

Dynamic Singularities in Free-floating Space Manipulators

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Abstract

Dynamic Singularities are shown for free-floating space manipulator systems where the spacecraft moves in response to manipulator motions without compensation from its attitude control system. At a dynamic singularity the manipulator is unable to move its end-effector in some inertial direction; thus dynamic singularities must be considered in the design, planning, and control of free-floating space manipulator systems. The existence and location of dynamic singularities cannot be predicted solely from the manipulator kinematic structure because they are functions of the dynamic properties of the system, unlike the singularities for fixed-base manipulators. Also analyzed are the implications of dynamic singularities to the nature of the system's workspace.

I. Introduction

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Robotic manipulators will play important roles in future space missions. The control of such space manipulators poses planning and control problems not found in terrestrial fixed-base manipulators due to the dynamic coupling between space

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manipulators and their spacecraft. A number of control techniques for such systems have been proposed; these schemes can be classified in three categories. In the first category, spacecraft position and attitude are controlled by reaction jets to compensate for any manipulator dynamic forces exerted on the spacecraft. In this case, control laws for earth-bound manipulators can be used, but the utility of such systems may be limited because manipulator motions can both saturate the reaction jet system and consume relatively large amounts of attitude control fuel, limiting the useful life of the system [1]. In the second category, the spacecraft attitude is controlled, although not its translation, by using reaction wheels or attitude control jets [2,4]. The control of these systems is somewhat more complicated than that of the first category, although a technique called the Virtual Manipulator (VM) can be used to simplify the problem [4-7]. The third proposed category assumes a free-floating system in order to conserve fuel or electrical power [4,6-11]. Such a system permits the spacecraft to move freely in response to manipulator motions. These too can be modeled using the VM approach [6,7]. Past work on the control of free-floating systems generally has proposed particular algorithms for free-floating systems and attempted to show their validity on a case by case basis [8-11]. However, algorithms which do not take into full account the spacecraft kinematics or dynamics have occasional problems [10,11]. This paper shows that these problems may be attributed to *dynamic singularities* which are not found in earth bound manipulators. These dynamic singularities must be considered in the design, planning, and control of these systems because of their important effects on the performance of free-floating space manipulators.

The existence of dynamic singularities is shown first by writing the kinematics and conservation equations in a compact, explicit form through the use of barycenters [12,13]. Then it is shown that the end-effector inertial linear and angular velocities can be expressed solely as a function of the velocities of the manipulator controlled joint angles, and that they do not depend upon the uncontrolled linear and angular velocity of the spacecraft. Next a Jacobian matrix, \mathbf{J}^* , is derived which relates the end-effector's linear and angular velocity in inertial space to the joint angular velocities. The rank of this Jacobian matrix is demonstrably deficient at given points in the manipulator's joint space which results in the manipulator being unable to move its end-effector in some direction in inertial space. These singular points cannot be determined solely from the kinematic structure of the system and instead depend upon a system's masses and inertias; hence they are called dynamic singularities. Dynamic singularities are *path dependent* because generally they are not fixed in a manipulator's inertial workspace. This is because the end-effector location in inertial space depends upon the *history* of the spacecraft attitude which is determined by the path taken by the end-effector. Finally, some regions in the inertial workspace exist, called the *Path Independent Workspace (PIW)*, where

dynamic singularities will not exist for any path taken within this region, as opposed to other parts of the workspace, called the *Path Dependent Workspace (PDW)*, where the occurrence of dynamic singularities depends upon the path taken by the manipulator's end-effector.

II. Jacobian Construction for Free-floating Manipulators

End-effector position and orientation can be obtained directly for a manipulator on a fixed-base or on a controlled vehicle as a function of a system's independent coordinates, namely of the manipulator joint angles and base position and attitude. However, end-effector position and orientation cannot be obtained directly in free-floating space manipulator systems because spacecraft position and attitude are coordinates which depend upon the history of a manipulator's motion. Still, provided that some realistic assumptions hold, a Jacobian \mathbf{J}^* can be written for the system and provide a linear relationship between the controlled manipulator's joint angular rates $\dot{\mathbf{q}}$ and the end-effector linear and angular inertial velocities, $\dot{\mathbf{r}}_E, \boldsymbol{\omega}_E$ such that:

$$\dot{\mathbf{x}} = [\dot{\mathbf{r}}_E, \boldsymbol{\omega}_E]^T = \mathbf{J}^* \dot{\mathbf{q}} \quad (1)$$

Dynamic singularities arise when \mathbf{J}^* becomes deficient. This Jacobian plays a similar role to Jacobians used by many fixed-base manipulator control algorithms which are functions of manipulator kinematics only. For example, Umetani and Yoshida proposed a resolved rate controller based on \mathbf{J}^* , called a Generalized Jacobian [9]. However, the construction of \mathbf{J}^* depends on a system's dynamics.

Here, the kinematic and dynamic relationships are formulated for the free-floating manipulator system depicted in Figure 1 and used to find an expression for \mathbf{J}^* , based on the use of the barycenters [12,13]. This approach has similarities to the VM method and has the advantage that the resulting dynamic equations are relatively general, compact and explicit. The body 0 in Figure 1 represents the spacecraft and the bodies k ($k=1, \dots, N$) represent the N manipulator links. Manipulator joint angles and velocities are represented by the $N \times 1$ column vectors \mathbf{q} and $\dot{\mathbf{q}}$. The spacecraft can translate and rotate in response to the manipulator movements. Finally, the manipulator is assumed to have revolute joints and an open-chain kinematic configuration so that a system with an N degree-of-freedom (DOF) manipulator will have $6+N$ DOF.

To derive \mathbf{J}^* , we must write $\dot{\mathbf{r}}_E$ and $\boldsymbol{\omega}_E$ as functions of the links and spacecraft inertial angular velocities $\boldsymbol{\omega}_i$ ($i=0, \dots, N$) and ultimately of the joint rates $\dot{\mathbf{q}}$. From

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Figure 1, it can be seen that the vector from the inertially fixed origin O to k body's center of mass (CM), $\underline{\mathbf{R}}_k$, is given by:

$$\underline{\mathbf{R}}_k = \underline{\mathbf{r}}_{cm} + \underline{\mathbf{p}}_k \quad k = 0, \dots, N \quad (2)$$

where $\underline{\mathbf{r}}_{cm}$ and $\underline{\mathbf{p}}_k$ are defined in Figure 1. The end-point position vector, $\underline{\mathbf{r}}_E$, can be derived from $\underline{\mathbf{R}}_N$ as:

$$\underline{\mathbf{r}}_E = \underline{\mathbf{r}}_{cm} + \underline{\mathbf{p}}_N + \underline{\mathbf{r}}_N \quad (3)$$

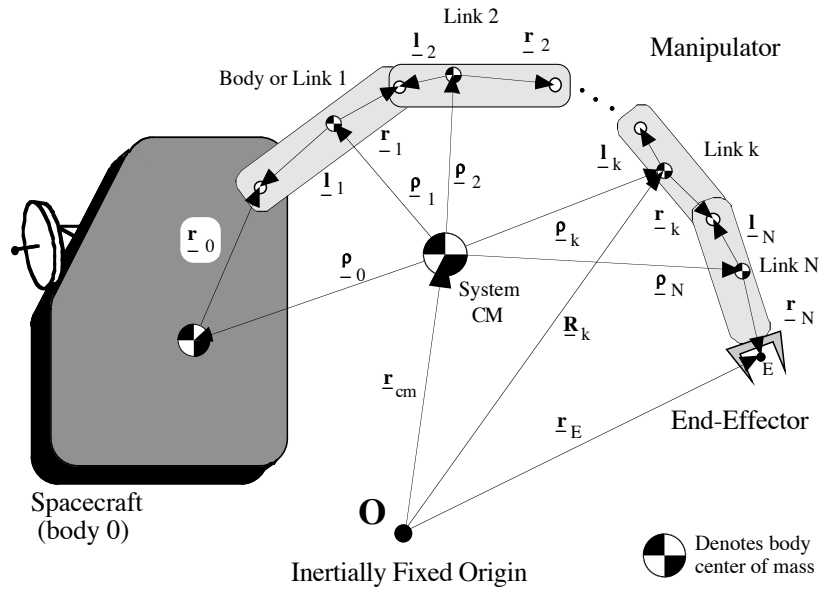


Figure 1. A free-floating space manipulator system.

