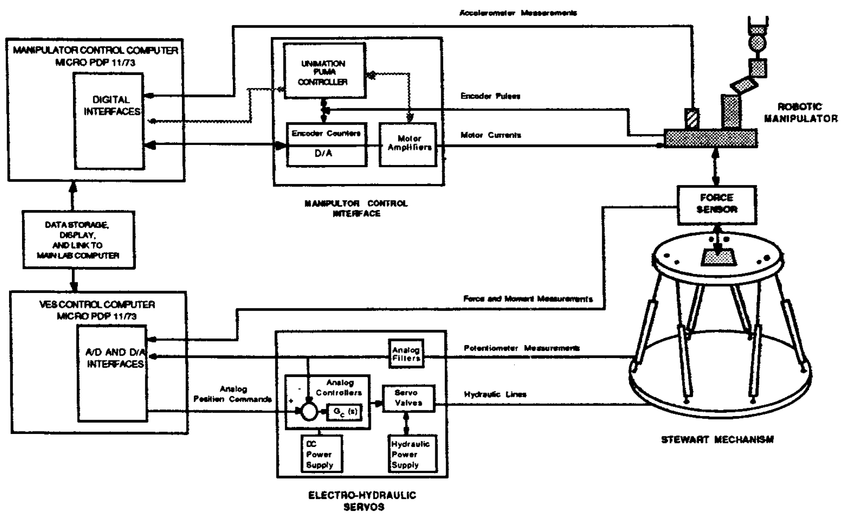

Figure 1. Overview of Space Vehicle Emulation System

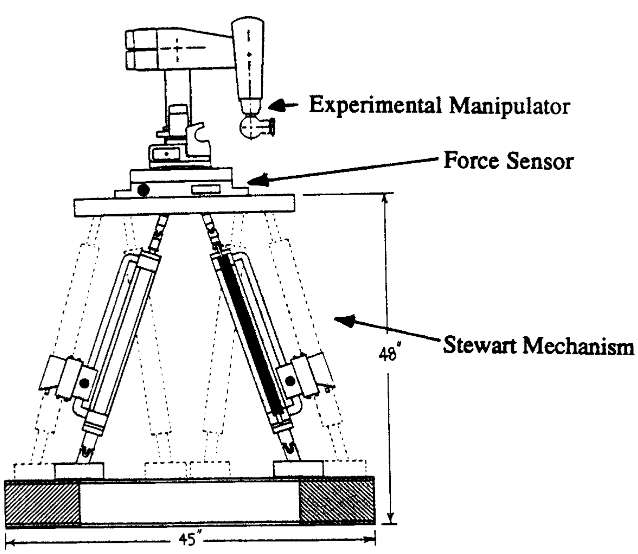

Figure 2. Mechanical Hardware Elements of Space Vehicle Emulation System

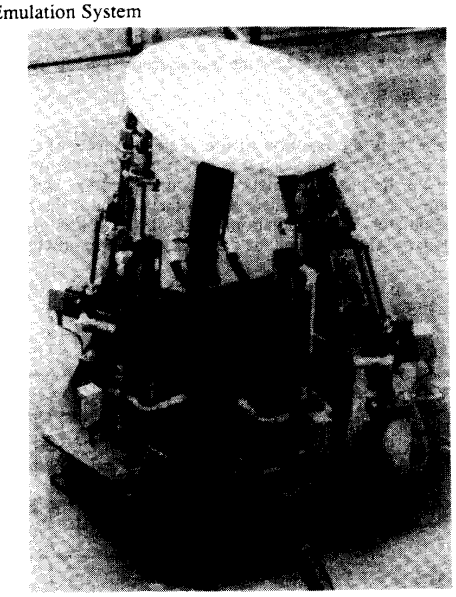

Figure 3. Photograph of Space Vehicle Emulation System

A micro-PDP11/73 is used for trajectory calculation, and also to provide position commands to the analog joint controllers of the Stewart mechanism. The microcomputer is also used to subtract the calculated gravitational load of the manipulator from the force measured by the force sensor for simulating weightless conditions. In addition to trajectory calculation and control, the computer performs a supervisory function checking for approaching violations of the kinematic constraints, and verifies that the joints are following the desired trajectory within allowed error bounds.

The dynamic reaction forces at the base of the manipulator are measured by the force sensor, and the motion of the base is controlled by the emulator system admittance controller to be the same as if the manipulator were free floating in space. The structure of the admittance control mode is shown in Figure 4. The output of the admittance model is integrated to give a series of desired positions for the emulator platform, and then an inverse kinematic model is used to calculate corresponding leg lengths of the Stewart mechanism.

