FAILURE RECOVERY CONTROL FOR SPACE ROBOTIC SYSTEMS

Evangelos Papadopoulos, Steven Dubowsky

Massachusetts Institute of Technology
Cambridge, MA 02139

ABSTRACT

The problem of controlling a failed joint of a space manipulator is addressed. It is shown that failure recovery control may be possible when dynamic coupling exists between the link whose joint has failed and some other link whose joint is working, and when the system inertia matrix is invariant with respect to the failed joint angle. A failure recovery control technique is developed and applied to two simple examples.

I. INTRODUCTION

In space systems, it is very important to be able to tolerate a component or subsystem failure, since these can jeopardize an entire mission. For example, imagine the consequences of a failure in a joint of the Space Shuttle manipulator, so that it cannot be driven back into its stowed position. While these systems are often designed with redundant elements to give them higher reliability, it is also highly desirable if they be able to operate after a failure, be it with reduced capabilities. In the above example, the ability to use the working manipulator joints to control the system partially and to stow it, is critical. In this study, the question of how to control a space manipulator system after a failure of one of its joints is addressed.

A possible scenario for failure recovery control is as follows: The i^{th} joint of a manipulator system fails in some way. Its actuator, controller or control circuitry fails, but its brakes and encoders still operate. Assume that the working joints can be used to control the i^{th} joint, and drive this joint to some position, such as its stow configuration, where it can be locked using the brakes. The manipulator can then be stowed using its other joints. The failed joint could also be driven and locked into a joint position which satisfies an optimality criterion, such as the maximization of the broken system's workspace. The system could then be used with a reduced number of Degrees-of-Freedom, (DOF), and hence the system would have "recovered" to some extent from the failure.

While substantial research has been done on the dynamics and control of space manipulator systems, [1-8], very little work related to manipulator joint failures exists. Reference [9] proposed a manipulator with passive joints which is controlled in distinct phases by using the passive joint brakes and the active joint actuators. Joint failures were considered in Reference [10], where their impact on a teleoperator's performance was addressed.

The basic thrust of this paper is to show that failure recovery control is possible and to identify sufficient conditions for its use. The controllability of a failed manipulator is first studied based on a dynamic model linearized around an equilibrium point. It is shown that a failed joint can be controlled if dynamic coupling between the link whose joint has failed and some other link whose joint is working exists. A failure recovery controller is designed and shown to be effective if the system's inertia matrix is invariant with respect to the failed joint.

II. CONTROLLABILITY OF A FAILED SPACE MANIPULATOR

It is assumed here that the i^{th} joint function has failed, and that the manipulator's joints are frictionless; then $\tau_i = 0$. An important question that is considered next is whether such a manipulator is controllable, i.e. whether we can drive its joints to any desired position and velocity.

To address this question, we consider the nonlinear dynamics of a rigid space manipulator whose equations of motion are written in the form:

$$\mathbf{H}(\mathbf{q})\,\ddot{\mathbf{q}} + \mathbf{C}(\mathbf{q},\dot{\mathbf{q}})\,\dot{\mathbf{q}} = \boldsymbol{\tau} \tag{1}$$

where $\mathbf{q}$ is the vector of the generalized system coordinates, $\mathbf{H}(\mathbf{q})$ is an inertia matrix, $\mathbf{C}(\mathbf{q},\dot{\mathbf{q}})$ is a matrix that contains the nonlinear Coriolis and centripetal terms, and $\boldsymbol{\tau}$ is the joint torque vector. It has been shown that Equation (1) will describe the dynamics of *free-floating* manipulators systems, consisting of a manipulator mounted on a uncontrolled spacecraft. It will also describe the dynamics of *free-flying* manipulator systems, consisting of a manipulator mounted on a spacecraft whose position and attitude are actively controlled, and remain fixed [11]. In free-floating systems, $\mathbf{q}$ consists only of the manipulator joint angles, and the spacecraft coordinates are eliminated, see References [6,11]. In such a case, the inertia matrix has the same structure as the inertia matrix of a fixed-based manipulator, albeit with different entries.