The $\underline{\mathbf{p}}_k$ vectors are defined uniquely by the free-floating system configuration and, thus, they can be expressed as sums of the weighted, body-fixed vectors $\underline{\mathbf{l}}_i$, and $\underline{\mathbf{r}}_i$ ($i=0, \dots, N$), defined in Figure 1. Indeed, from Figure 1 we have the following N equations in $N+1$ unknowns:

$$\underline{\mathbf{p}}_k - \underline{\mathbf{p}}_{k-1} = \underline{\mathbf{r}}_{k-1} - \underline{\mathbf{l}}_k \quad k = 1, \dots, N \quad (4)$$

Since the $\underline{\mathbf{p}}_k$ vectors are defined with respect to the system CM , it holds that:

$$\sum_{k=0}^N m_k \underline{\mathbf{p}}_k = 0 \quad (5)$$

where m_k is the mass of body k . Equations (4) and (5) can be solved for $\underline{\mathbf{p}}_k$ as a function of $\underline{\mathbf{r}}_k$ and $\underline{\mathbf{l}}_k$, yielding:

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$$\mathbf{p}_k = \sum_{i=1}^k (\mathbf{r}_{i-1} - \mathbf{l}_i) \mu_i - \sum_{i=k+1}^N (\mathbf{r}_{i-1} - \mathbf{l}_i) (1-\mu_i) \quad k = 0, \dots, N \quad (6)$$

where μ_i represents the mass distribution defined by:

$$\mu_i \equiv \left\{ \begin{array}{ll} 0 & i = 0 \\ \sum_{j=0}^{i-1} \frac{m_j}{M} & i = 1 \dots N \\ 1 & i = N+1 \end{array} \right\} \quad (7)$$

M is the total system mass. Equation (6) can be simplified using the notion of a barycenter (*BC*) [12,13]. The barycenter location for the i^{th} body with respect its *CM* is defined by the body fixed vector $\underline{\mathbf{c}}_i$ shown in Figure 2 and given by:

$$\underline{\mathbf{c}}_i = \mathbf{l}_i \mu_i + \mathbf{r}_i (1-\mu_{i+1}) \quad i = 0, \dots, N \quad (8)$$

The barycenter of the i^{th} body can be found equivalently by adding a point mass equal to $M\mu_i$ to joint i , and a point mass equal to $M(1-\mu_{i+1})$ to joint $i+1$, forming an *augmented body* [12,13]. The barycenter is then the center of mass of the augmented body as shown in Figure 2. Figure 2 also shows a set of *body-fixed* vectors which are defined by:

$$\underline{\mathbf{c}}_i^* = -\underline{\mathbf{c}}_i \quad (9a)$$

$$\underline{\mathbf{r}}_i^* = \mathbf{r}_i - \underline{\mathbf{c}}_i \quad (9b)$$

$$\underline{\mathbf{l}}_i^* = \mathbf{l}_i - \underline{\mathbf{c}}_i \quad (9c)$$

Using Equations (9), Equation (8) can be rewritten as:

$$M \mu_i \underline{\mathbf{l}}_i^* + M (1-\mu_{i+1}) \underline{\mathbf{r}}_i^* + m_i \underline{\mathbf{c}}_i^* = 0 \quad (10)$$

It can be shown then that Equation (6) can be written in a compact and general form as:

$$\mathbf{p}_k = \sum_{i=0}^N \mathbf{v}_{ik} \quad k = 0, \dots, N \quad (11)$$

where the barycentric vectors \mathbf{v}_{ik} are given by the following selection law:

$$\mathbf{v}_{ik} \equiv \left\{ \begin{array}{ll} \underline{\mathbf{r}}_i^* & i < k \\ \underline{\mathbf{c}}_i^* & i = k \\ \underline{\mathbf{l}}_i^* & i > k \end{array} \right\} \quad (12)$$

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See Reference [15]. Equation (11) reveals an interesting characteristic of space manipulators, namely that the position of the center of mass of link k in inertial space depends on the position of *all* links, including the ones *after* link k as well as on the position of the base. This is to be contrasted with the case of earth-bound manipulators where the position of a link depends on the positions of the *previous* links only.

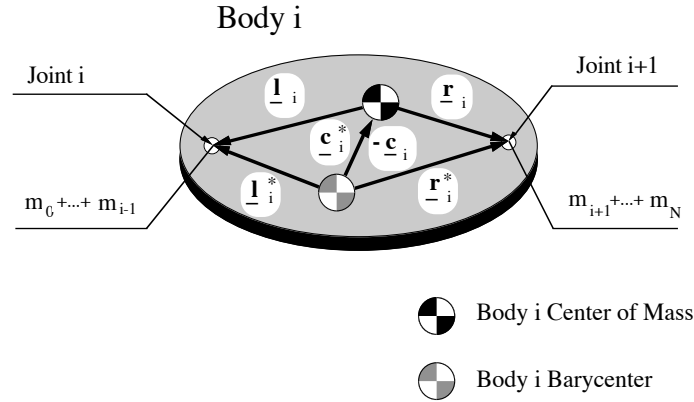


Figure 2. Definition of barycenters and vectors \underline{r}_i^* , \underline{l}_i^* , \underline{c}_i^* .

Since each \underline{v}_{ik} is defined by vectors fixed in body i which rotates with angular velocity $\underline{\omega}_i$, and because we assume that the manipulator has no prismatic joints, the time derivative of \underline{p}_k is simply given by:

$$\dot{\underline{p}}_k = \sum_{i=0}^N \underline{\omega}_i \times \underline{v}_{ik} \quad k = 0, \dots, N \quad (13)$$

Differentiating Equations (3) and combining the results with Equation (13) yields the following expression for the end-effector inertial velocity $\dot{\underline{r}}_E$:

$$\dot{\underline{r}}_E = \dot{\underline{r}}_{cm} + \sum_{i=0}^N \underline{\omega}_i \times \underline{v}_{iN} + \underline{\omega}_N \times \underline{r}_N \quad (14)$$

For this system the linear momentum vector with respect to the origin \mathbf{O} is:

$$\underline{p} = M \dot{\underline{r}}_{cm} = \sum_{k=0}^N m_k \dot{\underline{R}}_k \quad (15)$$

In the absence of external forces, and assuming zero initial CM velocity, \underline{p} is zero. Then $\dot{\underline{r}}_{cm}$ is zero and \underline{r}_{cm} is constant. We can assume that \underline{r}_{cm} is zero without loss

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of generality, which is equivalent to choosing the inertial origin, \mathbf{O} , to be at the CM . Consequently:

$$\dot{\mathbf{r}}_E = \sum_{i=0}^N \boldsymbol{\omega}_i \times \mathbf{v}_{iN} + \boldsymbol{\omega}_N \times \mathbf{r}_N \equiv \sum_{i=0}^N \boldsymbol{\omega}_i \times \mathbf{v}_{iN} \quad (16)$$

where \mathbf{v}_{iN} is equal to \mathbf{v}_{iN} for all i,k except for \mathbf{v}_{NN} , for which it is given by $\mathbf{v}_{NN} = \mathbf{v}_{NN} + \mathbf{r}_N$.