The force sensor measures the sum of the dynamic reaction forces and the gravitational reaction forces. As the position of the links and the orientation of the base of the manipulator change, the gravitational load also changes. This load must be subtracted from the load measured by the force sensor to give the dynamic reaction force at the base. The gravitational load of the manipulator on the force sensor can be calculated from the position of the links of the manipulator and the orientation of the base of the manipulator to the vertical, which are known, and the mass properties of the manipulator.

III. A ONE DEGREEE OF FREEDOM EXAMPLE

The method of calculating the mass properties of a manipulator from measurement of base reaction forces will be developed with reference to a simple one-degree-of-freedom example before the more general three-degree of freedom case is presented. The manipulator shown in Figure 5 is mounted on a fixed force sensor and can rotate about its z_1 axis only. The reference frame is attached to the center of the force sensor.

The moment measured by the force sensor is given by:

$$\mathbf{m} = {}^N\mathbf{r} \times \mathbf{f} + \mathbf{m}_0 \qquad (1)$$

where ${}^N\mathbf{r}$ is the position of the center of mass of the moving link in the force sensor reference frame, $\mathbf{f}$ is the gravitational load on the center of mass of the moving link, and $\mathbf{m}_0$ is the moment at the force sensor due to the weight of the base of the manipulator. The gravitational load on the link is given by the product of the mass and the acceleration due to gravity:

$$\mathbf{f} = \begin{bmatrix} 0 \\ 0 \\ M_1 g \\ 1 \end{bmatrix} \qquad (2)$$

where M_1 is the mass of the link. The moment at the force sensor consists of three terms:

$$\mathbf{m} = \begin{bmatrix} m_x \\ m_y \\ m_z \\ 1 \end{bmatrix} \qquad (3)$$

${}^N\mathbf{r}$, the position of the center of mass in the Newtonian frame can be described in terms of $\mathbf{r}_1$, the position of the center of mass in the link frame using the transformation matrices, $\mathbf{T}_0$, $\mathbf{T}_1$:

$${}^N\mathbf{r} = \mathbf{T}_0 \mathbf{T}_1 \mathbf{r}_1 \qquad (4)$$

where $\mathbf{T}_0$ is given by:

$$T_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & l_{0y} \\ 0 & -1 & 0 & -l_{0z} \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad (5)$$

and $\mathbf{T}_1$ is given by:

$$T_1 = \begin{bmatrix} C_1 & -S_1 & 0 & 0 \\ S_1 & C_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad (6)$$

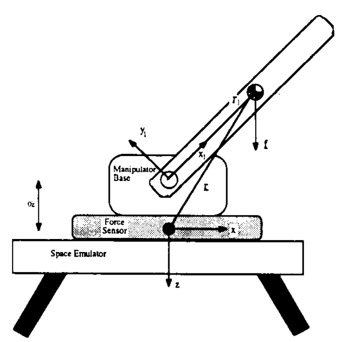

Figure 5. Simple One-Degree-of-Freedom Manipulator Mounted on a Force Sensor

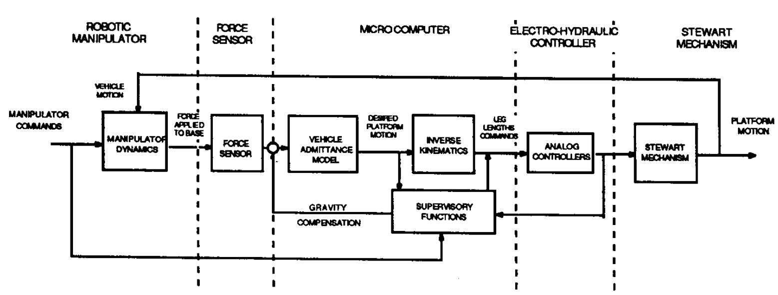

Figure 4. Admittance Control System

and $\mathbf{r}_1$ is given by:

$$\mathbf{r}_1 = \begin{bmatrix} r_{1x} \\ r_{1y} \\ r_{1z} \\ 1 \end{bmatrix} \tag{7}$$

Note that that the systems of frames shown in Figure 5 differ from the standard Denavit-Hartenberg notation; frame i is attached at joint i rather than at joint i+1. This notation is used to simplify the analysis.