It can be recognized that if one joint controller does not operate, it is not possible to use feedback linearization to convert Equation (1) to a linear double integrator system. Hence, the direct application of well established controllability tests cannot be used. Instead, the controllability of a space manipulator with a failed joint will be analyzed locally, by linearizing the system dynamic equations around an equilibrium point. This analysis will let us determine whether we can design a controller, with a linear error-feedback part, capable of driving the failed joint to a desired angle. To this end, we examine first the equilibria of the unforced system described by Equation (1). Note that this equation has infinite equilibrium solutions of the form $(\mathbf{q},\dot{\mathbf{q}}) = (\mathbf{q}_{des},\mathbf{0})$. In the neighborhood of such an equilibrium point, Equation (1) can be linearized and written as:

$$\mathbf{H}(\mathbf{q}_{des})\,\delta\ddot{\mathbf{q}} = \delta\boldsymbol{\tau} = \delta[\tau_1,\ldots,\tau_{i-1},0,\tau_{i+1},\ldots,\tau_N]^T \quad (2)$$

where τ_i, the failed joint torque, has been set to zero, and δ denotes a small quantity. Equation (2) can be written in the standard linear form as:

$$\frac{d}{dt}\begin{bmatrix}\delta\mathbf{q}\\ \delta\dot{\mathbf{q}}\end{bmatrix} = \begin{bmatrix}\mathbf{0} & \mathbf{1}\\ \mathbf{0} & \mathbf{0}\end{bmatrix}\begin{bmatrix}\delta\mathbf{q}\\ \delta\dot{\mathbf{q}}\end{bmatrix} + \begin{bmatrix}\mathbf{0}_i\\ \mathbf{H}^{-1}\mathbf{1}_i\end{bmatrix}\mathbf{u} \quad (3)$$

where $\mathbf{u} = \delta[\tau_1,\ldots,\tau_{i-1},\tau_{i+1},\ldots,\tau_N]^T \in \mathbf{R}^{N-1}$ and $\mathbf{1}_i$ and $\mathbf{0}_i$ are both N×(N-1) matrices obtained from the unit and zero N×N matrices respectively, after their i^{th} column, the one that corresponds to the failed joint, is removed. The controllability matrix for the linearized system, $\mathbf{F}$, is:

$$\mathbf{F} = \begin{bmatrix}\mathbf{0}_i & \mathbf{H}^{-1}\mathbf{1}_i\\ \mathbf{H}^{-1}\mathbf{1}_i & \mathbf{0}_i\end{bmatrix} \quad (4)$$

Clearly, since $\mathbf{F}$ has 2(N-1) columns, its rank is at most 2(N-1) and not 2N for the system to be controllable, and therefore the linearized system is *uncontrollable*. However, this system does have a 2(N-1) dimensional controllable subspace.

It should be recalled that our main interest is to control q_i and not necessarily to control the entire system configuration $\mathbf{q}$. Therefore, the attention is now focused on examining whether q_i is among the angles that can be controlled. If this task can be achieved, then the failed joint angle q_i can be locked at a desired angle, and normal operation can be resumed with the remaining DOF, or the system can be stowed. More formally, the question raised here is whether the system is output controllable when its output is $\mathbf{y}=[\delta q_i, \delta\dot{q}_i]^T$.

The output of the linearized system is written as:

$$\mathbf{y} = \begin{bmatrix}0\ldots1\ldots0 & 0\ldots0\ldots0\\ 0\ldots0\ldots0 & 0\ldots1\ldots0\end{bmatrix}\begin{bmatrix}\delta\mathbf{q}\\ \delta\dot{\mathbf{q}}\end{bmatrix} = \mathbf{C}_i\begin{bmatrix}\delta\mathbf{q}\\ \delta\dot{\mathbf{q}}\end{bmatrix} \quad (5)$$

where the 1's appear at the i^{th} and $(N+i)^{th}$ position of the first and second rows of $\mathbf{C}_i$ respectively. The output controllability matrix for this system, $\mathbf{F}_i$, is a 2×(N-1) matrix given by:

$$\mathbf{F}_i = \mathbf{C}_i\,\mathbf{F} =$$

$$= \begin{bmatrix}0\ldots0 & h'_{i1}\ldots h'_{ii-1}\ h'_{ii+1}\ldots h'_{iN}\\ h'_{i1}\ldots h'_{ii-1}\ h'_{ii+1}\ldots h'_{iN} & 0\ldots0\end{bmatrix} \quad (6)$$

where h'_{ij} (j=1,…,i-1,i+1,…N) are elements of the i^{th} row of $\mathbf{H}^{-1}$ with the i^{th} element eliminated. In order to be able to control the failed joint, $\mathbf{F}_i$ must have rank 2. However, if all the h_{ij} (j=1,…,i-1,i+1,…,N) entries of the inertia matrix $\mathbf{H}$ are zero, then both $\mathbf{H}$ and $\mathbf{H}^{-1}$ have the form:

$$\mathbf{H} = \begin{bmatrix}\mathbf{X}_1 & \mathbf{0}_1 & \mathbf{X}_2\\ 0\ldots0 & h_{ii} & 0\ldots0\\ \mathbf{X}_2^T & \mathbf{0}_2 & \mathbf{X}_3\end{bmatrix} \quad \mathbf{H}^{-1} = \begin{bmatrix}\mathbf{X}'_1 & \mathbf{0}_1 & \mathbf{X}'_2\\ 0\ldots0 & h'_{ii} & 0\ldots0\\ \mathbf{X}'^T_2 & \mathbf{0}_2 & \mathbf{X}'_3\end{bmatrix} \quad (7)$$

where $\mathbf{X}_k$ and $\mathbf{X}'_k$, (k=1,2,3), are matrices, and $\mathbf{0}_k$, (k=1,2), are zero columns of appropriate dimensions. Therefore, if $h_{ij}=0$, (j=1,…,i-1,i+1,…,N), then $h'_{ij}=0$ and hence the rank of $\mathbf{F}_i$ is also zero. On the other hand, if any h_{ij} is nonzero, the rank of $\mathbf{F}_i$ is 2 and hence, the failed joint can be driven to any desired q_i and $\dot{q}_i$. In other words, to control the angle of the failed joint i, there must be *dynamic coupling* between the link with the failed joint and a link with a working joint, j. Physically, this condition requires that the control input corresponding to some coordinate is able to affect the coordinate corresponding to the failed joint.

In many cases, this condition is satisfied. For example, all rotational DOF of a planar system are coupled. On the other hand, the second and third joints of the manipulator shown in Figure 1 are not coupled to the first one, and in this case $h_{1j} = 0$ (j=2,3). If the first joint of such a manipulator fails while being at rest, no motion of the second or third joints will affect the failed one. Similarly, by inspection of the inertia matrix of a free-flying space robotic system, it can be seen that the translational DOF that correspond to the system Center of Mass, (CM), are not coupled to the rotational DOF which correspond to the spacecraft attitude and to the manipulator joint revolutions [11]. This confirms that if a spacecraft's thrusters do not operate, it is impossible to control the translation of the system CM by using manipulator actuators or spacecraft reaction wheels [5].

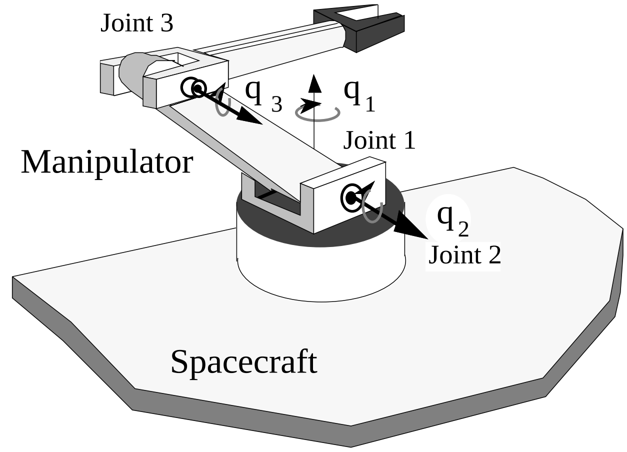

Figure 1. A space manipulator with a dynamically decoupled axis.

III. FAILURE RECOVERY CONTROL

In the previous section it was found that a failed joint may be controlled with a linear error-feedback controller if dynamic coupling between the failed joint and some other joint of a space manipulator exists. The objective of this section is to design such a controller capable of driving the failed joint angle q_i to a desired value $q_{i,des}$ in a stable manner. Here the full nonlinear dynamic equations of motion will be used. It is assumed that dynamic coupling exists and hence, for some joint j, $h_{ij} \neq 0$. Since only N-1 DOF can be controlled, we will control the failed joint angle i at the expense of angle j. To this end, the i^{th} equation of the N equations represented by Equation (1) is written as follows:

$$\sum_{m=1}^{N} h_{im}\ddot{q}_m + \sum_{m=1}^{N}\sum_{k=1}^{N} h_{imk}\dot{q}_m\dot{q}_k = 0 \qquad (8)$$