The end-effector inertial angular velocity required to find \mathbf{J}^* , see Equation (1), is the inertial velocity of the last body in the chain given by:

$$\boldsymbol{\omega}_E = \boldsymbol{\omega}_N \quad (17)$$

The inertial angular velocity $\boldsymbol{\omega}_j$ of the j^{th} body can be written as a function of the relative angular velocity of body i with respect to body $i-1$ (the joint velocity of joint i), called $\boldsymbol{\omega}_i^{i-1}$, as:

$$\boldsymbol{\omega}_j = \boldsymbol{\omega}_0 + \sum_{i=1}^j \boldsymbol{\omega}_i^{i-1} \quad j = 1, \dots, N \quad (18)$$

Equations (16), (17) and (18) relate the end-effector linear and angular velocities in inertial coordinates $\dot{\mathbf{r}}_E$ and $\boldsymbol{\omega}_E$ to the controlled relative angular velocities $\boldsymbol{\omega}_i^{i-1}$ and to the spacecraft inertial angular velocity, $\boldsymbol{\omega}_0$. Although $\boldsymbol{\omega}_0$ is uncontrolled, it can be found as a function of the controlled joint rates by using the principle of the angular momentum conservation. The system angular momentum vector with respect to the inertial origin is given by:

$$\mathbf{h} = \mathbf{r}_{cm} \times \mathbf{p} + \sum_{k=0}^N \{ \mathbf{I}_k \cdot \boldsymbol{\omega}_k + m_k \mathbf{p}_k \times \dot{\mathbf{p}}_k \} \quad (19)$$

where \mathbf{I}_k is the inertia dyadic of body k with respect to its center of mass. Since we assumed an initial zero linear momentum vector \mathbf{p} , the first term in the right side of Equation (19) is identically equal to zero and the angular momentum with respect to \mathbf{O} is equal to the angular momentum with respect to the system CM , \mathbf{h}_{cm} . Using Equations (11) and (13), Equation (19) can be written as [15]:

$$\mathbf{h} = \mathbf{h}_{cm} = \sum_{j=0}^N \sum_{i=0}^N \sum_{k=0}^N \mathbf{D}_{ijk} \cdot \boldsymbol{\omega}_j \quad (20)$$

where:

$$\mathbf{D}_{ijk} = \mathbf{I}_i \delta_{ij} \delta_{jk} + m_k \{ (\mathbf{v}_{ik} \cdot \mathbf{v}_{jk}) \mathbf{1} - \mathbf{v}_{jk} \mathbf{v}_{ik} \} \quad i, j, k = 0, \dots, N \quad (21)$$

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The dyadics $\underline{\mathbf{D}}_{ijk}$ are functions of the distribution of inertia through the system and are formed from the barycentric vectors $\underline{\mathbf{v}}_{ik}$. The terms δ_{ij} , δ_{jk} are Kronecker deltas.

It can be shown that the angular momentum given by Equation (20) can be written in the form:

$$\underline{\mathbf{h}}_{cm} = \sum_{j=0}^N \sum_{i=0}^N \underline{\mathbf{D}}_{ij} \bullet \underline{\boldsymbol{\omega}}_j \quad (22)$$

with $\underline{\mathbf{D}}_{ij}$ derived from Equation (21) with the help of Equation (10) and given by:

$$\underline{\mathbf{D}}_{ij} \equiv \left\{ \begin{array}{ll} -M \{(\underline{\mathbf{1}}_j^* \bullet \underline{\mathbf{r}}_i^*) \underline{\mathbf{1}} - \underline{\mathbf{1}}_j^* \underline{\mathbf{r}}_i^*\} & i < j \\ \underline{\mathbf{1}}_i + \sum_{k=0}^N m_k \{(\underline{\mathbf{v}}_{ik} \bullet \underline{\mathbf{v}}_{ik}) \underline{\mathbf{1}} - \underline{\mathbf{v}}_{ik} \underline{\mathbf{v}}_{ik}\} & i = j \\ -M \{(\underline{\mathbf{r}}_j^* \bullet \underline{\mathbf{1}}_i^*) \underline{\mathbf{1}} - \underline{\mathbf{r}}_j^* \underline{\mathbf{1}}_i^*\} & i > j \end{array} \right\} \quad (23)$$

where $\underline{\mathbf{1}}$ is the unit dyadic [15]. The angular momentum of the system is constant in the absence of external torques. We may assume that the initial angular momentum is zero (no initial spin); hence, $\underline{\mathbf{h}}_{cm}$ remains zero for all time. Equation (22) cannot be further integrated (with the exception of $N=1$) and must be carried along. Equations (16), (17), (18) and (22) are sufficient to describe the motion of the end-effector in inertial space as a function of a free-floating system's joint angular velocities, one in which the position and attitude of the spacecraft is not controlled.

The above vector formulation is independent of specific frames of reference. However, to construct the system Jacobian $\underline{\mathbf{J}}^*$, Equations (16), (17), (18) and (22) must be expressed in matrix form. For this purpose we assume all manipulator joints revolute; a reference frame with axes parallel to each body's principal axes is attached to each center of mass. The body inertia matrix expressed in this frame is diagonal. Bold lower case symbols represent column vectors, bold upper case matrices. Right superscripts must be interpreted as "with respect to," left as "expressed in frame." A missing left superscript implies a column vector expressed in the inertial frame.

The column vectors ${}^i \underline{\mathbf{v}}_{ik}$ expressed in frame i are transformed in the inertial frame as follows:

$$\underline{\mathbf{v}}_{ik} = \underline{\mathbf{T}}_i {}^i \underline{\mathbf{v}}_{ik} = \underline{\mathbf{T}}_0^0 \underline{\mathbf{v}}_{ik} \quad (24a)$$

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$${}^0\mathbf{v}_{ik} = {}^0\mathbf{T}_i {}^i\mathbf{v}_{ik} \quad (24b)$$

where \mathbf{T}_i is a transformation matrix that is given by:

$$\mathbf{T}_i(\mathbf{e}, n, q_1, \dots, q_i) = \mathbf{T}_0(\mathbf{e}, n) {}^0\mathbf{T}_i(q_1, \dots, q_i) \quad (25a)$$

$${}^0\mathbf{T}_i(q_1, \dots, q_i) = {}^0\mathbf{A}_1(q_1) \dots {}^{i-1}\mathbf{A}_i(q_i) \quad (25b)$$

Note that ${}^{i-1}\mathbf{A}_i(q_i)$ transforms a column vector expressed in frame i to a column vector in frame $i-1$ and is a function of the relative joint angle of the two frames, q_i . The inertia matrices \mathbf{D}_{ij} can be expressed in the spacecraft frame according to the following equation:

$${}^0\mathbf{D}_{ij} = \mathbf{T}_0^T \mathbf{D}_{ij} \mathbf{T}_0 \quad i, j = 1, \dots, N \quad (26)$$

The 3x3 transformation matrix \mathbf{T}_0 can be computed using the Euler parameters \mathbf{e} and n [16]:

$$\mathbf{T}_0(\mathbf{e}, n) = (n^2 - \mathbf{e}^T \mathbf{e}) \mathbf{1} + 2 \mathbf{e} \mathbf{e}^T + 2 n \mathbf{e}^\times \quad (27)$$

$$\mathbf{e}(\mathbf{a}, \theta) = \mathbf{a} \sin(\theta/2) \quad (28a)$$

$$n(\mathbf{a}, \theta) = \cos(\theta/2) \quad (28b)$$

where \mathbf{a} is the unit vector of the instant axis about which the spacecraft is rotated for an angle θ , the T superscript denotes transposition, and the $^\times$ superscript denotes a skew-symmetric matrix that is formed from an \mathbf{e} according to:

$$\mathbf{e}^\times = \begin{bmatrix} 0 & -e_z & e_y \\ e_z & 0 & -e_x \\ -e_y & e_x & 0 \end{bmatrix} \quad (29)$$

$\mathbf{1}$ is the 3x3 identity matrix.