Expanding equation (1) gives:

$$\begin{bmatrix} m_x \\ m_y \\ m_z \\ 1 \end{bmatrix} = \begin{bmatrix} m_{0x} \\ m_{0y} \\ m_{0z} \\ 1 \end{bmatrix} + \begin{bmatrix} (r_{1z} + l_{0y})\, M_1\, g \\ (-C_1\, r_{1x} + S_1\, r_{1y})\, M_1\, g \\ 0 \\ 1 \end{bmatrix} \tag{8}$$

If the link is moved through two more positions, we get two more sets of moments with their corresponding joint angles:

$$\begin{bmatrix} m_x' \\ m_y' \\ m_z' \\ 1 \end{bmatrix} = \begin{bmatrix} m_{0x} \\ m_{0y} \\ m_{0z} \\ 1 \end{bmatrix} + \begin{bmatrix} (r_{1z} + l_{0y})\, M_1\, g \\ (-C_1'\, r_{1x} + S_1'\, r_{1y})\, M_1\, g \\ 0 \\ 1 \end{bmatrix} \tag{9}$$

$$\begin{bmatrix} m_x'' \\ m_y'' \\ m_z'' \\ 1 \end{bmatrix} = \begin{bmatrix} m_{0x} \\ m_{0y} \\ m_{0z} \\ 1 \end{bmatrix} + \begin{bmatrix} (r_{1z} + l_{0y})\, M_1\, g \\ (-C_1''\, r_{1x} + S_1''\, r_{1y})\, M_1\, g \\ 0 \\ 1 \end{bmatrix} \tag{10}$$

The second rows of equations (8), (9) and (10) can be extracted to give:

$$\begin{bmatrix} -C_1 & S_1 & 1 \\ -C_1' & S_1' & 1 \\ -C_1'' & S_1'' & 1 \end{bmatrix} \begin{bmatrix} r_{1x}\, M_1\, g \\ r_{1y}\, M_1\, g \\ m_{0y} \end{bmatrix} = \begin{bmatrix} m_y \\ m_y' \\ m_y'' \end{bmatrix} \tag{11}$$

which can be solved for $r_{1x} M_1$, $r_{1y} M_1$, and m_{0y}. The product of the mass of the link and the distance of the center of mass from the joint is calculated but the mass of the link alone is not determined. The $r_{1z} M_1$ component of the position of the center of mass cannot be found from this set of simultaneous equations because terms involving it remain constant throughout the process of rotating the link. As long as the manipulator base remains horizontal the moment about the z axis remains constant. Hence the mass parameters that contribute to this moment cannot be calculated, nor are they needed for compensation of the gravitational load of the manipulator on the base.

By tilting the platform about the x axis, the m_x component of the reaction moment at the base can be varied, as illustrated in Figure 6.

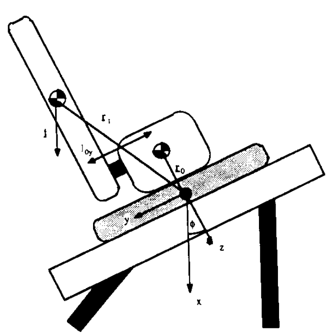

Figure 6. Simple Manipulator on a Tilted Platform

For the platform tilted at an angle ϕ the gravitational load at the center of mass is given by:

$$\mathbf{f} = \begin{bmatrix} 0 \\ M_1\, g \sin\phi \\ M_1\, g \cos\phi \\ 1 \end{bmatrix} \tag{12}$$

and the resulting moment measured at the base of the manipulator is:

$$\begin{bmatrix} m_x \\ m_y \\ m_z \\ 1 \end{bmatrix} = \mathbf{m}_0' + \begin{bmatrix} (r_{1z}+l_{0y})\, M_1 g\cos\phi + (S_1 r_{1x} + C_1 r_{1y} + l_{0z})\, M_1 g \sin\phi \\ (C_1\, r_{1x} - S_1\, r_{1y})\, M_1\, g \cos\phi \\ (C_1\, r_{1x} - S_1\, r_{1y})\, M_1\, g \sin\phi \\ 1 \end{bmatrix} \tag{13}$$

where $\mathbf{m}_0'$ is the moment of the tilted base and is given by:

$$\mathbf{m}_0' = \begin{bmatrix} r_{0y}\, M_0\, g \cos\phi - r_{0z}\, M_0\, g \sin\phi \\ -r_{0x}\, M_0\, g \cos\phi \\ r_{0x}\, M_0\, g \sin\phi \end{bmatrix} \tag{14}$$

and $\mathbf{r}_0$ is the location of the center of mass of the base relative to the force sensor frame.

By measuring the moments for three different base angles the mass parameters, $r_{0x} M_0$, $r_{0y} M_0$, and $(r_{0z} M_0 + (r_{1z} + l_{0y}) M_1)$ can be calculated. Since the position of the center of mass of link 1 cannot be varied in the direction parallel to the joint axis, it is not possible to distinguish between the mass parameters of the base and the link in this direction. However, since its is not possible to determine these individual terms, either by moving the link of the manipulator or its base, nor is it necessary to in order to compensate for the gravitational load of the manipulator on the base.