For $h_{ij} \neq 0$, Equation (8) can be solved for $\ddot{q}_j$ to yield:

$$\ddot{q}_j = -\frac{1}{h_{ij}}\left\{\sum_{m=1\; m\neq j}^{N} h_{im}\ddot{q}_m + \sum_{m=1}^{N}\sum_{k=1}^{N} h_{imk}\dot{q}_m\dot{q}_k\right\} \qquad (9)$$

Equation (9) is used to eliminate $\ddot{q}_j$ in the remaining N-1 equations of Equation (1) to yield N-1 equations of motion in the form:

$$\tilde{\mathbf{H}}(\mathbf{q})\ddot{z} + \tilde{\mathbf{C}}(\mathbf{q}, \dot{\mathbf{q}}) = \tilde{\boldsymbol{\tau}} \qquad (10)$$

where $\tilde{\mathbf{H}}$ is a (N-1)×(N-1) matrix, $\tilde{\boldsymbol{\tau}} \in \mathbf{R}^{N-1}$ contains all nonzero control torques, and $\tilde{\mathbf{C}}$ contains nonlinear terms. Finally $\mathbf{z} \in \mathbf{R}^{N-1}$ is the vector of all joints angles including the failed joint angle q_i, but excluding joint angle q_j:

$$\mathbf{z} = [q_1, \ldots, q_{j-1}, q_{j+1}, \ldots, q_N]^T \qquad (11)$$

Various error-feedback controllers can now be designed based on Proportional-Derivative control, computed torque, or their variants. One such a law is the following:

$$\tilde{\boldsymbol{\tau}} = \tilde{\mathbf{C}}(\mathbf{q}, \dot{\mathbf{q}}) + \tilde{\mathbf{H}}(\mathbf{q})\{\mathbf{K}_p(\mathbf{z}_{des} - \mathbf{z}) - \mathbf{K}_d\, \dot{z}\} \qquad (12)$$

where $\mathbf{K}_p$ and $\mathbf{K}_d$ are (N-1)×(N-1) positive definite diagonal matrices and where it is assumed implicitly that all the joint sensors, including those of the failed joint, still provide feedback. By applying control law (12) to the reduced equations (10), a stable linear decoupled system is obtained. For this system, $\mathbf{z}$ converges asymptotically to $\mathbf{z}_{des}$, and $\dot{z}$ to zero, if $\dot{q}_j$ converges to zero. Recall that q_i is an element of $\mathbf{z}$; therefore, if $\mathbf{z}$ converges to $\mathbf{z}_{des}$ the task of controlling the failed joint i is achieved. However, if the velocity $\dot{q}_j$ does not converge to zero, then $\tilde{\mathbf{C}}$ may be unbounded and the j^{th} DOF may be unstable. Therefore, the behavior of $\dot{q}_j$ must be studied.

For $\mathbf{z} \to \mathbf{z}_{des}$, $\dot{z} \to 0$, Equation (8) becomes:

$$h_{ij}\ddot{q}_j + h_{ijj}(\dot{q}_j)^2 \cong 0 \qquad (13)$$

where h_{ij}, h_{ijj}, are functions of q_j only. Equation (13) is in general *unstable*, unless $h_{ijj}\dot{q}_j/h_{ij}$ is always positive. This condition cannot be guaranteed, and hence $\dot{q}_j$ may not converge to zero. In other words, although the control law given by Equation (12) may drive q_i to a desired set point, it may also destabilize the j^{th} DOF. If a condition could be found to guarantee that $\dot{q}_j$ would converge to zero, then the system would remain stable. This issue is addressed next.

In the absence of gravity, a rigid body has zero potential energy, and hence, a system's Lagrangian is equal to its kinetic energy, T. In such a case, Equation (8) also can be written as:

$$\frac{d}{dt}\left\{\frac{\partial T}{\partial \dot{q}_i}\right\} - \frac{\partial T}{\partial q_i} = 0 \qquad (14)$$

Equation (14) is integrated and assuming that the system is initially at rest, the result is:

$$\sum_{m=1}^{N} h_{im}\dot{q}_m(t) = \int_0^t \frac{\partial T}{\partial q_i}\, dt = \int_0^t \frac{1}{2}\frac{\partial\{\dot{\mathbf{q}}^T\mathbf{H}(\mathbf{q})\dot{\mathbf{q}}\}}{\partial q_i}\, dt \qquad (15)$$