The scalar form of Equation (19) can now be written as:

$$\boldsymbol{\omega}_j = \boldsymbol{\omega}_0 + \boldsymbol{\omega}_j^0 = \boldsymbol{\omega}_0 + \mathbf{T}_0 \sum_{i=1}^j {}^0\mathbf{T}_i {}^i\mathbf{u}_i \dot{q}_i \quad (30a)$$

$$= \boldsymbol{\omega}_0 + \mathbf{T}_0 {}^0\mathbf{F}_j \dot{\mathbf{q}} \quad j = 1, \dots, N \quad (30b)$$

where ${}^i\mathbf{u}_i$ is the unit column vector in frame i parallel to the revolute axis through joint i , and ${}^0\mathbf{F}_j$ is a 3xN matrix given by:

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$${}^0\mathbf{F}_j \equiv [{}^0\mathbf{T}_1^1 \mathbf{u}_1, {}^0\mathbf{T}_2^2 \mathbf{u}_2, \dots, {}^0\mathbf{T}_j^j \mathbf{u}_j, \mathbf{0}] \quad j = 1, \dots, N \quad (31)$$

where $\mathbf{0}$ is a $3 \times (N-j)$ zero element matrix, and:

$$\mathbf{q} = [q_1, q_2, \dots, q_j, \dots, q_N]^T \quad (32)$$

Using Equations (25) through (32), Equations (16), (17) and (22) yield:

$$\dot{\mathbf{r}}_E = \mathbf{T}_0 \{ {}^0\mathbf{J}_{11} {}^0\boldsymbol{\omega}_0 + {}^0\mathbf{J}_{12} \dot{\mathbf{q}} \} \quad (33a)$$

$$\boldsymbol{\omega}_E = \mathbf{T}_0 \{ {}^0\boldsymbol{\omega}_0 + {}^0\mathbf{J}_{22} \dot{\mathbf{q}} \} \quad (33b)$$

$$\mathbf{0} = {}^0\mathbf{D} {}^0\boldsymbol{\omega}_0 + {}^0\mathbf{D}_q \dot{\mathbf{q}} \quad (33c)$$

where:

$${}^0\mathbf{J}_{11} \equiv -\sum_{i=0}^N [{}^0\mathbf{T}_i^i \mathbf{v}_{iN}]^\times \quad {}^0\mathbf{J}_{12} \equiv -\sum_{i=1}^N [{}^0\mathbf{T}_i^i \mathbf{v}_{iN}]^\times {}^0\mathbf{F}_i \quad {}^0\mathbf{J}_{22} \equiv {}^0\mathbf{F}_N \quad (34a)$$

$${}^0\mathbf{D}_j \equiv \sum_{i=0}^N {}^0\mathbf{D}_{ij} \quad (j=0, \dots, N) \quad {}^0\mathbf{D} \equiv \sum_{j=0}^N {}^0\mathbf{D}_j \quad {}^0\mathbf{D}_q \equiv \sum_{j=1}^N {}^0\mathbf{D}_j {}^0\mathbf{F}_j \quad (34b)$$

The term ${}^0\mathbf{D}_{ij}$ ($i, j=0, \dots, N$) represents inertia matrices, derived according to Equation (23); these are expressed in the spacecraft frame. Equations (33a) and (34b) reflect the fact that the motion of the end-effector is the vector sum of two partial velocities. The first is due to the motion of the joints, the second to the resulting motion of the spacecraft caused by dynamic coupling. Equation (33c) expresses the conservation of angular momentum. ${}^0\mathbf{J}_{11}$ is a skew-symmetric 3×3 matrix whose elements correspond to the vector from the system *CM* to the end-effector, expressed in the spacecraft frame. ${}^0\mathbf{J}_{12}$ is a $3 \times N$ matrix whose N columns are the components of vectors starting at the manipulator joints and ending at the end-effector. Along with ${}^0\mathbf{J}_{22}$, they correspond to the Jacobian of the end-effector Virtual Manipulator, with the first link fixed. (This is equivalent to a fixed attitude spacecraft). ${}^0\mathbf{D}$ is the 3×3 inertia matrix of the whole system expressed in the spacecraft frame at the system *CM*, while ${}^0\mathbf{D}_q$ is a $3 \times N$ matrix and corresponds to the inertia of the system's moving parts. All the matrices in Equations (34a-b) depend on the system configuration \mathbf{q} , only.

Equation (33c) can be used to eliminate the spacecraft angular velocity ${}^0\boldsymbol{\omega}_0$ from Equations (33a-b), and hence to derive the free-floating system Jacobian \mathbf{J}^* , defined in Equation (1) as:

$$\mathbf{J}^*(\mathbf{e}, \mathbf{n}, \mathbf{q}) = \text{diag}(\mathbf{T}_0, \mathbf{T}_0) \begin{bmatrix} {}^0\mathbf{J}_{11} & {}^0\mathbf{D}^{-1} {}^0\mathbf{D}_{\mathbf{q}} + {}^0\mathbf{J}_{12} \\ & -{}^0\mathbf{D}^{-1} {}^0\mathbf{D}_{\mathbf{q}} + {}^0\mathbf{J}_{22} \end{bmatrix} \quad (35a)$$

$$= \text{diag}(\mathbf{T}_0, \mathbf{T}_0) {}^0\mathbf{J}^*(\mathbf{q}) \quad (35b)$$

$${}^0\mathbf{J}^*(\mathbf{q}) \equiv \begin{bmatrix} -{}^0\mathbf{J}_{11} & {}^0\mathbf{D}^{-1} {}^0\mathbf{D}_{\mathbf{q}} + {}^0\mathbf{J}_{12} \\ & -{}^0\mathbf{D}^{-1} {}^0\mathbf{D}_{\mathbf{q}} + {}^0\mathbf{J}_{22} \end{bmatrix} \quad (35c)$$

Both \mathbf{J}^* and ${}^0\mathbf{J}^*$ are $6 \times N$ matrices. Note that if N is equal to six, then \mathbf{J}^* is square and, if not singular, can be inverted. Note also that $\text{diag}(\mathbf{T}_0, \mathbf{T}_0)$ is always non-singular, because \mathbf{T}_0 is always non-singular. If N is less than six, it is not possible to follow any given end-effector trajectory while, if N is greater than six, the manipulator is redundant and a generalised inverse technique can be used. We will assume in the rest of the paper that N is equal to six (no redundancy) unless it is otherwise noted. If \mathbf{J}^* is going to be used for planning, \mathbf{T}_0 must be updated as the system moves. The new \mathbf{e} and \mathbf{n} are computed according to Equation (36) given below, see [16]:

$$\dot{\mathbf{e}} = 1/2 [\mathbf{e}^\times + \mathbf{n}\mathbf{1}] {}^0\boldsymbol{\omega}_0 \quad (36a)$$

$$\dot{\mathbf{n}} = -1/2 \mathbf{e}^T {}^0\boldsymbol{\omega}_0 \quad (36b)$$

III. Dynamic Singularities

Now we have shown a systematic and efficient way of constructing the Jacobian \mathbf{J}^* that relates the motion of the end-effector as a function of the manipulator's controlled rates $\dot{\mathbf{q}}$ in spite of the uncontrolled motions of the spacecraft, and revealed the Jacobian's explicit structure. We will address the important question of when the Jacobian becomes singular. This is important for control and physical reasons, since nearly all planning algorithms as well as all resolved rate or acceleration control algorithms need to invert \mathbf{J}^* , given by Equation (35). Also the system Jacobian, for a manipulator position, must be invertible or of full rank in order *physically to move* the manipulator end-effector in all directions at that point in space.

Singularities occur for fixed-base non-redundant manipulators when end-effector velocity due to the motion of one joint is parallel to the velocity due to the motion of some other joint. At such points, at least one degree of freedom is lost and the rank of the manipulator Jacobian \mathbf{J} is reduced, accordingly becoming singular. Singular points for fixed-base manipulators occur at workspace boundaries or when there is alignment of joint axes. Given the kinematic structure of a manipulator, we can find all its singular configurations by solving the equation $\det[\mathbf{J}(\mathbf{q})]=0$. The

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literature usually describes singular points in terms of fixed-base manipulator workspace positions instead of singular configurations or of singularities in the joint space because at any singular set of joint angles \mathbf{q}_s , there corresponds a singular point in the six DOF workspace. The obvious benefit is that the manipulator path planner or controller can be designed to avoid these workspace points. Singularities of fixed-base manipulators are *kinematic*, because it is sufficient to analyze the *kinematic structure* of the manipulator in order to identify them.