IV. IDENTIFICATION OF MASS PROPERTIES

The method described in the previous section can be generalized to obtain all the mass properties necessary for gravity compensation of a manipulator. The knowledge of these properties will allow the calculation of the gravitationally induced moments that are transmitted to the base of the manipulator. For a PUMA 260, the most important contributions to these moments are due to the manipulator base and to its first three links so the wrist position will remain fixed.

The moment vector $\mathbf{m}$, with respect to the force sensor frame N, is given by (N is dropped)

$$\mathbf{m} = M_t\, \mathbf{r} \times \mathbf{g} = M_t\, g\, \mathbf{F}\, \mathbf{r} \tag{15}$$

where M_t is the total mass of the manipulator, $\mathbf{g}$ is the acceleration of gravity, $\mathbf{r}$ is the position vector of the manipulator center of mass (*cm*) with respect to N and **F** is a matrix whose elements f_i are the components of the unit vector parallel to $\mathbf{g}$ in N.

$$M_t = M_0 + M_1 + M_2 + M_3 \tag{16}$$

where M_0, M_1, M_2, and M_3 are the masses of the base and the first three links of the manipulator (the last link includes the wrist and gripper).

$$\mathbf{r} = \begin{bmatrix} r_x \\ r_y \\ r_z \end{bmatrix} \tag{17}$$

and,

$$\mathbf{F} = \begin{bmatrix} 0 & f_z & -f_y \\ -f_z & 0 & f_x \\ f_y & -f_x & 0 \end{bmatrix} \tag{18}$$

$M_t g$ represents the weight of the manipulator which can be obtained with one measurement. **F** can be readily obtained since it is a function of the orientation (direction) cosines of the base. So, in order to estimate the transmitted moments, two steps are required: (a) express $\mathbf{r}$ as a function of the unknown mass properties and the base and links positions and, (b) estimate the unknown parameters.

(a) Express $\mathbf{r}$ as a function of the mass properties and the base and links positions. The base and links positions can be expressed by the use of transformation matrices. The link frames used in this paper to describe the position of the manipulator are shown in Figure 7. Frame 0 is attached to the manipulator base, which has mass M_0 while its *cm* is at $\mathbf{r}_0$. In the same way, frame i (i=1,2,3) is attached to link i with mass m_i and *cm* at $\mathbf{r}_i$. Note that again the frames are not defined according to the usual Denavit-Hartenberg notation; frame i is attached at joint i rather than at joint i+1. This notation simplifies the subsequent analysis.

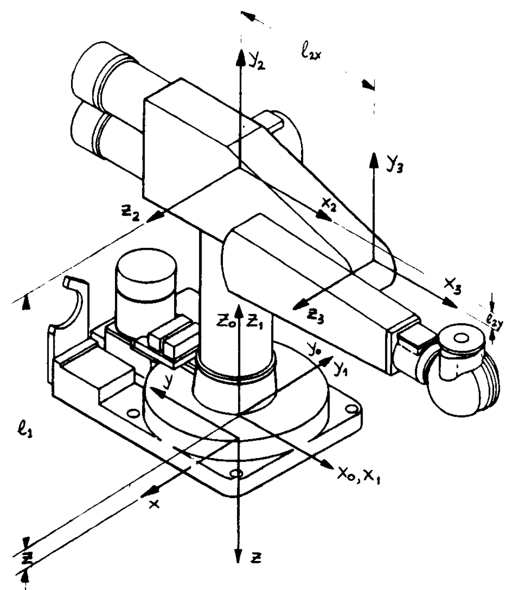

Figure 7. Link Frames

The A_i matrices transform position vectors $\mathbf{r}_i$ in frame i to vectors in frame i-1. Matrix A_0 transforms position vectors in the manipulator base frame to position vectors in the force sensor frame N.

$$A_0 = \begin{bmatrix} 0 & -1 & 0 & X \\ -1 & 0 & 0 & Y \\ 0 & 0 & -1 & Z \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad A_1 = \begin{bmatrix} c_1 & -s_1 & 0 & 0 \\ s_1 & c_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} c_2 & -s_2 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ s_2 & c_2 & 0 & l_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad A_3 = \begin{bmatrix} c_3 & -s_3 & 0 & l_{2x} \\ s_3 & c_3 & 0 & l_{2y} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{19}$$