Taking the limit of Equation (15) as $t \to \infty$ and setting $\mathbf{z}(\infty) = \mathbf{z}_{des}$, and $\dot{z}(\infty) = 0$, the following expression for the asymptotic behavior of $\dot{q}_j$ results:

$$\lim_{t\to\infty}(\dot{q}_j) = \frac{1}{h_{ij}}\int_0^\infty \frac{1}{2}\dot{\mathbf{q}}^T \frac{\partial \mathbf{H}(\mathbf{q})}{\partial q_i}\dot{\mathbf{q}}\, dt \qquad (16)$$

Expression (16) shows that $\dot{q}_j$ will reach zero if the integrand is identically zero. This is equivalent to requiring that q_i be *ignorable*, that is that the inertia matrix $\mathbf{H}$ is invariant with respect to this coordinate, [12]. Note that when q_i is ignorable, Equation (14) becomes an *integral of motion*, and the system order is N-1. In such a case $\dot{q}_j$ will converge to zero, the failure recovery controller given by Equation (12) will drive the failed joint at its desired angle, and the system response will be stable.

It remains to examine when the coordinate q_i of the failed joint can be ignorable. In some cases, this is a feature of the system itself. For example, the elements of the inertia matrix of a two DOF free-flying manipulator system with a controlled attitude spacecraft, shown in Figure 2, are given by Equations (A1), and are only a function of the second joint angle, q_2. This means that in the range where h_{12} is nonzero, one can control the first joint by using the second joint. In other cases, the invariance of the inertia matrix can be obtained by design, as was done in Reference [13] with the aim of designing a controller with configuration-independent dynamic behavior.

To conclude this section, we note that a failure recovery controller can be implemented if two requirements hold: (a) the existence of dynamic coupling between the failed joint and some other operating one, and (b) the invariance of the inertia matrix with respect to the failed joint. Recall that two minor requirements were also assumed: (a) there is no friction at the joints and (b) the system is initially at rest. In addition, in practice one more condition must be observed: (c) limits on joint motions do not restrict the motion required to position the failed joint. In the next section, the notion of failure recovery control is demonstrated using two examples.

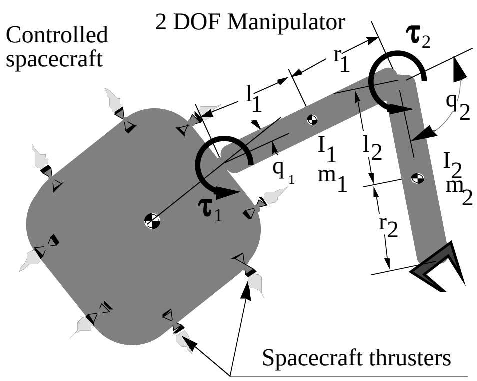

Figure 2. A space manipulator system with its attitude control system.

IV. EXAMPLES

In the first example, consider a two DOF manipulator mounted on a spacecraft whose position and attitude is fixed in inertial space by using jet actuators, see Figure 2. Note that any planar space system can be reduced to this case by fixing all its coordinates except two, which then become q_1 and q_2. Next, assume that the first joint has failed, that is $\tau_1 = 0$. The objective is to control the failed joint angle q_1. The equations of motion for this system are:

$$h_{11}\ddot{q}_1 + h_{11}\ddot{q}_2 - 2h\dot{q}_2^2 - 2h\dot{q}_1\dot{q}_2 = 0 \quad (17a)$$

$$h_{21}\ddot{q}_1 + h_{22}\ddot{q}_2 + h\dot{q}_1^2 = \tau_2 = \tau \quad (17b)$$

where h_{ij} and h are given in Appendix A. Obviously this system is dynamically coupled since h_{12} is nonzero except at two angles q_2. Hence, failure control may be possible in a region where q_2 does not approach these values. Equation (17a) can be solved for $\ddot{q}_2$ and then substituted in Equation (17b) to yield the only equation of motion for this system:

$$\tilde{H}\ddot{q}_1 + \tilde{C}(q_2, \dot{q}_1, \dot{q}_2) = \tau \quad (18)$$

where $\tilde{C}$ contains nonlinear terms and $\tilde{H}$ is a 1×1 matrix equal to:

$$\tilde{H} = -(h_{11}h_{22}-h_{12}^2)/h_{12} \quad (19)$$

Note that $(h_{11}h_{22}-h_{12}^2)$ is always positive, since it is equal to the determinant of the full system inertia matrix **H**, and therefore the sign of $\tilde{H}$ is opposite to the sign of h_{12}; $\tilde{H}$ is not positive definite. However, if h_{12} is nonzero, the control law given by Equation (12) can be applied:

$$\tau = \tilde{C}(q_2, \dot{q}_1, \dot{q}_2) - \tilde{H}\{K_p(q_{1,des}-q_1)-K_d\dot{q}_1\} \quad (20)$$