The singularities of \mathbf{J}^* for a free-floating space system are obtained by examining Equation (35). First, it can be seen that the term $\text{diag}(\mathbf{T}_0, \mathbf{T}_0)$ is always square and invertible. Thus, any singular points of \mathbf{J}^* are due to singular points of ${}^0\mathbf{J}^*(\mathbf{q})$ which can be found from the condition:

$$\det[{}^0\mathbf{J}^*(\mathbf{q})] = 0 \quad (37)$$

Equation (37) proves that all singularities are functions of the manipulator *configuration* with respect to its spacecraft, namely to the joint angles \mathbf{q} , not to the spacecraft attitude. These singularities correspond to singular points in the manipulator's *joint space*. From Equations (3), (11), (24) and (25), the position of the manipulator workspace follows as a function of both the joint angles \mathbf{q} and the spacecraft attitude \mathbf{e}, \mathbf{n} :

$$\mathbf{r}_E = \mathbf{p}_N + \mathbf{r}_N = \mathbf{T}_0(\mathbf{e}, \mathbf{n}) \sum_{i=0}^N {}^0\mathbf{T}_i(q_1, \dots, q_i) {}^i\mathbf{v}_{iN} \quad (38)$$

In general, due to the system's redundant nature, each point in the manipulator workspace can be reached with infinite system configurations \mathbf{q} and spacecraft attitudes (\mathbf{e}, \mathbf{n}) . Singular points in joint space cannot be mapped into unique points in the workspace. Furthermore, given an initial position of this system and both the final inertial position and orientation of its end-effector, both the final manipulator configuration and the spacecraft attitude is a function of the selected path to reach that end-effector position and orientation. This property is due to the *non-integrability* of the angular momentum equation as given by Equation (33c). Therefore, an end-effector position in the workspace can be singular or not depending on whether it reaches this point in a singular configuration. Thus the free-floating manipulator singularities in the workspace are *path dependent*.

In addition, ${}^0\mathbf{J}^*(\mathbf{q})$ in Equation (35c) depends on both the kinematic and mass properties expressed by the submatrices ${}^0\mathbf{J}_{12}$ and ${}^0\mathbf{J}_{22}$, and on the inertia distribution of the manipulator and the spacecraft, see Equations (23) and (34). As noted earlier, all ${}^0\mathbf{D}_{ij}$ matrices are functions of the system configuration and, hence, this distribution is configuration dependent. As a result, any singular configurations

cannot be predicted by examining the kinematic structure of the manipulator alone. Since the singularities of \mathbf{J}^* depend on the system's dynamic parameters, its mass and inertia properties, we call them *dynamic singularities*.

The dynamic singularities of a free-floating manipulator space manipulator system can be explained physically by noting that the end-effector velocity $\dot{\mathbf{x}}$, given by Equation (1), can be decomposed in two parts. The first part is due to the motion of the manipulator joints, the second is due to spacecraft motion. This second motion occurs because of the dynamic coupling of the spacecraft and the manipulator and is a function of the system masses and inertias. The matrix \mathbf{J}^* becomes singular when the end-effector velocity $\dot{\mathbf{x}}$, produced by the combined joint-spacecraft motion caused by the motion of a manipulator joint, is parallel to another $\dot{\mathbf{x}}$ produced by the by the same means by some other joint and the spacecraft. If the mass and inertia of the vehicle becomes very large, approximating a fixed-base manipulator, then all the dynamic terms in Equation (35) vanish and \mathbf{J}^* reduces to the fixed-base manipulator Jacobian, while the dynamic singularities reduce to the ordinary kinematic singularities.

The conclusion of this analysis is that if the spacecraft of a space manipulator system is not actively controlled but is free-floating, then dynamic singularities can occur. All resolved rate or resolved acceleration control schemes will fail because at these points, Equation (35) has no inverse. Control schemes that compute the desired joint torques by using a transposed Jacobian will fail to keep the desired end-effector velocity because dynamic singularities represent an inherent *physical* limitation. The manipulator will move with a velocity that is the projection of the desired velocity on the allowed direction: the result may be large end-effector errors.

IV. A Planar Example

Consider the simple planar free-floating space manipulator system shown in Figure 3. The system parameters are given in Table I. As shown in the Appendix, the system Jacobian is:

$$\mathbf{J}^*(\theta, \mathbf{q}) = \begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix} {}^0\mathbf{J}^*(\mathbf{q}) \quad (39a)$$

where:

$${}^0\mathbf{J}^*(\mathbf{q}) = \frac{1}{D} \begin{bmatrix} -(\beta s_1 + \gamma s_{12})D_0 & \beta s_1 D_2 - \gamma s_{12}(D_0 + D_1) \\ -\alpha(D_1 + D_2) + (\beta c_1 + \gamma c_{12})D_0 & -(\alpha + \beta c_1)D_2 + \gamma c_{12}(D_0 + D_1) \end{bmatrix} \quad (39b)$$

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where θ , q_1 and q_2 , are defined in Figure 3, $s_1 = \sin(q_1)$, $c_{12} = \cos(q_1+q_2)$ etc. The inertia scalar sums D , D_0 , D_1 and D_2 are defined in the Appendix, see Equation (A13), and $\alpha \equiv {}^0r_0^* = 0.426$ m, $\beta \equiv {}^1r_1^* = 0.894$ m, and $\gamma \equiv {}^2c_2^* + r_2 = 0.968$ m. Since each D_i ($i=0,1,2$) and D are functions of \mathbf{q} , the Jacobian elements are more complicated functions of the \mathbf{q} than their fixed-base counterparts. This Jacobian should be compared to the fixed-base manipulator Jacobian \mathbf{J} which is given by:

$$\mathbf{J}(\mathbf{q}) = \begin{bmatrix} -(l_1+r_1)s_1-(l_2+r_2)s_{12} & -(l_2+r_2)s_{12} \\ (l_1+r_1)c_1+(l_2+r_2)c_{12} & (l_2+r_2)c_{12} \end{bmatrix} \quad (40)$$

The same structure between \mathbf{J}^* or ${}^0\mathbf{J}^*(\mathbf{q})$ and $\mathbf{J}(\mathbf{q})$ can be seen.

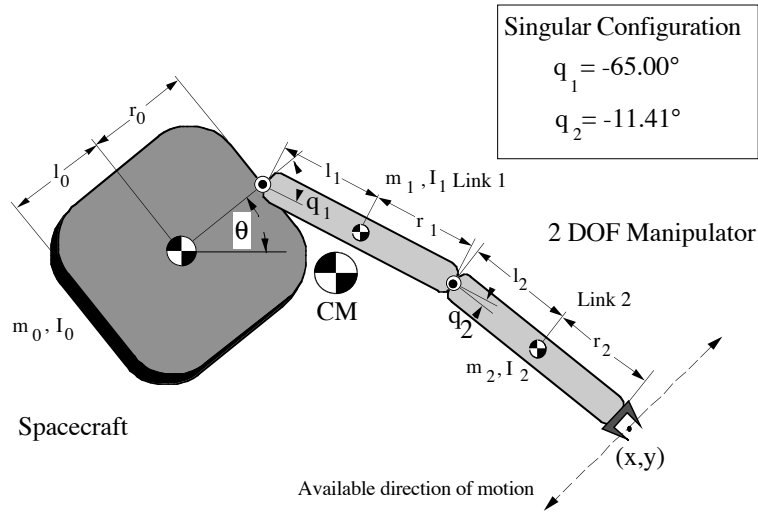


Figure 3. A planar free-floating manipulator system shown in a dynamically singular configuration.

Table I. The system parameters.

Body	l_i (m)	r_i (m)	m_i (Kg)	I_i (Kg m ²)
0	.5	.5	40	6.667
1	.5	.5	4	0.333
2	.5	.5	3	0.250

In order to invert \mathbf{J}^* given by Equation (38), the 2×2 matrix, ${}^0\mathbf{J}^*(\mathbf{q})$, must be inverted. First its determinant becomes zero when:

$$\alpha\beta D_2(q_1, q_2)\sin(q_1) + \beta\gamma D_0(q_1, q_2)\sin(q_2) - \alpha\gamma D_1(q_1, q_2)\sin(q_1 + q_2) = 0 \quad (41)$$

The values of q_1 and q_2 which satisfy Equation (41) and result in dynamically singular configurations can be plotted in joint space as shown in Figure 4. This Figure also shows that conventional kinematic singularities like $q_1 = k\pi$, $q_2 = k\pi$, $k=0, \pm 1, \dots$ still satisfy Equation (41). However, infinitely more dynamically singular configurations exist which cannot be predicted from the kinematic structure of the manipulator.