The next step is to formulate the manipulator T_i matrices that transform a position vector in frame i, in a position vector in frame N. These are given by

$$\mathbf{T_0} = \mathbf{A_0} \tag{20}$$

$$\mathbf{T_1} = \mathbf{T_0 A_1} = \begin{bmatrix} -s_1 & -c_1 & 0 & X \\ -c_1 & s_1 & 0 & Y \\ 0 & 0 & -1 & Z \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{21}$$

$$\mathbf{T_2} = \mathbf{T_1 A_2} = \begin{bmatrix} -s_1 c_2 & s_1 s_2 & c_1 & X \\ -c_1 c_2 & c_1 s_2 & -s_1 & Y \\ -s_2 & -c_2 & 0 & -l_1 + Z \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{22}$$

$$\mathbf{T_3} = \mathbf{T_2 A_3} = \begin{bmatrix} s_1(s_2 s_3 - c_2 c_3) & s_1(c_2 s_3 + s_2 c_3) & c_1 & s_1 s_2 l_{2y} - l_{2x} s_1 c_2 + X \\ c_1(s_2 s_3 - c_2 c_3) & c_1(c_2 s_3 + s_2 c_3) & -s_1 & -l_{2x} c_1 c_2 + c_1 s_2 l_{2y} + Y \\ -(s_2 c_3 + c_2 s_3) & s_2 s_3 - c_2 c_3 & 0 & -s_2 l_{2x} - l_1 - c_2 l_{2y} + Z \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{23}$$

If we denote by $\mathbf{r}_i$ the position vector of the i *cm* in frame i, where:

$$\mathbf{r}_i = \begin{bmatrix} r_{ix} \\ r_{iy} \\ r_{iz} \\ 1 \end{bmatrix} \quad (i = 0,1,2,3) \tag{24}$$

then the position vector of the center of mass of the whole manipulator, $\mathbf{r}$, can be expressed as:

$$\mathbf{r} = \frac{1}{M_t} \mathbf{T_L} \left(\sum_{i=0}^{3} M_i \, T_i \, \mathbf{r}_i \right) \tag{25}$$

where:

$$\mathbf{T_L} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \tag{26}$$

$\mathbf{T_L}$ is a 3x4 matrix that removes the last element from the 4x1 vectors in the summation. Combining equations (15) and (25) gives an expression for the moments at the base of the manipulator.

By varying the manipulator joint angles, using the VAL controller of the PUMA, and varying the orientation of the base, using the Space Vehicle Emulation System the mass properties of the manipulator can be identified. Knowing these properties permits the estimation of gravitational moments transmitted by the manipulator to its base, once its joint angles and base orientation are known.

Expanding eq. (25) reveals that all M_i's appear in products with the components of $\mathbf{r}_i$'s or with known lengths. Since these lengths cannot be altered, it is not possible to estimate these masses or lengths independently. Further, each $m_i \mathbf{r}_i$ term can not be identified independently. This can be seen if eq. (25) is written as follows,

$$\mathbf{r} = \frac{1}{M_t} \mathbf{T_L} [\; T_0 \quad T_1 \quad T_2 \quad T_3 \;] \begin{bmatrix} M_0 \mathbf{r}_0 \\ M_1 \mathbf{r}_1 \\ M_2 \mathbf{r}_2 \\ M_3 \mathbf{r}_3 \end{bmatrix} = \frac{1}{M_t} \mathbf{T_L T} \, (\mathbf{MR})' \tag{27}$$

$\mathbf{T_L T}$ is a 3x16 matrix, while the unknowns vector $(\mathbf{MR})'$ is 16x1, containing 12 unknowns. Unfortunately, some of the columns of $\mathbf{T_L T}$ are linearly dependent, because they are either the same or they are the weighted sum of other columns. This means that if one uses the above equation for identification, the resulting matrix will always be singular and not invertible. The result is that some unknowns can not be identified independently.

In order to overcome this problem, all dependent columns of $\mathbf{T_L T}$ are removed, the remaining matrix being **W**, and the unknown vector is modified accordingly and named $\mathbf{p}$. Thus, $\mathbf{r}$ is now decomposed in a set of two new matrices, **W** and $\mathbf{p}$. The matrix **W** has the special property that the dimension of its column space is equal to the number of its columns. It turns out that for the base and first three links of the PUMA $\mathbf{p}$ is a 9x1 vector. For each revolute link of a manipulator, only two mass properties can be identified, the ones that correspond to distances normal to the axis of rotation. Since the link *cm* can not move along its revolute joint axis, it is not possible to identify the mass properties in the direction of that axis. Three more mass properties are identifiable by rotating the base, these are combinations of the mass properties of all three links and the base. The 9×1 vector of measurable mass properties is given by:

$$\begin{bmatrix} p_1 \\ p_2 \\ p_3 \\ p_4 \\ p_5 \\ p_6 \\ p_7 \\ p_8 \\ p_9 \end{bmatrix} = \begin{bmatrix} m_3 r_{3x} \\ m_3 r_{3y} \\ m_2 r_{2x} + m_3 l_{2x} \\ m_2 r_{2y} + m_3 l_{2y} \\ -m_1 r_{1y} + m_2 r_{2z} + m_3 r_{3z} \\ m_1 r_{1x} \\ m_0(-r_{0y}+X) + (m_1+m_2+m_3)X \\ m_0(-r_{0x}+Y) + (m_1+m_2+m_3)Y \\ m_0(-r_{0z}+Z) + (m_1+m_2+m_3)Z - m_1 r_{1z} - (m_2+m_3) l_1 \end{bmatrix} \tag{28}$$

The moments equation (15) is then written as:

$$\mathbf{m} = M_t\, g\, \mathbf{F}\, \mathbf{r} = g\, \mathbf{F}\, \mathbf{W}\, \mathbf{p} \tag{29}$$

where W given is given by:

$$\mathbf{W} = \begin{bmatrix} s_1(s_2s_3-c_2c_3) & s_1(c_2s_3+s_2c_3) & -s_1c_2 & s_1s_2 & c_1 & -s_1 & 1 & 0 & 0 \\ c_1(s_2s_3-c_2c_3) & c_1(c_2s_3+s_2c_3) & -c_1c_2 & c_1s_2 & -s_1 & -c_1 & 0 & 1 & 0 \\ -(s_2c_3+c_2s_3) & s_2s_3-c_2c_3 & -s_2 & -c_2 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{30}$$

(b) Estimate the unknown parameters. So far, $\mathbf{r}$ has been expressed as a function of nine independent properties and of the link positions. However, in order to use (29) to calculate the moments, we must first estimate $\mathbf{p}$. For each set of joint angles and base orientation a set of three moments can be measured, the components of $\mathbf{m}$. Unfortunately, only two of the moments vector $\mathbf{m}$ components are independent. This is due to the fact that $\mathbf{m}$ always belongs to a plane perpendicular to the acceleration of gravity vector:

$$\mathbf{m} \cdot \mathbf{g} = M_t(\mathbf{r} \times \mathbf{g}) \cdot \mathbf{g} \equiv 0 \tag{31}$$

The physical explanation of this problem is that the location of the center of mass in the direction of the gravity vector can not be determined since this distance does not affect the moment at the base.

This condition leads to a non invertible $\mathbf{F}$ with rank 2; hence it is not possible to completely determine the position of the center of mass, $\mathbf{r}$, with one set of moment measurements $\mathbf{m}$. Clearly, three sets of measurements of the moments $\mathbf{m}$ corresponding to independent positions of the manipulator will not result in nine independent equations; Only six equations will be obtained. For this reason it will not be possible to invert the 9x9 matrix that would result from the three 3x9 **FW** matrices. A solution to this problem can be obtained by a least-squares approach. The moment equations is written in the form:

$$\mathbf{m}^{(i)} = m_t g \mathbf{F}^{(i)} \mathbf{r}^{(i)} = g \mathbf{F}^{(i)} \mathbf{W}^{(i)} \mathbf{p} \qquad (i = 1, \ldots, N) \tag{32}$$

where the superscript i corresponds to the i^{th} set of measurements. If N sets of measurements are taken, then the above equation, for i running from 1 to N, can be written as follows

$$\mathbf{m}^* = \begin{bmatrix} \mathbf{m}^{(1)} \\ \mathbf{m}^{(2)} \\ \vdots \\ \mathbf{m}^{(N)} \end{bmatrix} = g \begin{bmatrix} \mathbf{F}^{(1)} & & \mathbf{O} \\ & \mathbf{F}^{(2)} & \\ \mathbf{O} & & \mathbf{F}^{(N)} \end{bmatrix} \begin{bmatrix} \mathbf{W}^{(1)} \\ \mathbf{W}^{(2)} \\ \vdots \\ \mathbf{W}^{(N)} \end{bmatrix} \mathbf{p} = g \mathbf{F}^* \mathbf{W}^* \mathbf{p} \tag{33}$$

Since the dimension of the column space of $\mathbf{W}^{(i)}$ is nine, by taking measurements at independent sets of joint angles, the rank of $\mathbf{W}^*$ can always be made equal to nine. However, since the rank of $\mathbf{F}^{(i)}$ is only two, the rank of $\mathbf{F}^*$ is at most 2N. In order to be possible to recover $\mathbf{p}$, the product $\mathbf{F}^*\mathbf{W}^*$ must have rank 9, so the rank of $\mathbf{F}^*$ must be at least 9. It follows that N must be at least equal to 5 since

$$\text{rank}(\mathbf{F}^*\mathbf{W}^*) \le \text{rank}(\mathbf{F}^*) \le 2N \tag{34}$$