As explained in the previous section, to obtain a stable response, the failed joint angle q_1 must be ignorable. Since h_{ij} (i,j=1,2), given by Equations (A1), are not functions of q_1, this condition is satisfied. Then Equation (16a) can be written as:

$$\frac{d}{dt}\{h_{11}\dot{q}_1 + h_{12}\dot{q}_2\} = 0 \quad (21)$$

which can be integrated to yield:

$$h_{11}\dot{q}_1 + h_{12}\dot{q}_2 = 0 \quad (22)$$

where it was assumed that the system was initially at rest. Then, the control law given by Equation (20) guarantees that q_1 and $\dot{q}_1$ converge asymptotically to $q_{1,des}$ and to 0, respectively. Furthermore, since $\dot{q}_1$ converges to zero, it follows from Equation (22) that $\dot{q}_2$ also converges to zero, and that the system is stable.

In the second example, consider a free-floating space manipulator system, where the spacecraft thrusters are turned off. System parameters are given in Table I. Its inertia matrix is given in Appendix A and is a function of both manipulator joint angles q_1 and q_2. Assume that the second joint has failed, i.e. $\tau_2 = 0$, see Figure 3.

Table I. System parameters for the example.

Body	l_i (m)	r_i (m)	m_i (Kg)	I_i (Kg m^2)
0	.5	.5	40	6.667
1	.5	.5	4	0.333
2	.5	.5	3	0.250

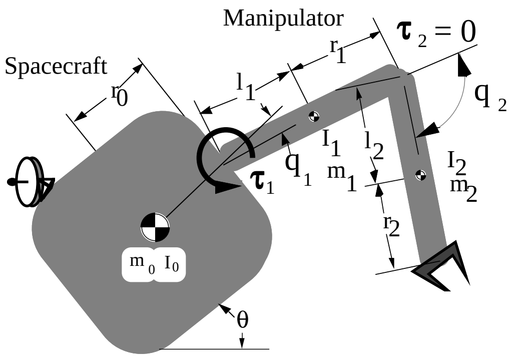

Figure 3. A free-floating space manipulator system with a failed actuator.

A simple Proportional-Derivative error-feedback controller is used in this example to control the failed second joint:

$$\tau_1 = K_p(q_{2,des}-q_2) - K_d \dot{q}_2 \qquad (23)$$

where K_p = 50 Nm/rad, and K_d = 45 Nmsec/rad, and q_2 is in radians. In order to maximize the failed system's reach, the desired position for the failed joint angle is set to be $q_{2,des} = 0^\circ$. The initial conditions are $(\theta,q_1,q_2) = (39.6^\circ,-134.2^\circ,134.4^\circ)$.

Figure 4 shows the time history of the joint angles and the spacecraft attitude. Since the failed joint q_2 is not ignorable, the angle q_1 drifts as predicted, although the failed joint approaches its set-point of 0°. Figure 5 shows the motion of the system in inertial space. The system is initially at rest and the end-effector is at point A. When control action starts, the end-effector follows the path shown in Figure 5 and continues to drift because the first joint is destabilized.

Close examination of the entries of the inertia matrix given by Equations (A2)-(A4), reveals the fact that if the center of mass of the second link lies on the second joint axis, or in other words if l_2 is zero, then all d_{2i} (i=0,1,2) are zero and q_2 is ignorable. Note that this can be achieved by changing the mass distribution of the second link, for example by using counterweights.

To illustrate this, the same motion as above is simulated, but this time l_2=0. As shown in Figure 6, q_2 converges to the desired set-point and the motion is stable, as predicted. Figure 7 depicts the same motion in inertial space. Note that forces transmitted through the second joint bearings can create no torques about the second link CM, as its location is on the joint axis. Therefore, the second link can only translate and that explains why its inertial orientation remains constant. The controller produces a torque that rotates the first link till q_2 becomes equal to the set-point, which in the case of Figure 6, is zero.