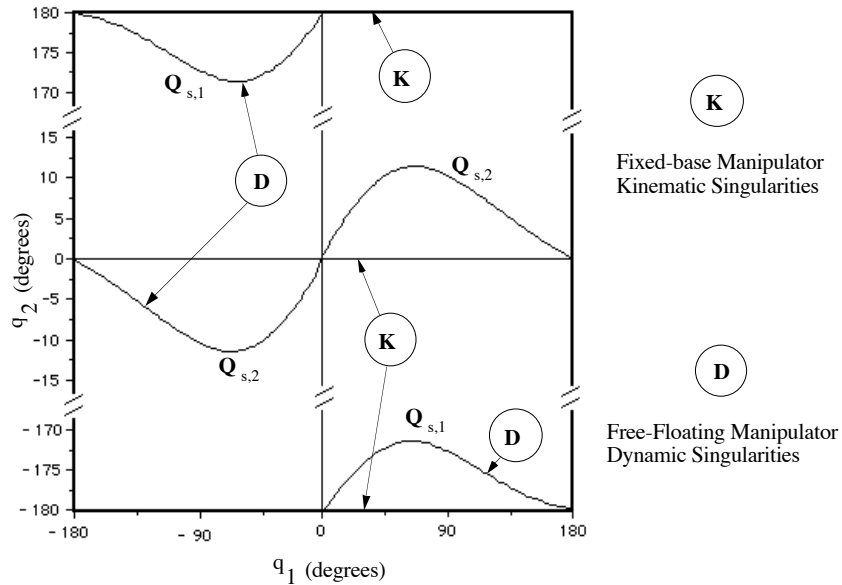


Figure 4. Dynamically singular configurations (q_1, q_2) for the two link manipulator in joint space.

Figure 3 shows the manipulator in the singular configuration at $q_1 = -65^\circ$, $q_2 = -11.41^\circ$: spacecraft attitude at $\theta = 40^\circ$. This figure also shows the only available direction for the end-effector motion. The inertial motion of the end-effector in this configuration will be the shown, no matter how the joint actuators are driven. The best a control algorithm can do is to follow the available direction. *All* algorithms that use a Jacobian inverse, such as the resolved rate or resolved acceleration control algorithms, fail at such a point. Ones that use a pseudoinverse Jacobian or a Jacobian transpose will likely follow the available direction, but may result in large unrecoverable errors.

To demonstrate this problem, the manipulator end-point is commanded to reach the workspace point (1.5,1.5) starting from the initial location of (2,0) with initial attitude θ equal to 21° using a simple Transposed Jacobian Control algorithm,

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augmented by a velocity feedback term for increased stability margins [17]. This control algorithm assumes that the end-effector inertial position and velocity, \mathbf{x} and $\dot{\mathbf{x}}$, can be calculated or measured directly. Assuming \mathbf{x} and $\dot{\mathbf{x}}$ are measured, the control law is:

$$\boldsymbol{\tau} = \mathbf{J}^{*T} \{ \mathbf{K}_p (\mathbf{x}_{des} - \mathbf{x}) - \mathbf{K}_d \dot{\mathbf{x}} \} \quad (42)$$

where \mathbf{x}_{des} is the inertial desired point location. The matrices \mathbf{K}_p and \mathbf{K}_d are diagonal. Note that this algorithm specifies the desired end-effector location; the path of the end-effector to this desired location is not specified in advance. If the control gains are large enough, then the motion of the end-point will be a straight line. The torque vector $\boldsymbol{\tau}$ is non-zero until the $(\mathbf{x}_{des} - \mathbf{x})$ and $\dot{\mathbf{x}}$ are zero or until the vector in the brackets in Equation (42) is in the null space of \mathbf{J}^{*T} .

Figure 5 shows the motion of the end-effector from the initial location at point A (2,0), towards the final location at point D (1.5,1.5). The control gain matrices are $\mathbf{K}_p = \text{diag}(5,5)$ and $\mathbf{K}_d = \text{diag}(15,15)$. Initially the end-effector path is initially almost a straight line. However, once the manipulator assumes a dynamically singular configuration at point B in Figure 5, the end-effector cannot move towards its desired position; rather it moves along the available direction converging finally to point C, for which $(\mathbf{x}_{des} - \mathbf{x})$ is in the null space of \mathbf{J}^{*T} . Any further motion beyond C is impossible. Figure 6 shows the time history of the spacecraft attitude and manipulator joint angles. The system reaches a dynamically singular configuration in about 5 seconds and thereafter oscillates about singular configurations until it finally converges to point C. Note again that an algorithm using a Jacobian inverse would fail at a location like point B.

Finally, it is interesting to note that when both m_0 and I_0 approach infinity, \mathbf{J}^* approaches \mathbf{J} , the Jacobian derived for the same manipulator on a fixed-base, without any change in matrix size. To show this note that if the spacecraft is massive, $\beta \rightarrow l_1 + r_1$, $\gamma \rightarrow l_2 + r_2$, approaching the manipulator link lengths, $m_0/M \rightarrow 1$, $m_1/M \rightarrow 0$, $m_2/M \rightarrow 0$, $D_0/D \rightarrow 1$, $D_1/D \rightarrow 0$ and $D_2/D \rightarrow 0$. \mathbf{T}_0 becomes a constant transformation from the manipulator base frame to the inertial frame, usually the unit matrix.

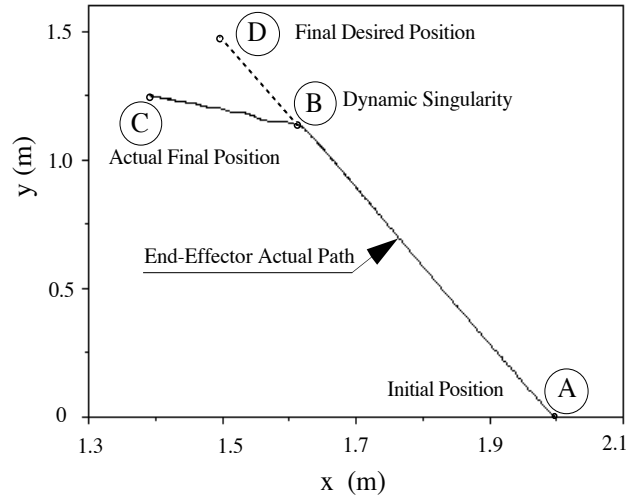


Figure 5. Dynamic Singularities result in large tracking errors.

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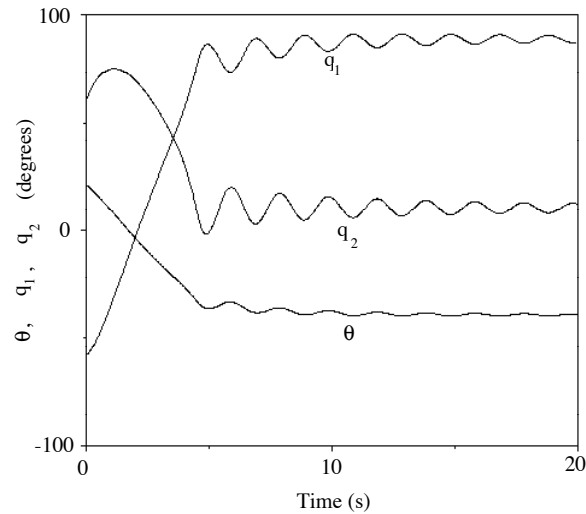


Figure 6. The spacecraft attitude θ and the manipulator joint angles q_1 and q_2 as a function of time

V. Space Manipulator Workspaces

Space manipulators have more complex workspace characteristics than fixed-base manipulators, as shown by using the concept of the Virtual Manipulator. Vafa describes a *constrained* workspace, one where all points can be reached if the attitude of the spacecraft is controlled, but not its position [5]. This workspace is a sphere with its center at the system's *CM*. However, it can be shown that if the attitude is not controlled, as for a free-floating system, then points in this space can still always be reached by a suitable path selection [15]. For this reason we prefer to call this workspace the *reachable* workspace. What follows below shows that the nature of this workspace is related to a system's dynamic singularities.