This analysis suggests that one needs at least five independent sets of measurements. This leads to a non-square (15x9) matrix for $\mathbf{F}^*\mathbf{W}^*$, which is not invertible. Instead, a least squares inversion can be employed as follows:

$$\mathbf{p} = \frac{1}{g} (\mathbf{W}^{*T}\mathbf{F}^{*T}\mathbf{F}^*\mathbf{W}^*)^{-1} (\mathbf{F}^*\mathbf{W}^*)^T \mathbf{m}^* \tag{35}$$

Provided that the matrix, $\mathbf{F}^*\mathbf{W}^*$, is of full row rank, which is typically the case, the above formula will yield the true value for $\mathbf{p}$. Alternatively, one or two rows from each set of equation (33) can be discarded to give a square $\mathbf{F}^*\mathbf{W}^*$ matrix. However, in general a redundancy of measurements reduces the estimation errors, so all measurements were used.

An estimate of the magnitude of the error in $\mathbf{p}$ due to measurement errors is given by the condition number of the matrix, $\mathbf{F}^*\mathbf{W}^*$. It is known that if $\|\delta \mathbf{m}^*\|$ is the magnitude of the error vector corresponding to $\|\mathbf{m}^*\|$, then the error in $\|\mathbf{p}\|$ is bounded according to:

$$\|\delta \mathbf{p}\| \le C\, \|\mathbf{p}\| \frac{\|\delta \mathbf{m}^*\|}{\|\mathbf{m}^*\|} \tag{36}$$

where C is the condition number of $\mathbf{F}^*\mathbf{W}^*$. The implication is that if C is large, then some of the parameters in $\mathbf{p}$ may be affected greatly by some minor errors in $\mathbf{m}^*$. It is desirable for C to be as small as possible, ideally equal to 1. Matrices $\mathbf{F}^*$ and $\mathbf{W}^*$ depend on the orientation of the base and on the joint angles of the manipulator, solely, and hence the experiments may be designed in such a way as to minimize C and thus obtain a better estimate of $\mathbf{p}$. As a rule of thumb, the condition number is reduced by choosing angles that are different by as close to 90° as possible, and taking more sets of measurements than the required five.

V. EXPERIMENTAL RESULTS

The mass parameters of PUMA 260 robot were experimentally measured using the Space Vehicle Emulation System described in section II. The wrist is fixed with joint angles of $\theta_4=90°$, $\theta_5=90°$, $\theta_6=60°$, as given by the VAL controller. During the experiments, the three components F_i of the force vector (weight) and the three components of the moment vector, $\mathbf{m}$, were measured. All measurements were relative to the force sensor frame N. The joint angles of the manipulator were taken from the VAL display. The elements of the $\mathbf{F}$ matrices were obtained by measuring the three forces (F_i, i = x,y,z) transmitted by the manipulator to its base:

$$f_i = F_i / \sqrt{F_x^2 + F_y^2 + F_z^2} = F_i / M_t\, g \qquad (i = x, y, z) \tag{37}$$

Thirty sets of measurements were taken for different joint angles and for two different manipulator base orientations.The condition number of the resulting $\mathbf{F}^*\mathbf{W}^*$ matrix was 10.8 which was considered sufficiently low for our purposes. The condition number could be further reduced by tilting the manipulator more than the 25° y-axis tilt that was used during our experiments, but was not tried for safety reasons. The basic geometric parameters and total mass for our PUMA 260 are:

Z	=	-0.036	m
l_1	=	0.330	m
l_{2x}	=	0.198	m
l_{2y}	=	-0.019	m
M_t	=	10.6	kg

The estimates of the mass properties of the manipulator were calculated using the above analysis in section IV and found to be:

p_1	=	0.158	Kgm
p_2	=	0.010	Kgm
p_3	=	0.382	Kgm
p_4	=	-0.015	Kgm
p_5	=	0.437	Kgm
p_6	=	0.008	Kgm
p_7	=	-0.050	Kgm
p_8	=	0.275	Kgm
p_9	=	-2.445	Kgm

where the elements of $\underline{\mathbf{p}}$ have been defined in equation (28).

Using these mass property values, the gravitational forces and moments in the x and y directions and the force in the z direction were predicted to within an accuracy of 1% throughout the workspace. The moment in the z direction was predicted to within an accuracy of 10%. The accuracy of measurement of the moment in the z direction was limited by the low signal to noise ratio, and could be improved by increasing the angle of y-axis tilt.