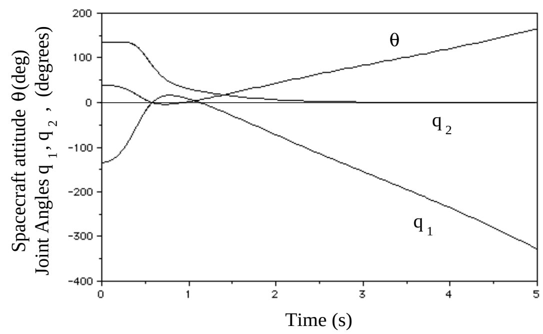

Figure 4. Unstable spacecraft attitude and joint angles motions.

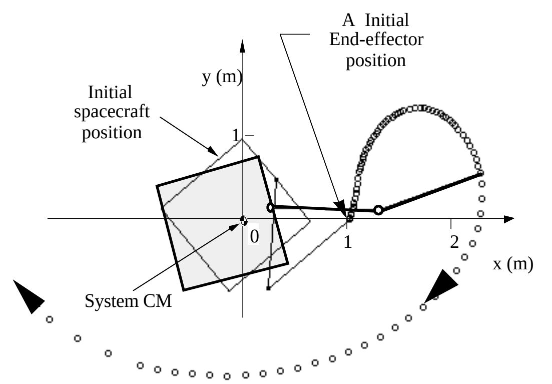

Figure 5. Unstable system motion in inertial space.

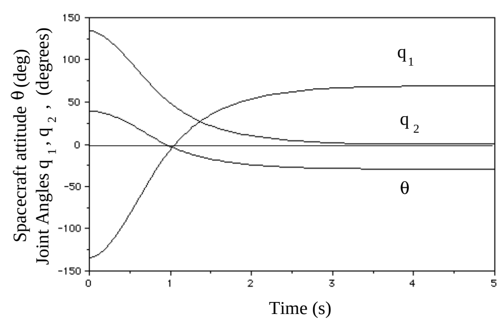

Figure 6. Stable spacecraft attitude and joint angles motions.

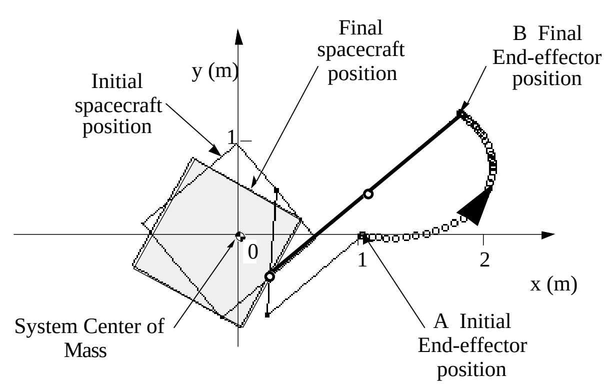

Figure 7. Stable system motion in inertial space.

V. CONCLUSIONS

This paper shows that under some conditions, it is possible to use the working joints of a space system to control a joint which has failed. This is a problem that belongs to a more general class of problems, which is the control of systems with fewer actuators than DOF. Results showed that in order to be able to design a failure recovery controller, dynamic coupling and invariance of the inertia matrix with respect to the failed joint angle must exist. The failure recovery technique developed was illustrated for two example systems and the importance of the derived conditions was demonstrated.

VI. ACKNOWLEDGMENTS

The support of this work by NASA (Langley Research Center, Automation Branch) under Grant NAG-1-801 is acknowledged.

REFERENCES

[1] Dubowsky, S., Vance, E., and Torres, M., "The Control of Space Manipulators Subject to Spacecraft Attitude Control Saturation Limits," *Proc. of the NASA Conf. on Space Telerobotics*, JPL, Pasadena, CA, Jan. 31-Feb. 2, 1989.

[2] Alexander, H., and Cannon, R., "Experiments on the Control of a Satellite Manipulator," *Proc. of the Amer. Control Conf.*, Seattle, WA, June 1987.

[3] Masutani, Y., Miyazaki, F., and Arimoto, S., "Sensory Feedback Control for Space Manipulators," *Proc. of the IEEE Int. Conf. on Robotics and Automation*, Scottsdale, AZ, May 1989.

[4] Umetani, Y., and Yoshida, K., "Experimental Study on Two Dimensional Free-Flying Robot Satellite Model," *Proc. of the NASA Conf. on Space Telerobotics*, Pasadena, CA, January 1989.