We have proven already that a system's dynamic singularities are a unique function of the configuration and that their occurrence at a particular inertial workspace location is path dependent. Here we are interested in finding regions in the reachable workspace in which dynamic singularities will never occur.

Recall that dynamically singular configurations can be found from Equation (37). Its solution represents a family of hypersurfaces $\mathbf{Q}_{s,i}$ ($i=1,2,\dots$) in the manipulator *joint space*. These hypersurfaces are collections of points \mathbf{q}_s that result in dynamically singular configurations. Further note that the transformation matrix \mathbf{T}_0 does not change the length of a vector; hence, the distance R of the end-effector location from the system *CM* can be written using Equation (38) as a function of the system's configuration \mathbf{q} only:

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$$R = R(\mathbf{q}) = \left\| \sum_{i=0}^N {}^0\mathbf{T}_i(q_1, \dots, q_i) {}^i\mathbf{v}_{iN} \right\| \quad (43)$$

The symbol $\|\bullet\|$ denotes a vector's length. Equation (43) also defines a spherical shell in inertial space with its center at the *CM* and with a radius R . Hence, each singular configuration \mathbf{q}_s is mapped according to Equation (43) to a spherical shell in inertial space. By the same token, each hypersurface $\mathbf{Q}_{s,i}$ is mapped according to Equation (43) to a volume contained within the spherical shells with radii:

$$R_{\min,i} = \min_{\mathbf{q} \in \mathbf{Q}_{s,i}} R(\mathbf{q}) \quad (i=1,2,\dots) \quad (44a)$$

$$R_{\max,i} = \max_{\mathbf{q} \in \mathbf{Q}_{s,i}} R(\mathbf{q}) \quad (i=1,2,\dots) \quad (44b)$$

All workspace points that belong in this volume can be singular if they are reached in singular configurations \mathbf{q}_s . As shown earlier, this may happen or not depending on the path taken by the manipulator's end-effector. If there is more than one singular hypersurfaces, then there are more such volumes containing points that can lead to singular configurations. We call the union of all these volumes a *Path Dependent Workspace (PDW)*. The Path Dependent Workspace contains all reachable workspace locations that may be reached in singular configurations, depending upon the path taken by the end-effector. It follows that locations in the PDW can be reached with some paths but not with others; this justifies their name. In order to reach points belonging to the PDW, carefully selected paths must be employed.

Subtracting the PDW from the reachable workspace results in the *Path Independent Workspace (PIW)*. Due to its construction, this workspace region contains all reachable workspace locations that will *never* lead to dynamically singular configurations. It follows that all points in the Path Independent Workspace can be reached by any path, assuming that this path lies entirely in the PIW. It can be shown that the PIW is a subset of the *free workspace* defined by Vafa [15,5]. PIW or PDW spaces may reduce to zero depending on the case. A clear goal for the designer is to reduce the PDW and increase the PIW.

The construction of the PIW and PDW workspaces is demonstrated using the system illustrated in Figure 3. The distance R of the end-effector from the system *CM* given by Equation (43) is written as:

$$R = R(\mathbf{q}) = \sqrt{\alpha^2 + \beta^2 + \gamma^2 + 2\alpha\beta\cos(q_1) + 2\alpha\gamma\cos(q_1+q_2) + 2\beta\gamma\cos(q_2)} \quad (45)$$

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For this example, there are two hypersurfaces \mathbf{Q}_s which are lines in the joint space (see Figure 4), and are found according to Equation (41). Each of these lines corresponds to pairs of q_1 and q_2 , which are substituted in Equation (45). Then, the conditions in Equations (44a-b) result in two Path Dependent Workspaces, constrained by $(R_{\min,1}, R_{\max,1})$ and $(R_{\min,2}, R_{\max,2})$ respectively:

$$R_{\min,1} = 0.352 \text{ m} = \alpha + \beta - \gamma \quad (46a)$$

$$R_{\max,1} = 0.500 \text{ m} = \alpha + \gamma - \beta \quad (46b)$$

$$R_{\min,2} = 1.436 \text{ m} = \beta + \gamma - \alpha \quad (47a)$$

$$R_{\max,2} = 2.288 \text{ m} = \alpha + \beta + \gamma \quad (47b)$$

The PIW is then found by subtracting the two PDW regions defined above from the reachable workspace, see Figure 7. In general, the PIW is smaller than the free workspace defined in Reference [5], although in this case it is equal to it. When the end-effector path has points belonging to the PDW, such as path A in Figure 7, the manipulator may assume a dynamically singular configuration because points in the PDW region can be dynamically singular, depending on the path. On the other hand, paths totally within the PIW region, such as path B, can never lead to dynamically singular configurations.

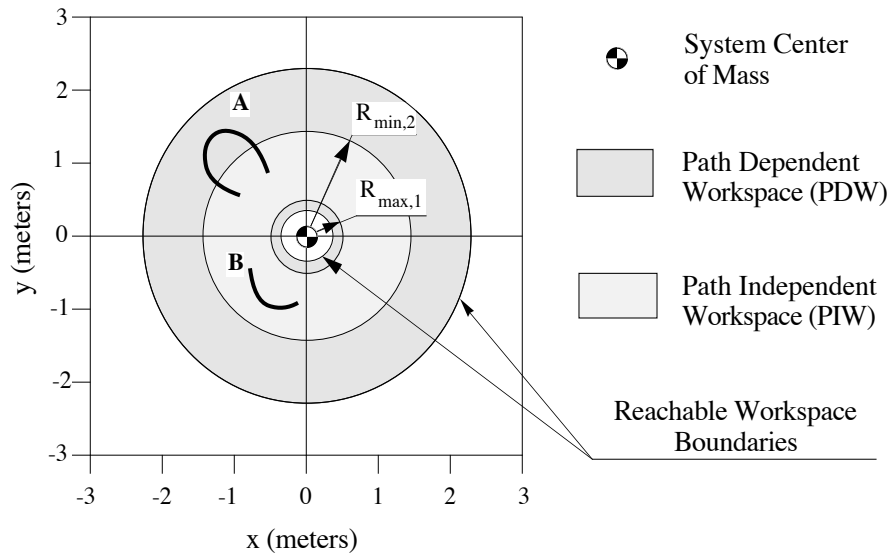


Figure 7. The Reachable, the Path Dependent, and Path Independent Workspaces.

VI. Reducing the Effect of Dynamic Singularities

Maximizing the PIW clearly reduces the impact of dynamic singularities on a system's effectiveness. This can be achieved by recalling that dynamic singularities occur because the spacecraft is free to rotate as a result of manipulator motions, see Equation (33). If the spacecraft attitude is kept constant, $\boldsymbol{\omega}_0$ is zero, and the only singular points are due to the kinematic singularities; the PIW is maximum [15]. However, this method requires the active control of the spacecraft attitude which can increase system complexity and cost and reduce the system's useful life.

The PIW can also be maximized using manipulator redundancy. If the manipulator is at a singular configuration, the redundant degrees of freedom may be used to achieve the necessary end-effector velocity. This is an area which requires additional research.

If the spacecraft is made to be massive compared to the manipulator, \mathbf{I}_0 and \mathbf{D}_0 become large. For example, it can be seen from Equation (41) that if \mathbf{I}_0 approaches infinity, the only singular configurations are the kinematic ones ($q_2 = \pm 0^\circ, \pm 180^\circ$). This means that if the inertia of the spacecraft is infinite, then no dynamic singularities occur and the PIW is equal to the maximum workspace. Although it is desirable in most cases to make the spacecraft as light as possible for a number of reasons, such as launch weight, a system's designer has the freedom to increase the system's inertia keeping its mass constant. Such a design would result in an increase in the system's PIW.