V. CONCLUSIONS

In the future, robotic manipulators will be increasingly used in applications in which the base of the manipulator is not fixed but is free floating in space. In order to develop algorithms for the control of free floating manipulators, a Space Vehicle Emulation System has been constructed. Emulation of space conditions requires accurate compensation for the gravitational load of the manipulator on the base of the emulator. A procedure has been developed for calculating the mass properties of a manipulator using measurements of the force and moments at the base of the manipulator. As in other mass property measurement techniques, not all of the mass properties are identified independently. However, the mass properties are sufficiently determined to compensate for the gravitational load of the manipulator on the Space Vehicle Emulation System. This procedure has been used to calculate the mass properties of the PUMA 260, and has been used experimentally to predict the gravitational load at the base of the manipulator.

VI. ACKNOWLEDGEMENTS

The support of this work by NASA (Langley Research Center Automation Branch) and the DARPA (US Army Human Engineering Laboratory and the Oak Ridge National Laboratory as agents) is acknowledged. The force sensor and platform controller were developed by Nathan Stelman. In addition, a number of MIT students have made significant contributions to this work. They are M. Fresko, N. Hootsmans, V. K. Krish, and N. Ismail.

REFERENCES

[1] Akin, D.L., Minsky, M.L., Thiel, E.D., and Kurtzman, C.R., 1983, Space Applications of Automation, Robotics and Machine Intelligence Systems (ARAMIS), MIT report, NASA Contract NAS8-34381, Cambridge, MA.

[2] An, C. H., C. G. Atkeson, and J. M. Hollerbach, "Estimation of Inertial Parameters of Rigid Body Links of Manipulators," Proc. 24th IEEE Conf. Decision and Control, Fort Lauderdale, December, 1985.

[3] Alexander, H.L. and Cannon, R.H., "Experiments on the Control of a Satellite Manipulator," Proc. of ACC, Minneapolis, MN, 1987.

[4] Armstrong, B., O. Khatib, and J. Burdick, "The Explicit Dynamic Model and Inertial Parameters of the PUMA 560 Arm," Proc. IEEE Int Conf. Robotics and Automation, San Francisco, April 1986.

[5] Atkeson, C. G., C. H. An, and J. M. Hollerbach, "Rigid Body Load Identification for Manipulators ," Proc. 24th IEEE Conf. Decision and Control, Fort Lauderdale, December, 1985.

[6] Dubowsky, S., I. Paul, and H. West, "An Analytical and Experimental Program to Develop Control Algorithms for Mobile Manipulators", Proc. RoManSy '88, Udine, Italy, 1988.

[7] Dubowsky, S. and Vafa, Z., 1987, "A Virtual Manipulator Model for Space Robotic Systems," Proceedings of the NASA Workshop on Space Telerobotics, Jet Propulsion Lab., CALTECH., Pasadena, CA.

[8] Fresko, M., The Design and Implementation of a Computer Controlled Platform With Variable Admittance, M.S. Thesis, Dept. of Mech. Eng. MIT, Cambridge, MA January, 1986.

[9] Olsen, H. B. and G. A. Bekey, "Identification of Parameters in Models of Robots with Rotary Joints," IEEE Int. Conf on Robotics and Automation, St. Louis, Missouri, 1985.

[10] Slotine, J.-J. E. and Li Weiping, "Adaptive Manipulator Control: A Case Study,"IEEE Int. Conf. on Robotics and Automation, Raleigh, NC 1987.

[11] Stelman, N., Design and Control of a Six-Degree-of-Freedom Platform with Variable Admittance, S.M. Thesis, Dept. of Mech. Eng., MIT, Cambridge, MA.

[12] Vafa, Z., 1987, Dynamics of Manipulators in Space, Ph.D. Thesis, MIT, Cambridge, MA.

[13] Vafa, Z., and Dubowsky, S., 1987, "On the Dynamics of Manipulators in Space Using the Virtual Manipulator Approach," Proceedings of the IEEE International Conference on Robotics and Automation, Raleigh, NC.

[14] Vafa, Z., Dubowsky, S., 1987, "Kinematic and Dynamic Models of Manipulators for use in Space Environments: The Concept of the Virtual Manipulator," Proc. of the Seventh World Congress on Theory of Machines and Mechanisms, Sevilla, Spain.

[15] Vafa, Z. and Dubowsky, S., "Minimization of Spacecraft Disturbances in Space Robotic Systems," Proc. of the 11th AAS Guidance and Control Conference, Keystone, CO, 1988.