[5] Vafa, Z., and Dubowsky, S., "The Kinematics and Dynamics of Space Manipulators: The Virtual Manipulator Approach," *International Journal of Robotics Research*, Vol. 9, No. 4, August 1990.

[6] Papadopoulos, E., and Dubowsky, S., "On the Nature of Control Algorithms for Space Manipulators," *Proc. of the IEEE Int. Conf. on Robotics and Automation*, Cincinnati, OH, May 1990.

[7] Papadopoulos, E., and Dubowsky, S., "On the Dynamic Singularities in the Control of Free-Floating Space Manipulators," *Proc. of the ASME WAM*, San Fransisco, CA, December 1989.

[8] Spofford, J., and Akin, D., "Redundancy Control of a Free-Flying Telerobot," *Proc. of the AIAA Guidance, Navigation and Control Conf.*, Minneapolis, MN, August 1988.

[9] Arai, H., and Tachi, S., "Position Control of a Manipulator with Passive Joints Using Dynamic Coupling," *Proc. of the 20th Int. Symposium on Industrial Robots*, Tokyo, Japan, October 1989.

[10] Draper, J. V., Handel S., and Hood C.C., "The Impact of Partial Joint Failure on Teleoperator Task Performance," *Proc. of the Fourth ANS Topical Meeting on Robotics and Remote Systems*, Albuquerque, NM, February 1991.

[11] Papadopoulos, E., "On the Dynamics and Control of Space Manipulators," Ph.D. Thesis, Department of Mechanical Engineering, MIT, October 1990.

[12] Goldstein, H., *Classical Mechanics*, Addison-Wesley, Reading, MA, 1980.

[13] Asada, H., and Youcef-Toumi, K., *Direct-Drive Robots: Theory and Practice*, MIT Press, Cambridge, MA, 1987.

APPENDIX A.

The elements of the inertia matrix of a space manipulator on an inertially fixed spacecraft, shown in Figure 2, are:

$$h_{11}=I_1+m_1l_1^2+m_2(l_1+r_1)^2+2m_2l_2(l_1+r_1)\cos(q_2)+I_2+m_2l_2^2$$

$$h_{12} = h_{21} = m_2l_2(l_1+r_1)\cos(q_2)+I_2+m_2l_2^2$$

$$h_{22} = I_2+m_2l_2^2$$

$$h = m_2l_2(l_1+r_1)\sin(q_2) \tag{A1}$$

where l_i is the distance of the CM of the i^{th} link from the left revolute joint and r_i the distance from the right joint, m_i and I_i are the mass and inertia of the i^{th} link.

The inertia matrix of a free-floating manipulator, shown in Figure 3, is derived in reference [6] and is used here:

$$\mathbf{H(q)}=\begin{bmatrix} d_{11}+2d_{12}+d_{22}-\dfrac{(D_1+D_2)^2}{D} & d_{12}+d_{22}-\dfrac{D_2(D_1+D_2)}{D} \\ d_{12}+d_{22}-\dfrac{D_2(D_1+D_2)}{D} & d_{22}-\dfrac{D_2^2}{D} \end{bmatrix} \tag{A2}$$

where:

$$D_j = d_{0j} + d_{1j} + d_{2j} \qquad j = 1, 2 \tag{A3a}$$

$$D = D_0 + D_1 + D_2 \tag{A3b}$$

$$d_{00} = I_0 + \frac{m_0(m_1+m_2)}{M} r_0^2$$

$$d_{10} = \frac{m_0r_0}{M}\left\{l_1(m_1+m_2) + r_1m_2\right\}\cos(q_1) = d_{01}$$

$$d_{20} = \frac{m_0m_2}{M} r_0l_2\cos(q_1+q_2) = d_{02}$$

$$d_{11} = I_1 + \frac{m_0m_1}{M} l_1^2 + \frac{m_1m_2}{M} r_1^2 + \frac{m_0m_2}{M}(l_1+r_1)^2$$

$$d_{21} = \left\{\frac{m_1m_2}{M} r_1l_2 + \frac{m_0m_2}{M} l_2(l_1+r_1)\right\}\cos(q_2) = d_{12}$$

$$d_{22} = I_2 + \frac{m_2(m_0+m_1)}{M} l_2^2 \tag{A4}$$

where M is the total system mass and all other symbols are defined in Figure 3.