Finally, for the case where the manipulator acts in a plane, it can be shown that if the manipulator is mounted at the spacecraft's center of mass, the PIW is equal to the reachable workspace and the PDW is eliminated [14,15]. For the example discussed in Section IV, if ${}^0r_0^*$ or α are zero, the only singular configuration that exists is at q_2 equal to $k\pi$ ($k=0,\pm 1,\dots$), see Equation (41). This is a kinematic singularity and corresponds to the reachable workspace boundaries. If ${}^0r_0^*$ or α approach zero, then the two circles that define the PIW, shown in Figure 7, approach the reachable workspace boundaries, see Equations (46); hence the dynamic singularities become less important. In some cases it may be possible to use combinations of the various techniques discussed. For example, a system may be designed to have a large moment of inertia about one axis while the manipulator arm is mounted near the spacecraft *CM* in the other two dimensions.

V. Conclusions

A general formulation describing the motion of a space manipulator system is presented. The system Jacobian is derived for a free-floating system where spacecraft position and attitude are not controlled. This Jacobian can be singular in

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configurations that are distinct from the usual kinematically singular configurations: a free-floating manipulator system exhibits singularities due to the dynamic coupling between link motions and the spacecraft. These singularities are called *dynamic singularities* and can be a serious problem for all planning and control algorithms that do not assume active control of spacecraft attitude. Consequently, their effects must be considered in the design of such systems.

Additionally, a workspace point may be singular or not depending on the end-effector path used to reach this point. Thus a manipulator's reachable workspace is divided in two regions. In the first, called a Path Independent Workspace (PIW), no dynamic singularities can occur; in the second, called a Path Dependent Workspace (PDW), dynamic singularities may occur depending on the path taken by the end-effector in the inertial space. Some notions are presented that may help in maximizing the PIW.

Acknowledgements

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Appendix A

The planar two link system shown in Figure 3 assumes the two coordinates of the end-effector, x and y , are controlled by the two manipulator joint angles, q_1 and q_2 . End-effector orientation is not controlled for this two DOF system ($N=2$), hence Equation (1) for this system is simply:

$$\dot{\mathbf{x}} = \dot{\mathbf{r}}_E = \frac{d}{dt} [x, y]^T = \mathbf{J}^* \dot{\mathbf{q}} \quad (\text{A1})$$

with:

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$$\mathbf{x} = \mathbf{r}_E = [x, y]^T \quad (\text{A2})$$

$$\mathbf{q} = [q_1, q_2]^T \quad (\text{A3})$$

while \mathbf{J}^* given by Equation (35), becomes:

$$\mathbf{J}^*(\theta, \mathbf{q}) = \mathbf{T}_0(\theta) {}^0\mathbf{J}^*(\mathbf{q}) = \mathbf{T}_0 [{}^0\mathbf{J}_{11} {}^0\mathbf{D}^{-1} {}^0\mathbf{D}_q + {}^0\mathbf{J}_{12}] \quad (\text{A4})$$

where θ denotes the spacecraft attitude, as shown in Figure 3. For this example, the transformation matrix from the spacecraft frame to the inertial frame, \mathbf{T}_0 , is given by:

$$\mathbf{T}_0(\theta) = \text{Rot}(\theta) = \begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix} \quad (\text{A5})$$

Only the planar sub-part of the transformation matrices is used for simplicity. The transformation matrices ${}^0\mathbf{T}_i$ are found according to Equation (25):

$${}^0\mathbf{T}_1 = \text{Rot}(q_1)$$

$${}^0\mathbf{T}_2 = \text{Rot}(q_1) \text{Rot}(q_2) \quad (\text{A6})$$

The following demonstrates the construction of the system inertia matrix. The matrices in Equation (A4) are assembled by first expressing all \mathbf{v}_{ik} in Equation (12) in the frame of the i^{th} body, according to Equations (7)-(9). For the sake of simplicity we assume that all \mathbf{r}_i and \mathbf{l}_i are parallel to the x axis of the i^{th} frame. Hence, only the x-component of the barycentric vectors ${}^i\mathbf{v}_{ik}$ is non-zero and given by:

$${}^0r_0^* = \frac{1}{M} r_0 m_0$$

$${}^0c_0^* = -\frac{1}{M} r_0 (m_1 + m_2)$$

$${}^0l_0^* = -\frac{1}{M} r_0 (m_1 + m_2) - l_0$$

$${}^1r_1^* = \frac{1}{M} \{r_1(m_0 + m_1) + l_1 m_0\}$$

$${}^1c_1^* = \frac{1}{M} (l_1 m_0 - r_1 m_2)$$

$${}^1l_1^* = -\frac{1}{M} \{l_1(m_1 + m_2) + r_1 m_2\}$$

$${}^2r_2^* = \frac{1}{M} l_2(m_0 + m_1) + r_2$$

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$$\begin{aligned} {}^2c_2^* &= \frac{1}{M} l_2(m_0+m_1) \\ {}^2l_2^* &= -\frac{1}{M} l_2m_2 \end{aligned} \quad (A7)$$

where the total mass of the system, M , is given by:

$$M = m_0 + m_1 + m_2 \quad (A8)$$

For the planar case, the inertia matrices ${}^0D_{ij}$ corresponding to Equation (3) reduce to the scalars ${}^0d_{ij}$ and are given by:

$$\begin{aligned} {}^0d_{00} &= I_0 + \frac{m_0(m_1+m_2)}{M} r_0^2 \\ {}^0d_{10} &= \frac{m_0r_0}{M} \{l_1(m_1+m_2) + r_1m_2\} \cos(q_1) = {}^0d_{01} \\ {}^0d_{20} &= \frac{m_0m_2}{M} r_0l_2 \cos(q_1+q_2) = {}^0d_{02} \\ {}^0d_{11} &= I_1 + \frac{m_0m_1}{M} l_1^2 + \frac{m_1m_2}{M} r_1^2 + \frac{m_0m_2}{M} (l_1+r_1)^2 \\ {}^0d_{21} &= \left\{ \frac{m_1m_2}{M} r_1l_2 + \frac{m_0m_2}{M} l_2(l_1+r_1) \right\} \cos(q_2) = {}^0d_{12} \\ {}^0d_{22} &= I_2 + \frac{m_2(m_0+m_1)}{M} l_2^2 \end{aligned} \quad (A9)$$

Both ${}^i\mathbf{u}_i$ ($i=1,2$) vectors in Equation (17) are equal to $[0 \ 0 \ 1]^T$; the 0F_i matrices reduce to:

$$\begin{aligned} {}^0F_1 &= [1 \ 0] \\ {}^0F_2 &= [1 \ 1] \end{aligned} \quad (A10)$$

For simplicity, set:

$$\alpha \equiv {}^0r_0^* \quad \beta \equiv {}^1r_1^* \quad \gamma \equiv {}^2c_2^* + r_2 \quad (A11)$$

Then the matrices in Equation (34) are given by:

$${}^0J_{11} = \begin{bmatrix} -\beta s_1 - \gamma s_{12} \\ \alpha + \beta c_1 + \gamma c_{12} \end{bmatrix}, \quad {}^0J_{12} = \begin{bmatrix} -\beta s_1 - \gamma s_{12} & -\gamma s_{12} \\ \beta c_1 + \gamma c_{12} & \gamma c_{12} \end{bmatrix}, \quad {}^0J_{22} = {}^0F_2 \quad (A12)$$

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$${}^0\mathbf{D}_j \equiv \mathbf{D}_j = \sum_{i=0}^2 {}^0d_{ij} \quad (j=0,1,2), \quad {}^0\mathbf{D} \equiv \mathbf{D} = \mathbf{D}_0 + \mathbf{D}_1 + \mathbf{D}_2, \quad {}^0\mathbf{D}_q = [\mathbf{D}_1 + \mathbf{D}_2 \quad \mathbf{D}_2] \quad (\text{A13})$$

where $s_1 \equiv \sin(q_1)$, $c_{12} \equiv \cos(q_1 + q_2)$ etc. Finally, the system Jacobian \mathbf{J}^* is assembled from Equations (A4) and (A7)-(A13) and given as Equation (38).